

# NORMAL MODE DECOMPOSITION OF SMALL-SCALE OCEANIC MOTIONS

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# Abstract

Small-scale oceanic motions are expected to contain both gravity waves and vortical motion. The vortical motion carries the perturbation potential vorticity of the system. Using eigenvectors of the linear equations of motion, the gravity mode and the vortical mode are defined. The vortical mode carries the linear perturbation potential vorticity and is horizontally nondivergent, whereas the gravity mode does not carry the linear perturbation potential vorticity. In an unforced, inviscid, linearized system, the gravity mode reduces to free linear gravity waves and the vortical mode to a steady geostrophic flow.

An attempt to separate oceanic measurements into the gravity and vortical modes can be conveniently made using fields of horizontal divergence  $HD$ , vortex stretching  $VS$ , and relative vorticity  $RV$ . Spectra of  $HD$ ,  $VS$ , and  $RV$  are estimated using measurements of horizontal velocity and temperature from IWEX. Frequency spectral estimates of area-averaged horizontal divergence  $\overline{HD}$  and relative vorticity  $\overline{RV}$  represent the result of both attenuation and contamination horizontal wavenumber array response functions. The attenuation array response function describes the unresolvable nature of small-scale fluctuations of  $HD$  and  $RV$ , and the contamination array response function describes the contamination between  $HD$  and  $RV$ . These two potential problems inhibit the estimation of fluctuations of  $HD$  and  $RV$  separately.

Observed frequency spectra of  $\overline{HD}$  are well represented by the GM-76 model (Cairns and Williams, 1976), whereas significant disagreements are found between spectral estimates of  $\overline{RV}$  and the GM model at small horizontal scales. Frequency spectra of  $HD$  and  $RV$  of the GM model are very sensitive to the high wavenumber cutoff. Since the cutoff is not well determined to date, observed discrepancies do not conclusively imply the failure of linear internal wave theory.

strikingly well with the GM-76 spectrum model. This agreement suggests that fluctuations at vertical scales greater than 68 m (the smallest resolvable scale) are mainly the gravity mode component. Vertical wavenumber spectra of  $\widehat{VS}$  and  $\widehat{TR}$  also agree with previous observations by Gregg (1977) and by Gargett et al. (1981).

A general scheme to separate the relative vorticity and horizontal kinetic energy spectra into gravity and vortical modes is proposed. This requires horizontal wavenumber-frequency spectra of uncontaminated  $HD$  and  $RV$  which can be obtained by measuring horizontal velocity components along a closed contour, or with a horizontal space lag smaller than the scale of  $HD$  and  $RV$ .