Homework #7

Complete the Matlab scripts below for the mode 1, mode 2 and mode 3 displacement discontintuities, and run the scripts to get the displacement and stress fields . The lines to complete are marked by question marks, for example: xn = ????????;
Start with the mode 3 displacement discontintuity first – it is the easiest one, having only 9 lines to complete. The mode 1 and mode 2 solutions each require completing 19 lines. For each plot, also answer the following questions, intended to help you evaluate whether the solution makes sense and to build your intuition. Don't give one-word answers where a sentence or two is more appropriate.

Mode 3: 3 plots **2pts/plot = 6pts total** **16 pts possible**

Describe the displacements. Are they in the right direction? **(2 pts)**

Are the symmetries correct for the stresses and the displacements? **(1 pt/plot)**

Which, if any, stress components are discontinuous across the displacement discontinuity? **(1 pt/plot)**

Describe the stress component at the right end of the displacement discontinuity. **(1 pt/plot)**

Does anything look funny, and if so, why? **(1 pt)**

Mode 1: 4 plots **4pts/ plot = 16pts total** **28 pts possible**

Describe the displacements. Are they in the right direction? **(2 pts)**

Are the symmetries correct for the stresses and the displacements? **(1 pt/plot)**

Which, if any, stress components are discontinuous across the disp. discontinuity?
(1 pt/plot)

Describe the stress component at the right end of the displacement discontinuity. **(1 pt/plot)**

Does anything look funny, and if so, why? **(1 pt)**

Mode 2: 4 plots **4pts/ plot = 16pts total** **28 pts possible**

Describe the displacements. Are they in the right direction? **(2 pts)**

Are the symmetries correct for the stresses and the displacements? **(1 pt/plot)**

Which, if any, stress components are discontinuous across the displacement discontinuity? **(1 pt/plot)**

Describe the stress component at the right end of the displacement discontinuity. **(1 pt/plot)**

Does anything look funny, and if so, why? **(1 pt)**

```
function [ux,uy,sxx,sxy,syy] = GG711_mode1dd(G,k,PR,x,y)
% Returns Cartesian displacements and stresses at grid points
% for a mode 1 displacement discontinuity
% with a unit Burger's vector (b=1) and unit half length (a=1).
% The displacement discontintuity extends along the x-axis
% from x = -a to x = +a.
% The solution is based on equations of Tables 8.1 and 9.1 of Barber (1992)
% for the following POLAR stress function:
% phi1 = r*logr*cost (mode  I dislocation)
% ux = displacement in the z-direction
% uy = displacement in the y-direction
% sxx = sigma xx
% sxy = sigma xy
% syy = sigma yy
% Parameters G, k, and PR are elastic parameters (see Barber, 1992)
% x,y = coordinates of observation gridpoints,
% IMPORTANT: In Barber's solutions, a dislocation (d) cut extends
% from the origin to the right along the x-axis (not the left).  So to form
% a displacement discontintuity (dd) from x = -a to x = +a, dd = dn - dp,
where
% dn is the dislocation that extends right from the negative end of "dd",
% and dp is the dislocation that extends right from the positive end of
"dd".
%  Example:
%  [x,y] = meshgrid(-1.95:0.1:1.95);
%  [ux,uy,sxx,sxy,syy] = GG711_mode1dd(1,2,0.25,x,y);
% Last revised on 3/04/03%

b = 1;                  % Unit strength displacement discontinuity
a = 1;                  % Unit half-length displacement discontinuity

% Find geometric terms from positive (p) and negative (n) element ends to
obs. pts.
% Positive end
xp    = x-a;
rp    = sqrt(xp.^2 + y.^2);
costp = xp./rp;
sintp = y./rp;
tp    = atan2(y,xp);            % tp = theta at positive end
logrp = log(rp);
axrp  = xp./rp;
ayrp  = y./rp;
axtp  = -ayrp;
aytp  = axrp;
% Negative end
xn    = ????????;
rn    = ????????;
costn = ????????;
```

```
sintn = ????????;
tn    = ????????;        % tp = theta at negative end
logrn = ????????;
axrn  = ????????;
ayrn  = ????????;
axtn  = ????????;
aytn  = ????????;

%  Calculate the lead coefficient C3 (Barber, 1992, p. 169) for By =1
C3 = ????????;

%  Calculate polar DISPLACEMENT components in an element-end-based rt
%  reference frame due to unit NORMAL disp. discontinuity
%  for dislocations extending right from xb = -a and xb = +a.
urp = C3*0.5*(  (k+1).*tp.*sintp - costp + (k-1).*logrp.*costp);
urn   = ????????;
utp   = ????????;
utn   = ????????;
%  Convert displacements from local rt coordinates to local xy coordinates
ux    = (axrn.*urn + axtn.*utn) - (axrp.*urp + axtp.*utp);
uy    = (ayrn.*urn + aytn.*utn) - (ayrp.*urp + aytp.*utp);

%  Calculate polar rt STRESS components due to unit NORMAL displacement
discontinuity
%   Positive end                                  Negative end
      srrp  = (+C3/(2*G))*costp./rp;              srrn  = ????????;
      srtp  = ????????;                           srtn  = ????????;
      strp  = srtp;                               strn  = ????????;
      sttp  = srrp;                               sttn  = ????????;
%  Convert stresses from local rt coordinates to local xy coordinates
% Normal discontintuity, Positive end
[sxxp,sxyp,syxp,syyp] =
stress_trans(srrp,srtp,strp,sttp,axrp,axtp,ayrp,aytp);
% Normal discontintuity, Negative end
[sxxn,sxyn,syxn,syyn] =
stress_trans(srrn,srtn,strn,sttn,axrn,axtn,ayrn,aytn);
% Normal discontintuity, Both ends (note minus sign)
% Superpose dislocations extending right from x = -a and x = +a.
sxx = ????????;    sxy = ????????;    syy = ????????;

% Plots
figure(1);  quiver(x,y,ux,uy);
    xlabel('x');  ylabel('y');      axis('equal');    title ('Displacement');
figure(2);  v1 = -0.6:0.2:0.6;  c1 = contour(x,y,sxx,v1); clabel(c1);
    xlabel('x');    ylabel('y');    title('sxx')
figure(3);  v2 = -0.3:0.1:0.3;  c2 = contour(x,y,sxy,v2); clabel(c2);
    xlabel('x');    ylabel('y');    title('sxy')
figure(4);  v3 = -0.3:0.1:0.3;  c3 = contour(x,y,syy,v3); clabel(c3);
```

```
xlabel('x');    ylabel('y');    title('syy')
```

```
function [ux,uy,sxx,sxy,syy] = GG711_mode2dd(G,k,PR,x,y)
% Returns Cartesian displacements and stresses at grid points
% for a mode 2 displacement discontinuity
% with a unit Burger's vector (b=1) and unit half length (a=1).
% The displacement discontintuity extends along the x-axis
% from x = -a to x = +a.
% The solution is based on equations of Tables 8.1 and 9.1 of Barber (1992)
% for the following POLAR stress function:
% phi2 = r*logr*sint (mode II dislocation)
% ux = displacement in the z-direction
% uy = displacement in the y-direction
% sxx = sigma xx
% sxy = sigma xy
% syy = sigma yy
% Parameters G, k, and PR are elastic parameters (see Barber, 1992)
% x,y = coordinates of observation gridpoints,
% IMPORTANT: In Barber's solutions, a dislocation (d) cut extends
% from the origin to the right along the x-axis (not the left). So to form
% a displacement discontintuity (dd) from x = -a to x = +a, dd = dn - dp,
where
% dn is the dislocation that extends right from the negative end of "dd",
% and dp is the dislocation that extends right from the positive end of
"dd".
%  Example:
%  [x,y] = meshgrid(-1.95:0.1:1.95);
%  [ux,uy,sxx,sxy,syy] = GG711_mode2dd(1,2,0.25,x,y);
% Last revised on 3/04/03%

b = 1;          % Unit strength displacement discontinuity
a = 1;          % Unit half-length displacement discontinuity

% Find geometric terms from positive (p) and negative (n) element ends to
obs. pts.
% Positive end
xp    = x-a;
rp    = sqrt(xp.^2 + y.^2);
costp = xp./rp;
sintp = y./rp;
tp    = atan2(y,xp);                % tp = theta at positive end
logrp = log(rp);
axrp  = xp./rp;
ayrp  = y./rp;
axtp  = -ayrp;
aytp  = axrp;
% Negative end
xn    = ????????;
rn    = ????????;
costn = ????????;
```

```
sintn = ????????;
tn    = ????????;              % tp = theta at negative end
logrn = ????????;
axrn  = ????????;
ayrn  = ????????;
axtn  = ????????;
aytn  = ????????;

% Calculate the lead coefficient C3 (Barber, 1992, p. 169) for Bx =1
C4 = ????????;

% Calculate polar DISPLACEMENT components in an element-end-based rt
% reference frame due to unit SHEAR disp. discontinuity
% for dislocations extending right from xb = -a and xb = +a.
urp   = C4*0.5*( -(k+1).*tp.*costp - sintp + (k-1).*logrp.*sintp );
urn   = C4*0.5*( -(k+1).*tn.*costn - sintn + (k-1).*logrn.*sintn );
utp   = C4*0.5*(  (k+1).*tp.*sintp + costp + (k-1).*logrp.*costp );
utn   = C4*0.5*(  (k+1).*tn.*sintn + costn + (k-1).*logrn.*costn );
% Convert displacements from local rt coordinates to local xy coordinates
ux    = (axrn.*urn + axtn.*utn) - (axrp.*urp + axtp.*utp);
uy    = (ayrn.*urn + aytn.*utn) - (ayrp.*urp + aytp.*utp);

% Calculate polar rt STRESS components due to unit SHEAR displacement
discontinuity
%  Positive end                                   Negative end
      srrp  = (+C4/(2*G))*sintp./rp;              srrn  = ????????;
      srtp  = ????????;                           srtn  = ????????;
      strp  = srtp;                               strn  = ????????;
      sttp  = srrp;                               sttn  = ????????;
% Normal discontintuity, Positive end
[sxxp,sxyp,syxp,syyp] =
stress_trans(srrp,srtp,strp,sttp,axrp,axtp,ayrp,aytp);
% Normal discontintuity, Negative end
[sxxn,sxyn,syxn,syyn] =
stress_trans(srrn,srtn,strn,sttn,axrn,axtn,ayrn,aytn);
% Normal discontintuity, Both ends (note minus sign)
% Superpose dislocations extending right from x = -a and x = +a.
sxx = ????????;    sxy = ????????;    syy = ????????;

% Plots
figure(1);  quiver(x,y,ux,uy);
    xlabel('x');  ylabel('y');      axis('equal');    title ('Displacement');
figure(2);  v1 = -0.6:0.2:0.6;  c1 = contour(x,y,sxx,v1); clabel(c1);
    xlabel('x');    ylabel('y');    title('sxx')
figure(3);  v2 = -0.3:0.1:0.3;  c2 = contour(x,y,sxy,v2); clabel(c2);
    xlabel('x');    ylabel('y');    title('sxy')
figure(4);  v3 = -0.3:0.1:0.3;  c3 = contour(x,y,syy,v3); clabel(c3);
    xlabel('x');    ylabel('y');    title('syy')
```

```
function [uz,sxz,syz] = GG711_mode3dd(G,x,y)
%  Returns Cartesian displacements and stresses at grid points
%  for an anti-plane strain (mode 3) displacement discontinuity
%  with a unit Burger's vector (b=1) and unit half length (a=1).
%  The displacement discontintuity extends along the x-axis
%  from x = -a to x = +a.
%  The solution is based on equations on pages 5-9 and 5-10
%  of the notes from GG711c.
%  uz = displacement in the z-direction
%  sxz = sigma xz
%  syz = sigma yz
%  G = shear modulus;
%  x,y = coordinates of observation gridpoints,
%   Example:
%   [x,y] = meshgrid(-1.95:0.1:1.95,-2:0.1:2);
%   [uz,sxz,syz] = GG711_mode3dd(1,x,y);
%  Last revised on 3/04/03
b = 1;                  % Unit strength displacement discontinuity
a = 1;                  % Unit half-length displacement discontinuity
b_2pi = b/(2*pi);             % Algebraic coefficient
Gb_2pi = G*b/(2*pi);          % Algebraic coefficient
% Contribution from "positive" end
xp    = x-a;
yp    = y;
rp2   = xp.*xp + yp.*yp;
uzp   = b_2pi*atan2(yp,xp);
sxzp  = -Gb_2pi.*(yp./rp2);
syzp  = Gb_2pi.*(xp./rp2);
% Contribution from "negative" end
xn    = ????????;
yn    = ????????;
rn2   = ????????;
uzn   = ????????;
sxzn  = ????????;
syzn  = ????????;
% Total contribution
uz    = ????????;
sxz   = ????????;
syz   = ????????;
% Plots
figure(1);  v1 = -0.8:0.2:0.8;  c1 = contour(x,y,uz,v1); clabel(c1);
    xlabel('x');    ylabel('y');    title('uz')
figure(2);  v2 = -0.4:0.1:0.4;  c2 = contour(x,y,sxz,v2); clabel(c2);
    xlabel('x');    ylabel('y');    title('sxz')
figure(3);  v3 = -0.4:0.1:0.4;  c3 = contour(x,y,syz,v3); clabel(c3);
    xlabel('x');    ylabel('y');    title('syz')
```