

Homework #6

The purpose of this homework is to give you some experience determining stresses and displacements from stress functions and to start to retrain your intuition so that you can explain some mechanical results that at first glance might seem incorrect.

Problem 1 Displacements for the stress function $\phi = x^2$ 20 pts

- A From this stress function find the stresses and, from them, the strains assuming plane stress. Then by integrating the normal strains find the displacements – including functions of x and y that arise from the integrations – and then use the shear strains to solve for these functions (see attached example). Disregard any terms that just yield a rigid body translation or rigid body rotation. If symmetry conditions require that certain terms be equal to each other or equal to zero, note that. **10 pts**
- B Check your work by taking the derivatives of the displacements to make sure you recover the strains. **3 pts**
- C Complete the attached Matlab script to plot the displacements. **2 pts**
- D Explain in words why the pattern looks the way it does. **5 pts**

Problem 2 Displacements for the stress function $\phi = xy$ 20 pts

- A From this stress function find the stresses and, from them, the strains assuming plane stress. Then by integrating the normal strains find the displacements – including functions of x and y that arise from the integrations – and then use the shear strains to solve for these functions (see attached example). Disregard any terms that just yield a rigid body translation or rigid body rotation. If symmetry conditions require that certain terms be equal to each other or equal to zero, note that. **10 pts**
- B Check your work by taking the derivatives of the displacements to make sure you recover the strains. **3 pts**
- C Complete the attached Matlab script to plot the displacements. **2 pts**
- D Explain in words why the pattern looks the way it does. **5 pts**

Problem 3 Stresses around a pressurized hole 16 pts

Consider a circular hole of radius “ a ” in an infinitely large plate, with the walls of the hole experiencing a constant normal traction S acting on them. No shear traction acts on the walls of the hole, and no tractions acts on the “edges” of the plate at an infinite distance from the hole. In other words, the boundary conditions for the problem are:

$$\text{At } r = a: \sigma_{rr} = S \quad \sigma_{r\theta} = 0$$

$$\text{At } r = \infty: \sigma_{rr} = 0 \quad \sigma_{r\theta} = 0$$

These stresses depend only on r – there is no dependence of the stresses on θ .

- A Draw a neatly labeled picture showing the geometry of the problem and the boundary conditions. **2 pts**
- B After inspecting Table 8.1 in Barber, write down the four terms in the general polar solution of the stress function (eq. 8.54 of Barber) that form a possible solution of this problem. **2 pts**
- C Based on the terms of the stress function in B, use Table 8.1 in Barber to write down the possible solution for the stresses σ_r , $\sigma_{r\theta}$, $\sigma_{\theta\theta}$. **3 pts**
- D By applying the boundary conditions, solve for the four coefficients in B. Show your work. You can use section 12.2 in Barber as a guide. **6 pts**
- E Write down the solution for the stresses σ_r , $\sigma_{r\theta}$, $\sigma_{\theta\theta}$. **3 pts**

Displacements for the stress function $\phi = x^2y$

$$\phi = x^2y \quad (1)$$

$$\sigma_{xx} = 0 \quad (2)$$

$$\sigma_{xy} = -2x \quad (3)$$

$$\sigma_{yy} = 2y \quad (4)$$

$$e_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu\sigma_{yy}}{E} = -\frac{\nu}{E}2y = \frac{\partial u_x}{\partial x} \quad (5)$$

$$e_{yy} = \frac{\sigma_{yy}}{E} - \frac{\nu\sigma_{xx}}{E} = \frac{2y(1-\nu)}{E} = \frac{\partial u_y}{\partial y} \quad (6)$$

$$e_{xy} = \sigma_{xy} \frac{(1+\nu)}{E} = -\frac{2x(1+\nu)}{E} = \frac{1}{2} \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) \quad (7)$$

$$u_x = \int \frac{\partial u_x}{\partial x} dx = \int e_{xx} dx = \frac{-\nu}{E} 2xy + f(y) \quad (8)$$

$$u_y = \int \frac{\partial u_y}{\partial y} dy = \int e_{yy} dy = \frac{1-\nu}{E} y^2 + g(x) \quad (9)$$

Inserting (8) and (9) into (7)

$$e_{xy} = -\frac{2x(1+\nu)}{E} = \frac{1}{2} \left(\frac{-\nu}{E} 2x + f'(y) + g'(x) \right) \quad (10)$$

This can be simplified:

$$-4x(1+\nu) = (-\nu 2x + Ef'(y) + Eg'(x)) \quad (11)$$

Now the left side of (11) has no y term, so $f'(y)$ must be zero, so $f(y)$ must be a constant (A). So by (8), $f(y)$ corresponds to a constant rigid body translation in the x-direction, and it can be set to zero.

$$-4x(1+\nu) = -\nu 2x + Eg'(x) \quad (12)$$

Solving for $g'(x)$

$$g'(x) = \frac{-4x(1+\nu) + 2x\nu}{E} = \frac{-2x}{E}(\nu+2) \quad (13)$$

$$g(x) = \frac{-x^2}{E}(\nu+2) + B = \frac{-x^2}{E}(\nu+2) \quad (14)$$

B corresponds to a rigid displacement, so that is why it is set to zero.

$$u_x = \frac{-\nu}{E} 2xy \quad (15)$$

$$u_y = \frac{1-\nu}{E} y^2 + \frac{-x^2}{E}(\nu+2) \quad (16)$$

Check

$$\frac{\partial u_x}{\partial x} = \frac{-\nu}{E} 2y = e_{xx} \quad \frac{\partial u_y}{\partial y} = \frac{2y}{E}(1-\nu) = e_{yy}$$

$$\frac{1}{2} \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) = \frac{1}{2} \left(\frac{-\nu}{E} 2x - \frac{2x}{E}(\nu+2) \right) = \frac{-2x}{E}(1+\nu) = e_{xy}$$

These all check

GG711c

function GG711c_HW6(PR,E)

% function for Homework 6 of GG711c.

x=-10:1:10;

y=x;

[X,Y] = meshgrid(x,y);

% Part one

% Plot the plane stress displacements

% corresponding to the Airy stress function

% $\phi = x^2$

% PR = Poisson's ratio

ux1 = ;

uy1 = ;

figure(1)

clf

quiver(X,Y,ux1,uy1)

hold on

plot([0 0 max(x) max(x) 0], [0 max(y) max(y) 0 0])

xlabel('x')

ylabel('y')

title('Displacement field for the stress function $\phi = x^2$ ')

axis('equal')

% Part two

% Plot the plane stress displacements

% corresponding to the Airy stress function

% $\phi = xy$

% PR = Poisson's ratio

ux3 = ;

uy3 = ;

figure(2)

clf

quiver(X,Y,ux2,uy2)

hold on

plot([0 0 max(x) max(x) 0], [0 max(y) max(y) 0 0])

xlabel('x')

ylabel('y')

title('Displacement field for the stress function $\phi = xy$ ')

axis('equal')