Homework #5

Use the equations for a plate under a uniaxial tension with a hole to model the stresses in the plate. Use a unit value for the tension (i.e., Sxx infinity = 1), let the radius "a" of the hole = 1, and let the grid be defined by:

    x=-2.99:0.02:2.99;

    y=x;

    [X,Y] = meshgrid(x,y);

The function you are to complete will run by typing (**note the semicolon at the end!**):

    [Srr,Srt,Stt,Sxx,Syx,Syy,taumax] = uniaxialhole2(1,X,Y);

Plot **labeled contour lines** of the stresses below and report on how you checked your plot (see page 2):

	Symmetry/Anti-symmetry check	Boundary value check
Srr		
Srt		
Stt		
Sxx		
Sxy		
Syy		
taumax *		

        * taumax is the maximum shear stress

After running uniaxialhole2, obtain the labeled contour plot for taumax, for example, by typing:

    c=contour(X,Y,taumax);

    clabel(c)

    axis('equal')

    xlabel('x')

    ylabel('y')

    title('Maximum shear stress')

You will probably want to investigate the contour and clabel functions on Matlab to get plots that lood good.

To minimize wasted time on your part, I've included a copy of the key, with the "minor" change that YOU need to fill in the correct expressions for almost each line. This serves as a good outline for how to complete the script. <u>Make sure the lines in your script end with semicolons (;)</u>

to stop the calculated values from printing out when you run the function, and make sure to use .* and ./ to multiply or divide matrices on an element-by-element basis.

When people code up equations, they need to check the results. Otherwise, how can a person have confidence in the results? The issue then turns to how to devise good checks. One simple check is to see if the results are consistent with the boundary conditions. In the problem here, do the plots reflect a traction-free hole? Are the concentrations in the tangential normal stress at the hole consistent with the equations? Along the x-axis, the radial normal stress σ_{rr} should equal σ_{xx}, the tangential normal stress $\sigma_{\theta\theta}$ should equal σ_{yy}, and the shear stress $\sigma_{r\theta}$, should equal σ_{xy}. Is that the case with your plots?

Another useful check is to utilize whatever symmetry (or antisymmetry) is available. In the example below, of a diamond-shaped plate under a uniaxial tension, planes connecting the vertices of the diamond are mirror planes. Both the loads and the plate geometry are symmetric about the x- and y-axes. As the inset "stress boxes" show, the normal stress arrows in the first quadrant are symmetrically reflected into the second and fourth quadrants, and the normal stress arrows in the second quadrant are symmetrically reflected into the third quadrant. So the normal stresses are symmetric about the x- and y-axes. In contrast, the shear stress arrows in the first quadrant are anti-symmetrically reflected into the second and fourth quadrants, and the shear stress arrows in the second quadrant are anti-symmetrically reflected into the third quadrant. So the shear stresses are anti-symmetric about the x- and y-axes. The displacements reflect as well, and certain displacement components are symmetric, whereas others are anti-symmetric.
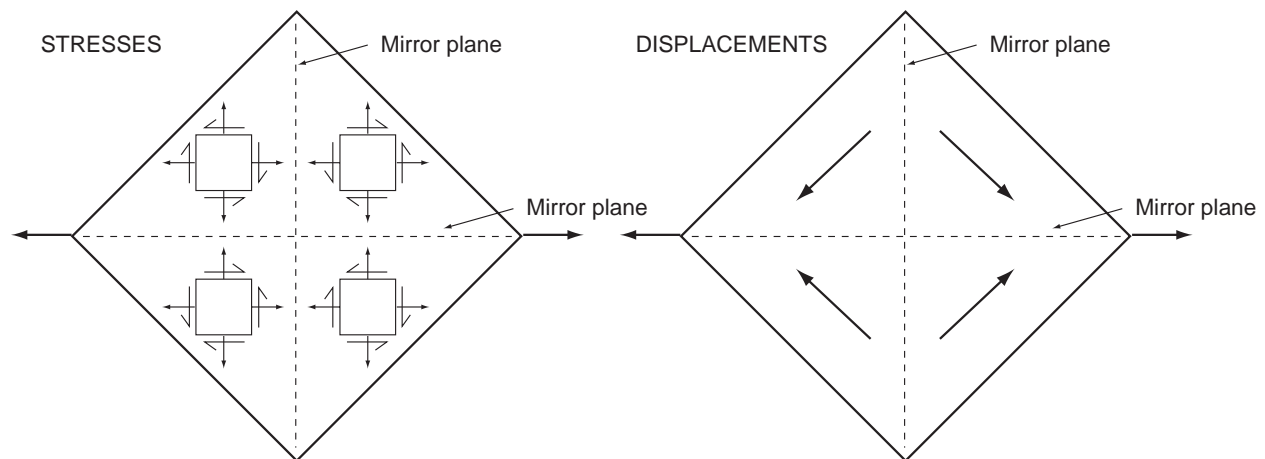
For each of your solutions, perform a boundary value check (if possible) and a symmetry check.

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function [Srr,Srt,Stt,Sxx,Sxy,Syy,taumax] = uniaxialhole2(S,X,Y)
% Stresses around a hole of unit radius in a plate under a uniaxial tension
% The solution is from Barber (1992, p. 94).
% S is the value of the uniaxial far-field tension along the x-axis
% X is the matrix (or vector) of x-coordinates of observation points
% Y is the matrix (or vector) of y-coordinates of observation points
% For a grid of points, X and Y can be found easily using
% Matlab's meshgrid command: [X,Y] = meshgrid(x,y)
% Type
%              help meshgrid
% for how to use the meshgrid command
% Example: [Srr,Srt,Stt,Sxx,Syx,Syy,taumax] = uniaxialhole2(S,X,Y)

% Calculate the radial distance to the first, second, and fourth powers
% for each point on the observation grid
r = sqrt(X.*X + Y.*Y);
r2 = X.*X + Y.*Y;
r4 = r2.*r2;
% Calculate theta for each point on the observation grid
theta = atan2(Y,X);
% Calculate the trig functions of theta for each point on the grid
cost = cos(theta);        % cosine of angle between x and r directions
sint = ;
cos2t = ;
sin2t = ;
cost2 = ;
sint2 = ;
axr = ;            % cosine of angle between x and r directions
ayr = ;            % cosine of angle between y and r directions
axt = ;            % cosine of angle between x and t directions
ayt = ;            % cosine of angle between y and t directions

% Calculate the stresses in a polar reference frame at the grid points
Srr = ;
Srt = ;
Str = ;
Stt = ;

% Because some grid points might be inside the circle, which is
% traction-free, check for points inside the circle and set the
% stresses at those points to zero
index = find(r<1);
Srr(index) = zeros(size(index));
Srt(index) = zeros(size(index));
Str(index) = zeros(size(index));
Stt(index) = zeros(size(index));

% Calculate the stresses in an x,y reference frame at the grid points
% See equation 5.30 in Chou and Pagano, or eqs. 1.7-1.9 of Barber,
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% or equations 4.3-4.9 of course notes for GG711
Sxx = ;
Sxy = ;
Syy = ;

% Calculate the maximum shear stress
% See equation 1.15 of Chou and Pagano
taumax = ;