# Homework #4

The purpose of this homework is to give you some experience with the kinds of stress distributions that can exist in a body and to see how traction boundary conditions can be used to determine the stresses inside a body.

Complete the attached tables for the six stress functions listed below. I've included an example to follow for $\phi = Cx^2y$. All these functions obey the biharmonic equation ($\nabla^2\phi = 0$) and hence yield admissible stress states. You should be able to recover the stress functions by integrations along a contour from point A [at the lower left corner of the box (at the origin)] through Point B [at the lower right corner of the box] to Point C [at the upper right corner of the box]. Start by finding the stresses from the stress functions. Then find the tractions along the sides of the box: enter the tractions for the AB leg and the BC leg in the table, and represent the normal and shear tractions in some clear manner on all four sides of the boxes at the top of the tables (see the example). On lines (j) and (k) find the partial derivatives of $\phi$ by integrating the tractions rather than by taking derivatives of the stress function.

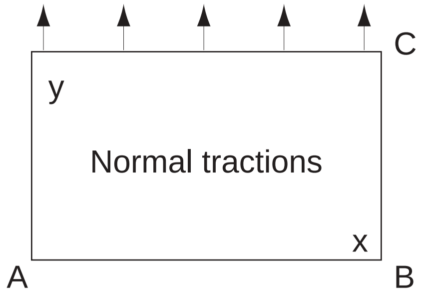
The integrals (see the left hand column) go from Point s to Point s*. Point s here is the same as Point A. Point s* is the endpoint for the integration, and we will let it be between points A and B [column (2)], at Point B [column (3)], between points B and C [column (4)], and at Point C [column (5)]. Column (3) shows the values of the integrals at Point B, and this value is then modified as one proceeds from Point B to Point C. I've "boxed" the values of the integrals at point B in column (3) and their counterparts in column (4) to make this point.

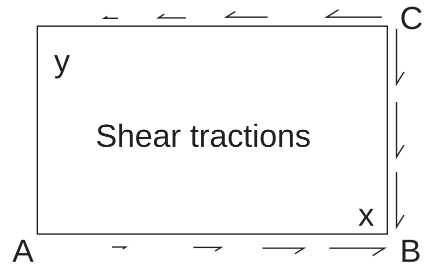
On a related issue, you might note that for a starting point at the origin (Point A) and for the polynomial stress functions selected here, $\partial\phi/\partial x$ and $\partial\phi/\partial y$ equal zero at Point A, just as $\phi$ equals zero at Point A.

Finally, don't forget the minus sign in front of the integral in line (j)!

$\phi1 = Cx^2$
$\phi2 = Cy^2$
$\phi3 = Cxy$
$\phi4 = Cx^3$
$\phi5 = Cy^3$
$\phi6 = Cxy^2$

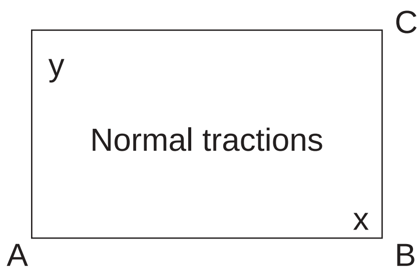
<u>Example</u>: $\phi = Cx^2y$

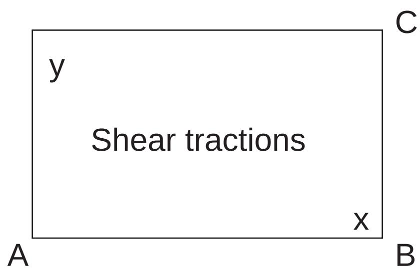



| | (1) Point A | (2) Leg AB | (3) B | (4) Leg BC | (5) C |
|---|---|---|---|---|---|
| (a) $\sigma_{xx}$ | XXXXXXXX | 0 | XXXXXXXX | 0 | XXXXXXXX |
| (b) $\sigma_{xy}$ | XXXXXXXX | -2Cx | XXXXXXXX | -2Cx | XXXXXXXX |
| (c) $\sigma_{yx}$ | XXXXXXXX | -2Cx | XXXXXXXX | -2Cx | XXXXXXXX |
| (d) $\sigma_{yy}$ | XXXXXXXX | 2Cy=0 | XXXXXXXX | 2Cy | XXXXXXXX |
| (e) $n_x$ | XXXXXXXX | 0 | XXXXXXXX | 1 | XXXXXXXX |
| (f) $n_y$ | XXXXXXXX | -1 | XXXXXXXX | 0 | XXXXXXXX |
| (g) $t_x$ (ae+bf) | XXXXXXXX | 2Cx | XXXXXXXX | 0 | XXXXXXXX |
| (h) $t_y$ (ce+df) | XXXXXXXX | -2Cy=0 | XXXXXXXX | -2Cx | XXXXXXXX |
| (i) ds | XXXXXXXX | dx | XXXXXXXX | dy | XXXXXXXX |
| (j) (from h and i)<br>$\partial\phi/\partial x = -\int_s^{s*} t_y ds$ | XXXXXXXX | Between A<br>and B<br>0 | At B<br>$\boxed{0}$ | Between B<br>and C<br>$\boxed{0}$ +2Cxy =<br>2Cxy | At C<br>2Cxy |
| (k)(from g and i)<br>$\partial\phi/\partial y = \int_s^{s*} t_x ds$ | XXXXXXXX | Between A<br>and B<br>$Cx^2$ | At B<br>$\boxed{Cx^2}$ | Between B<br>and C<br>$\boxed{Cx^2}$ + 0 =<br>$Cx^2$ | At C<br>$Cx^2$ |
| (l) dx/ds | XXXXXXXX | 1 | XXXXXXXX | 0 | XXXXXXXX |
| (m) dy/ds | XXXXXXXX | 0 | XXXXXXXX | 1 | XXXXXXXX |
| (n) $d\phi/ds$<br>(jl+km) | XXXXXXXX | 0 | XXXXXXXX | $Cx^2$ | XXXXXXXX |
| (o) (from i and n)<br>$\phi = \int_s^{s*} \frac{d\phi}{ds} ds$ | 0 | Between A<br>and B<br>0 | At B<br>$\boxed{0}$ | Between B<br>and C<br>$\boxed{0}$ + $Cx^2y$ =<br>$Cx^2y$ | At C<br>$Cx^2y$ |

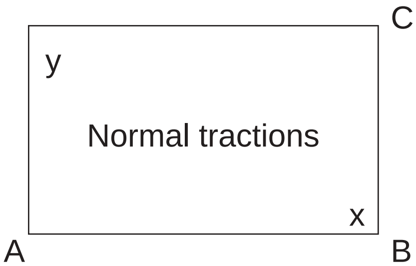
$\phi = Cx^2$

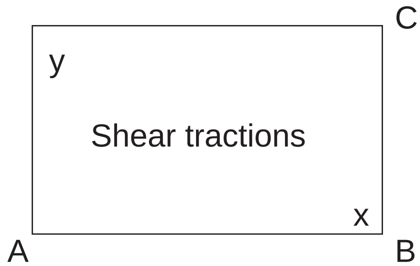



| | (1) Point A | (2) Leg AB | (3) B | (4) Leg BC | (5) C |
|---|---|---|---|---|---|
| (a) $\sigma_{xx}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (b) $\sigma_{xy}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (c) $\sigma_{yx}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (d) $\sigma_{yy}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (e) $n_x$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (f) $n_y$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (g) $t_x$ (ae+bf) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (h) $t_y$ (ce+df) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (i) ds | XXXXXXXX | dx | XXXXXXXX | dy | XXXXXXXX |
| (j) (from h and i) $\partial\phi/\partial x = -\int_s^{s*} t_y ds$ | XXXXXXXX | Between A and B | At B | Between B and C | At C |
| (k)(from g and i) $\partial\phi/\partial y = \int_s^{s*} t_x ds$ | XXXXXXXX | Between A and B | At B | Between B and C | At C |
| (l) dx/ds | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (m) dy/ds | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (n) $d\phi/ds$ (jl+km) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (o) (from i and n) $\phi = \int_s^{s*} \frac{d\phi}{ds} ds$ | 0 | Between A and B | At B | Between B and C | At C |

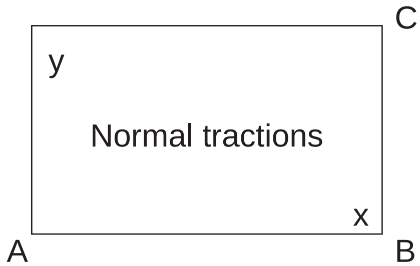
$\phi = Cy^2$

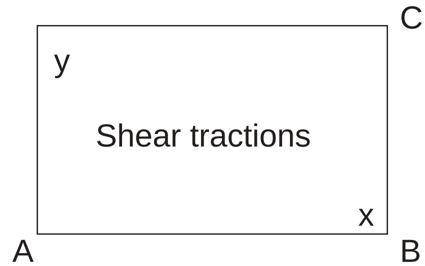



| | (1) Point A | (2) Leg AB | (3) B | (4) Leg BC | (5) C |
|---|---|---|---|---|---|
| (a) $\sigma_{xx}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (b) $\sigma_{xy}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (c) $\sigma_{yx}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (d) $\sigma_{yy}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (e) $n_x$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (f) $n_y$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (g) $t_x$ (ae+bf) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (h) $t_y$ (ce+df) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (i) ds | XXXXXXXX | dx | XXXXXXXX | dy | XXXXXXXX |
| (j) (from h and i)<br>$\partial\phi/\partial x = -\int_s^{s*} t_y ds$ | XXXXXXXX | Between A and B | At B | Between B and C | At C |
| (k) (from g and i)<br>$\partial\phi/\partial y = \int_s^{s*} t_x ds$ | XXXXXXXX | Between A and B | At B | Between B and C | At C |
| (l) dx/ds | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (m) dy/ds | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (n) $d\phi/ds$<br>(jl+km) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (o) (from i and n)<br>$\phi = \int_s^{s*} \frac{d\phi}{ds} ds$ | 0 | Between A and B | At B | Between B and C | At C |

$\phi = Cxy$




| | (1) Point A | (2) Leg AB | (3) B | (4) Leg BC | (5) C |
|---|---|---|---|---|---|
| (a) $\sigma_{xx}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (b) $\sigma_{xy}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (c) $\sigma_{yx}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (d) $\sigma_{yy}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (e) $n_x$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (f) $n_y$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (g) $t_x$ (ae+bf) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (h) $t_y$ (ce+df) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (i) ds | XXXXXXXX | dx | XXXXXXXX | dy | XXXXXXXX |
| (j) (from h and i)<br>$\partial\phi/\partial x = -\int_{s}^{s*} t_y ds$ | XXXXXXXX | Between A and B | At B | Between B and C | At C |
| (k)(from g and i)<br>$\partial\phi/\partial y = \int_{s}^{s*} t_x ds$ | XXXXXXXX | Between A and B | At B | Between B and C | At C |
| (l) dx/ds | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (m) dy/ds | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (n) $d\phi/ds$<br>(jl+km) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (o) (from i and n)<br>$\phi = \int_{s}^{s*} \frac{d\phi}{ds} ds$ | 0 | Between A and B | At B | Between B and C | At C |

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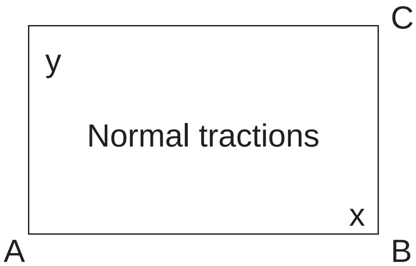
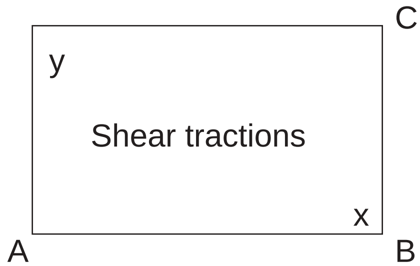
$\phi = Cx^3$

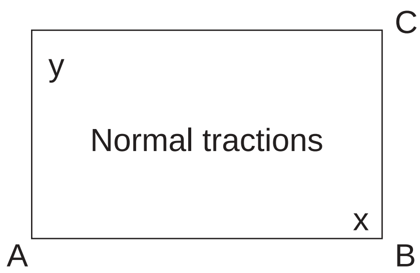


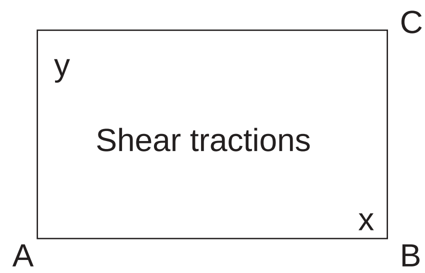




| | (1) Point A | (2) Leg AB | (3) B | (4) Leg BC | (5) C |
|---|---|---|---|---|---|
| (a) $\sigma_{xx}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (b) $\sigma_{xy}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (c) $\sigma_{yx}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (d) $\sigma_{yy}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (e) $n_x$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (f) $n_y$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (g) $t_x$ (ae+bf) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (h) $t_y$ (ce+df) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (i) ds | XXXXXXXX | dx | XXXXXXXX | dy | XXXXXXXX |
| (j) (from h and i) $\partial\phi/\partial x = -\int_s^{s*} t_y ds$ | XXXXXXXX | Between A and B | At B | Between B and C | At C |
| (k)(from g and i) $\partial\phi/\partial y = \int_s^{s*} t_x ds$ | XXXXXXXX | Between A and B | At B | Between B and C | At C |
| (l) dx/ds | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (m) dy/ds | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (n) $d\phi/ds$ (jl+km) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (o) (from i and n) $\phi = \int_s^{s*} \frac{d\phi}{ds} ds$ | 0 | Between A and B | At B | Between B and C | At C |

GG711c

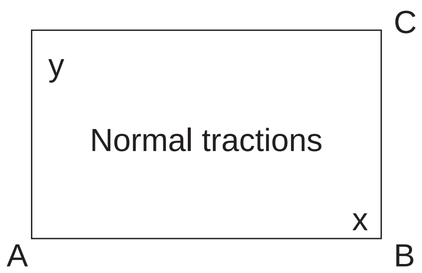
$\phi = Cy^3$

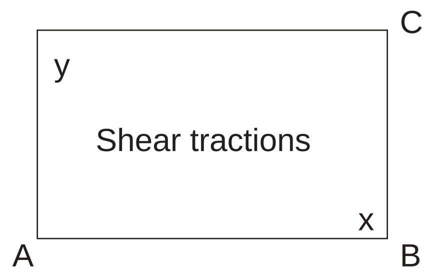



| | (1) Point A | (2) Leg AB | (3) B | (4) Leg BC | (5) C |
|---|---|---|---|---|---|
| (a) $\sigma_{xx}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (b) $\sigma_{xy}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (c) $\sigma_{yx}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (d) $\sigma_{yy}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (e) $n_x$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (f) $n_y$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (g) $t_x$ (ae+bf) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (h) $t_y$ (ce+df) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (i) ds | XXXXXXXX | dx | XXXXXXXX | dy | XXXXXXXX |
| (j) (from h and i)<br>$\partial\phi/\partial x = -\int_s^{s*} t_y ds$ | XXXXXXXX | Between A and B | At B | Between B and C | At C |
| (k)(from g and i)<br>$\partial\phi/\partial y = \int_s^{s*} t_x ds$ | XXXXXXXX | Between A and B | At B | Between B and C | At C |
| (l) dx/ds | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (m) dy/ds | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (n) $d\phi/ds$ (jl+km) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (o) (from i and n)<br>$\phi = \int_s^{s*} \frac{d\phi}{ds} ds$ | 0 | Between A and B | At B | Between B and C | At C |

$\phi = Cxy^2$




| | (1) Point A | (2) Leg AB | (3) B | (4) Leg BC | (5) C |
|---|---|---|---|---|---|
| (a) $\sigma_{xx}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (b) $\sigma_{xy}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (c) $\sigma_{yx}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (d) $\sigma_{yy}$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (e) $n_x$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (f) $n_y$ | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (g) $t_x$ (ae+bf) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (h) $t_y$ (ce+df) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (i) ds | XXXXXXXX | dx | XXXXXXXX | dy | XXXXXXXX |
| (j) (from h and i)<br>$\partial\phi/\partial x = -\int_s^{s*} t_y ds$ | XXXXXXXX | Between A and B | At B | Between B and C | At C |
| (k) (from g and i)<br>$\partial\phi/\partial y = \int_s^{s*} t_x ds$ | XXXXXXXX | Between A and B | At B | Between B and C | At C |
| (l) dx/ds | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (m) dy/ds | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (n) $d\phi/ds$<br>(jl+km) | XXXXXXXX | | XXXXXXXX | | XXXXXXXX |
| (o) (from i and n)<br>$\phi = \int_s^{s*} \frac{d\phi}{ds} ds$ | 0 | Between A and B | At B | Between B and C | At C |