Homework #3

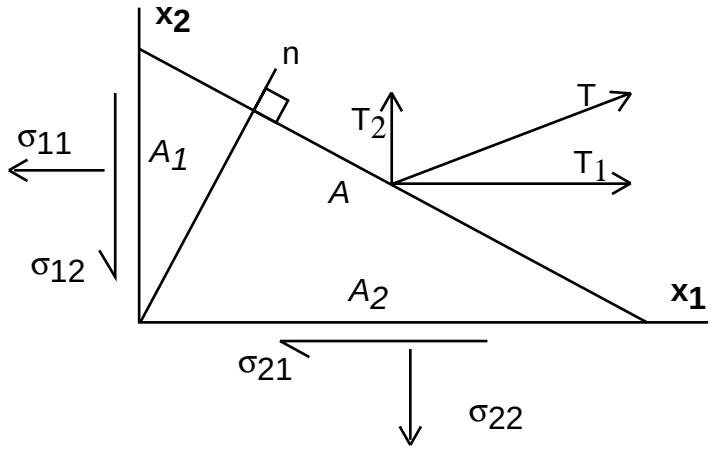
Problem 1 Cauchy's Law

Assuming σ_{11} = 0 Mpa; σ_{12} = +12 Mpa; σ_{21} = +12 Mpa; σ_{22} = 0 Mpa (the stresses here are given in tensor notation!!), and $\theta_{n1} = 62^{\circ}$, $\theta_{n2} = 28^{\circ}$, do the following:

1a Find the magnitude of the traction components T_1 and T_2 that act on the plane below. Do this problem by hand. (**4 pts**)

1b Find the magnitude and orientation of the traction vector T. (**2 pts**)

1c Repeat parts a and b using Matlab. Include a printout of your results. Use the atan2 function for the orientation. (**6 points**).



Problem 2 Stress Transformation

Suppose the following regional stresses act in the volcanically active area shown below:
σ_{11} = -10 Mpa; σ_{12} = +12 Mpa; σ_{21} = +12 Mpa; σ_{22} = -10 Mpa.
The stresses here are given in tensor notation. Note that the orientation of the x_1 and x_2 axes correspond to the x_3 pointing into the page.

2a Draw and label these stress components on the appropriate sides of the box at left.(**4 pts**)

2b By using the eig and acos commands in Matlab {e.g., [V,D] = eig(sigma)}, find the magnitude and orientation of the principal stresses acting within the region. (**2 points**)

2c Find the 4 sets of direction cosines between the x1 and x2 axes and the x_1' and x_2' axes, where the x1' axis trends northeast, and the x_2' axis trends due southeast.(**4 pts**)

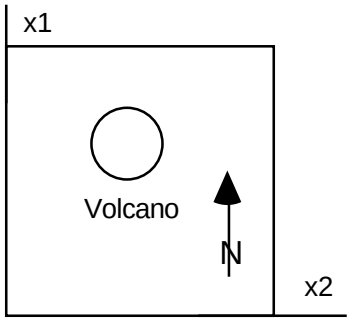
2d Derive by hand using tensor notation $\sigma_{1'1'}$, $\sigma_{1'2'}$, $\sigma_{2'1'}$, $\sigma_{2'2'}$.(**8 pts**)

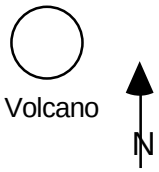
2e Draw a box around the right-hand volcano with the box sides being the principal planes.(**4)**

2f Draw the principal stresses acting on your box and label their magnitudes. (**4 pts**)

2g Draw a Mohr circle diagram that shows the stress state within the region. Mark the points on the Mohr circle that correspond to the stresses on planes A_1, A_2, A_1' and A_2', which are normal to the x_1, x_2, x_1' and x_2' axes, respectively. You might want to look at my notes for lecture 17 of GG303 for more information on the Mohr circle diagram. (**5 points)**

2h In the diagram at right, draw and label the most likely orientation of a potential dike, assuming that the dike develops perpendicular to the least compressive stress. (**2 pts**)





Problem 3 Lateral Confinement

In the absence of tectonic stresses, we may wonder what the state of stress in the earth should be. A common assumption for near the surface is that the earth is laterally constrained, so that particles can only be displaced up or down. Suppose that the x_1 and x_2 axes define the ground surface and the x_3 axis points down. Assuming that the earth behaves as an isotropic elastic material, if $\sigma_3 = \rho g x_3$, and ε_{11} and ε_{22} are zero, find the expression for the lateral stresses σ_{11} and σ_{22} as a function of depth. Hint: the lateral stress will increase linearly with depth, and use eqs. 6.8-6.13 to arrive at your answer. **(5 points)**

Problem 4 Strain Rosettes

Solve Problem 3-1 of Chou and Pagano. You need to use equation 2.3 of Chou and Pagano (or equation 1.37 of Barber) three times to write equations for e11, e22, and e33, then solve for exx, eyy, and exy. Use those results to solve for the principal strains, and use those results to solve for the principal stresses (see p. 248-254 of Middleton and Wilcock, 1994). **(10 points)**