GG612
Structural Geology Section
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Lecture 4
Isostasy
Rheology
Strike-view Cross Sections
Fault Mechanics

Isostasy

- Refers to gravitational equilibrium
- Provides a physical rationale for the existence of mountains
- Based on force balance and buoyancy concepts

$$P = \int_0^h \rho(h) g(h)\, dh$$

P = pressure (convention: compression is positive)
ρ = density
g = gravitational acceleration

For constant ρ and constant g,
P = ρgh

http://en.wikipedia.org/wiki/File:Iceberg.jpg

Isostasy

- Assumes a "compensation depth" at which pressures beneath two prisms are equal and the material beneath behaves like a static fluid, where $P_1 = P_2$
- Flexural strength of crust not considered
- Gravity measurements yield crustal thickness and density variations
- Complemented by seismic techniques

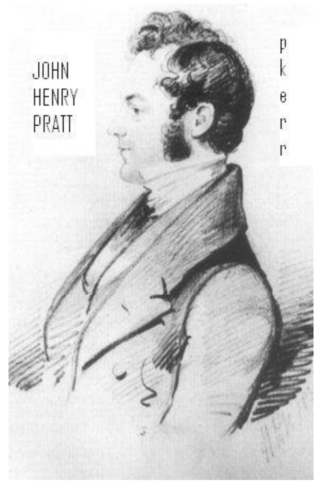
http://en.wikipedia.org/wiki/File:Iceberg.jpg

Isostasy
History

- Roots go back to da Vinci
- Term coined by Clarence Edward Dutton (USGS)
- Post-1800 interest triggered by surveying errors in India
- Two main models: Pratt, Airy

John Henry Pratt (6/4/1809-12/28/1871)

- Pratt, J.H., 1855, On the attraction of the Himalaya Mountains, and of the elevated regions beyond them, upon the Plumb-line in India. Philosophical Transactions of the Royal Society of London, v. 145, p. 53-100.
- British clergyman and mathematician
- Archdeacon of India



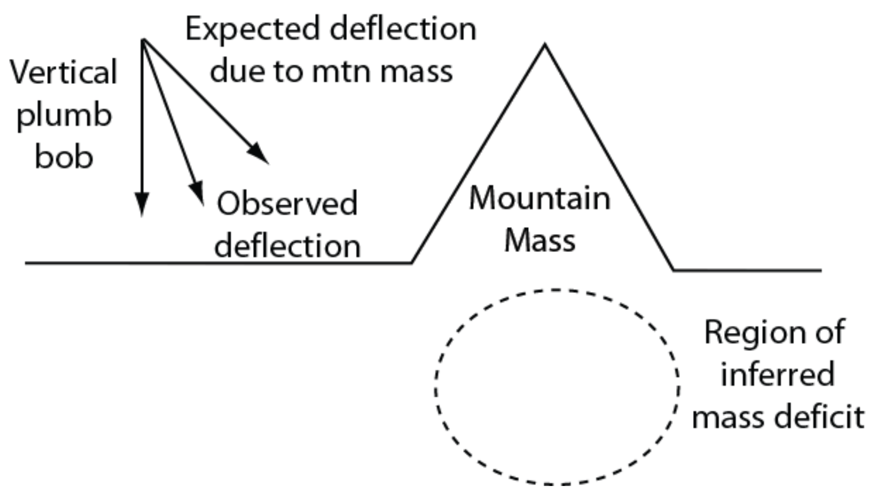
http://sphotos.ak.fbcdn.net/photos-ak-snc1/v2100/67/88/730660017/n730660017_5593665_6871.jpg

Sir George Biddell Airy (7/27/1801 - 1/2/1892

- Airy, G.B., 1855, On the computation of the Effect of the Attraction of Mountain-masses, as disturbing the Apparent Astronomical Latitude of Stations in Geodetic Surveys. Philosophical Transactions of the Royal Society of London. v. 145, p.101-104.
- British Royal Astronomer from 1835-1881
- Determined the mean density of the Earth from pendulum experiments in mines
- Contributor to elasticity theory (telescope deformation)
- Opponent of Charles Babbage from 1842 to ??

http://www.computerhistory.org/babbage/georgeairy/img/5-2-1.jpg

Isostasy and Gravity Measurements



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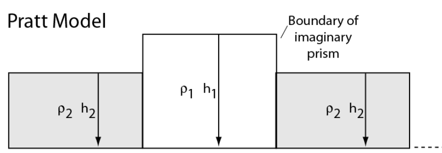
Comparison of Isostatic Models

Pratt Model



$$\rho_1 < \rho_2$$

$$\rho_1 g h_1 = \rho_2 g h_2$$

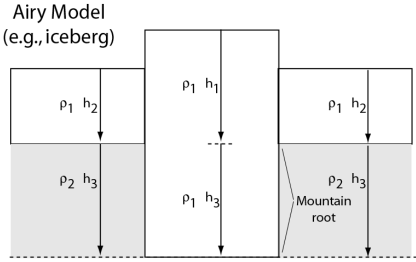
$$\rho_1 / \rho_2 = h_2 / h_1$$

$$(h_1)(\rho_1 / \rho_2) = h_2$$

Depth of compensation = h2

At isostatic equilibrium, P = constant at depth of compensation, so no flow

Airy Model
(e.g., iceberg)



$$\rho_1 < \rho_2$$

$$\rho_1 g h_1 + \rho_1 g h_3 = \rho_1 g h_2 + \rho_2 g h_3$$

$$\rho_1 (h_1 + h_3) = \rho_1 h_2 + \rho_2 h_3$$

$$(h_1 + h_3) = h_2 + (\rho_2 / \rho_1) h_3$$

$$(h_1 + h_3) - (\rho_2 / \rho_1) h_3 = h_2$$

Depth of compensation = h2+h3

At isostatic equilibrium, P = constant at depth of compensation, so no flow

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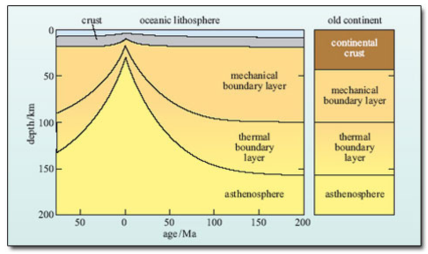
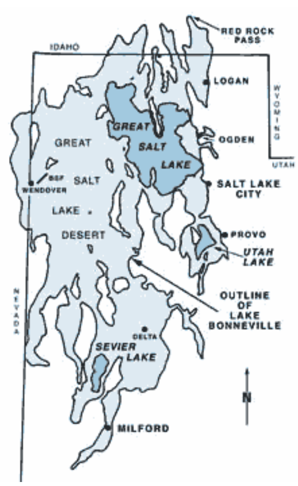
Thermal Isostasy (e.g., Turcotte and Schubert, 2002)

- Oceanic crust thickens and increases in density as it cools with time
- Oceanic crust thickens and increases in density with distance from ridge



http://openlearn.open.ac.uk/file.php/2717/!via/oucontent/course/414/s279_1_014i.jpg

Isostatic Rebound: Lake Bonneville



http://geology.utah.gov/online/pi-39/images/pi39-01.gif

Shorelines of Lake Bonneville Tilt Away from Lake

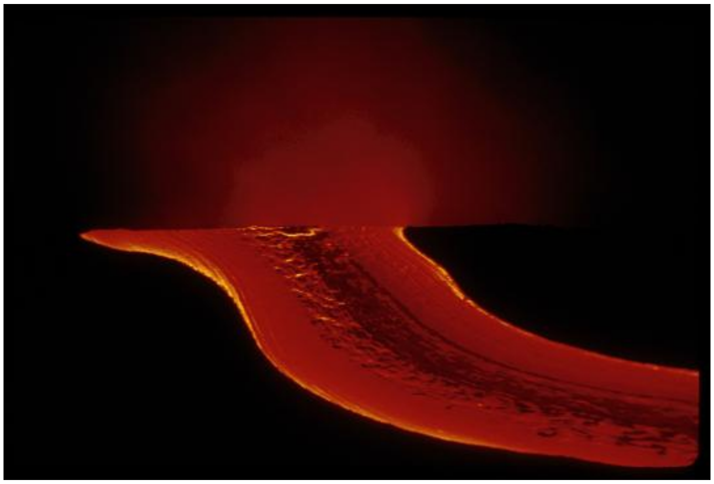
http://k43.pbase.com/g6/93/584893/2/79634985.GXiIakLZ.jpg

20. Rheology & Linear Elasticity

I Main Topics
- A Rheology: Macroscopic deformation behavior
- B Linear elasticity for homogeneous isotropic materials

20. Rheology & Linear Elasticity

Viscous (fluid) Behavior



http://manoa.hawaii.edu/graduate/content/slide-lava

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Ductile (plastic) Behavior

http://www.hilo.hawaii.edu/~csav/gallery/scientists/LavaHammerL.jpg

http://hvo.wr.usgs.gov/kilauea/update/images.html

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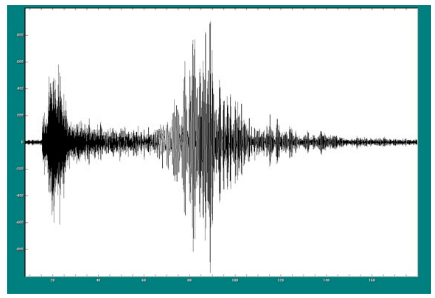
Elastic Behavior



https://thegeosphere.pbworks.com/w/page/24663884/Sumatra

http://www.earth.ox.ac.uk/__data/assets/image/0006/3021/seismic_hammer.jpg

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Brittle Behavior (fracture)

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20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior

A Elasticity

1. Deformation is reversible when load is removed
2. Stress (σ) is related to strain (ε)
3. Deformation *is not time dependent if load is constant*
4. *Examples: Seismic (acoustic) waves, rubber ball*

http://www.fordogtrainers.com

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20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior

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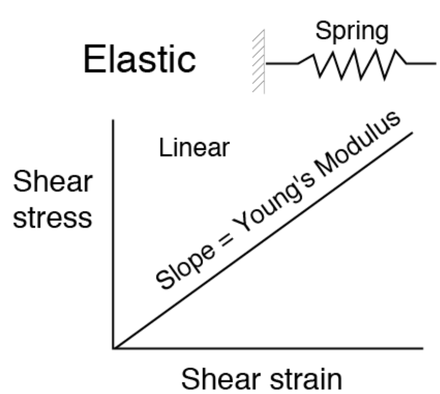

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20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior
  B Viscosity
    1 Deformation is irreversible when load is removed
    2 Stress (σ) is related to strain rate (ε)
    3 Deformation is time dependent if load is constant
    4 Examples: Lava flows, corn syrup

http://wholefoodrecipes.net

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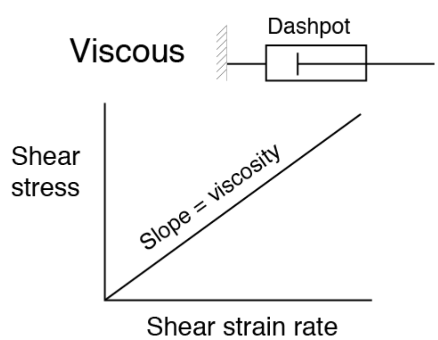
20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior
  B Viscosity
    1 Deformation is irreversible when load is removed
    2 Stress (σ) is related to strain rate (ε)
    3 Deformation is time dependent if load is constant
    4 Examples: Lava flows, corn syrup



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20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior
- C Plasticity
  - 1 No deformation until yield strength is locally exceeded; then irreversible deformation occurs under a constant load
  - 2 Deformation can increase with time under a constant load
  - 3 Examples: plastics, soils

http://www.therapyputty.com/images/stretch6.jpg

20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior
- C Brittle Deformation
  - 1 Discontinuous deformation
  - 2 Failure surfaces separate

http://www.thefeeherytheory.com

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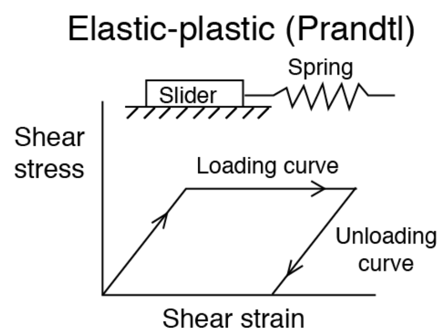
II Rheology: Macroscopic deformation behavior

D Elasto-plastic rheology



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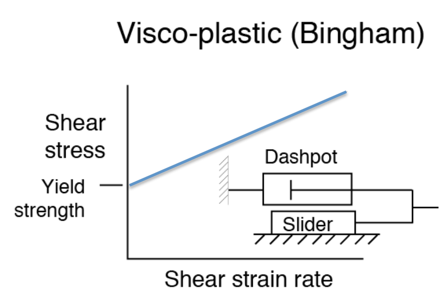
II Rheology: Macroscopic deformation behavior

E Visco-plastic rheology



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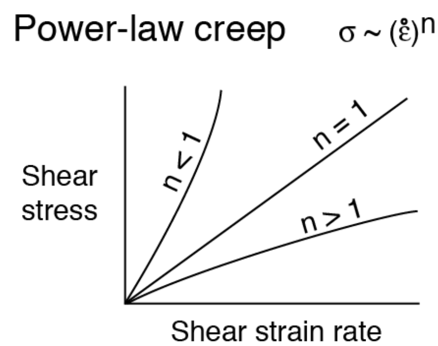
II Rheology: Macroscopic deformation behavior

F Power-law creep

1 $\dot{e} = (\sigma_1 - \sigma_3)^n e^{(-Q/RT)}$

2 Example: rock salt



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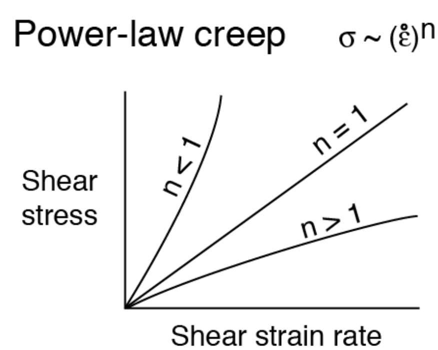
II Rheology: Macroscopic deformation behavior

G Linear vs. nonlinear behavior



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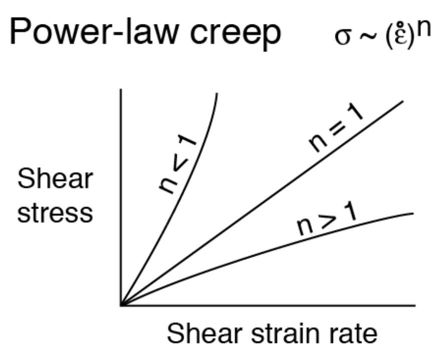
II Rheology: Macroscopic deformation behavior

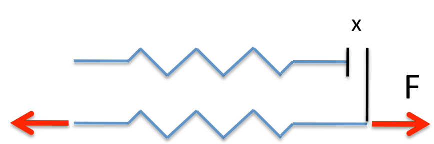
- H Rheology=f (σ_{ij} ,fluid pressure, strain rate, chemistry, temperature)
- I Rheologic equation of real rocks = ?

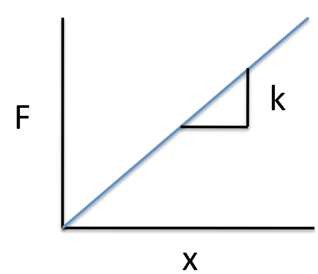


20. Rheology & Linear Elasticity

III Linear elasticity

- A Force and displacement of a spring (from Hooke, 1676): F= kx
  1. F = force
  2. k = spring constant
     Dimensions:F/L
  3. x = displacement
     Dimensions: length L)





20. Rheology & Linear Elasticity

III Linear elasticity (cont.)

B Hooke's Law for uniaxial stress: $\sigma = E\varepsilon$

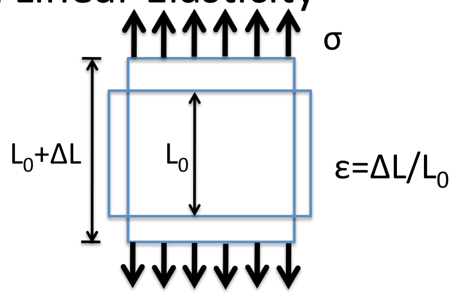
1 σ = uniaxial stress

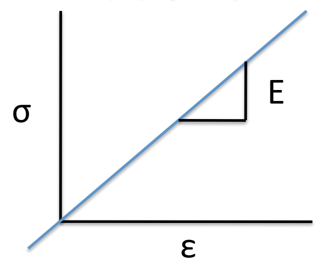
2 E = Young's modulus

Dimensions: stress

3 ε = strain

Dimensionless





20. Rheology & Linear Elasticity

III Linear elasticity (cont.)

B Hooke's Law for uniaxial stress (cont.): $\varepsilon_1 = \sigma_1/E$

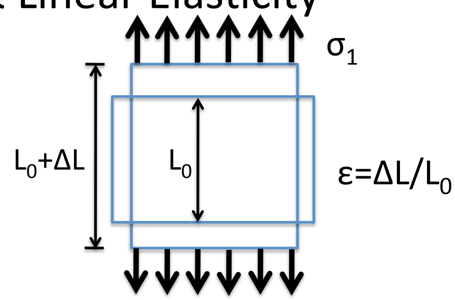
1 $\sigma_2 = \sigma_3 = 0$

2 $\varepsilon_2 = \varepsilon_3 = -\nu\varepsilon_1$

a ν = Poisson's ratio

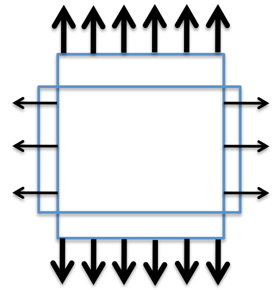
b ν is dimensionless

c Strain in one direction tends to induce strain in another direction



ε

20. Rheology & Linear Elasticity



III Linear elasticity (cont.)

C Linear elasticity in 3D for homogeneous isotropic materials

By superposition:

1 $\varepsilon_{xx} = \sigma_{xx}/E - (\sigma_{yy}+\sigma_{zz})(\nu/E)$

2 $\varepsilon_{yy} = \sigma_{yy}/E - (\sigma_{zz}+\sigma_{xx})(\nu/E)$

3 $\varepsilon_{zz} = \sigma_{zz}/E - (\sigma_{xx}+\sigma_{yy})(\nu/E)$

ε

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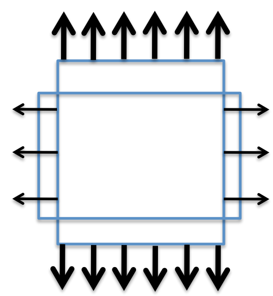
20. Rheology & Linear Elasticity



III Linear elasticity (cont.)

C Linear elasticity in 3D for homogeneous isotropic materials (cont.)

4 Directions of principal stresses and principal strains coincide

5 Extension in one direction can occur without tension

6 Compression in one direction can occur without shortening

ε

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20. Rheology & Linear Elasticity

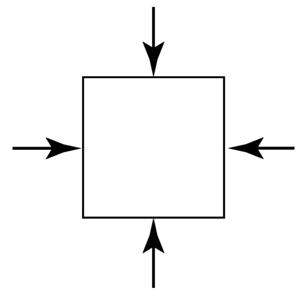
III Linear elasticity

E Special cases

1 Isotropic (hydrostatic) stress

a $\sigma_1 = \sigma_2 = \sigma_3$

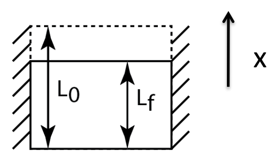
b No shear stress



2 Uniaxial strain

a $\varepsilon_{xx} = \varepsilon_1 \neq 0$

b $\varepsilon_{yy} = \varepsilon_{zz} = 0$



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20. Rheology & Linear Elasticity

III Linear elasticity

D Relationships among different elastic moduli

1 $G = \mu$ = shear modulus

$G = E/(2[1+\nu])$

$\varepsilon_{xy} = \sigma_{xy}/2G$

2 λ = Lame' constant

$\lambda = E\nu/([1 + \nu][1 - 2\nu])$

3 K = bulk modulus

$K = E/(3[1 - 2\nu])$

4 β = compressibility

$\beta = 1/K$

$\Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = -p/K$

p = pressure

5 P-wave speed: V_p

$$V_p = \sqrt{\left(K + \frac{4}{3}\mu\right)\Big/\rho}$$

6 S-wave speed: V_s

$$V_s = \sqrt{\mu/\rho}$$

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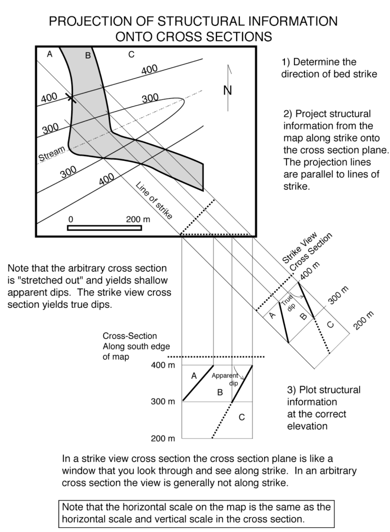
Strike-view Cross Sections

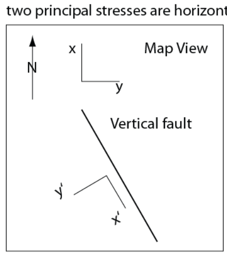
- Prepared by projecting features along strike onto a cross section plane, where the cross section plane is perpendicular to strike
- Shows the true inclination and thickness of features
- Lines of strike lie in geologic planes and connect points of equal elevation

Strike-View Cross Sections



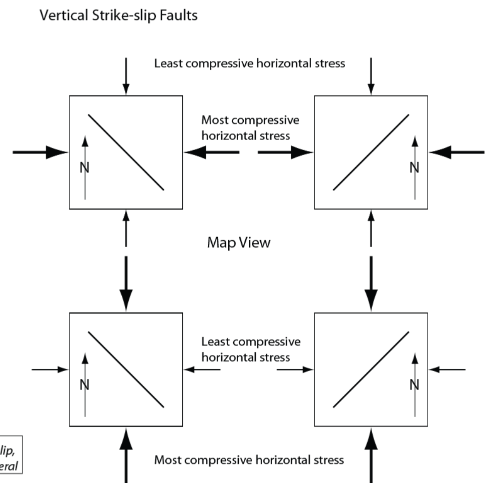
Fault Mechanics: Vertical Strike-slip Faults

Assume one principal stress is vertical, two principal stresses are horizontal



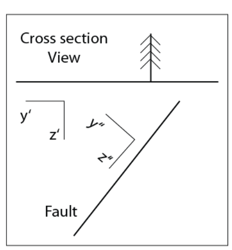
The x' axis is parallel to fault strike
The y'axis is normal to fault strike
The z' axis points down
If $\sigma_{y'x'} > 0$, left-lateral faulting
If $\sigma_{y'x'} = 0$, no faulting
If $\sigma_{y'x'} < 0$, right-lateral faulting

Draw the arrows showing how the faults would slip, and determine whether the slip is right- or left-lateral



Fault Mechanics: Dip-slip Faults

Assume one principal stress is vertical, two principal stresses are horizontal, and the horizontal principal stresses are parallel and normal to fault strike.



The x" axis is parallel to fault strike (at you)
The y" axis is normal to the fault
The z'' azis points down-dip
If $\sigma_{y''z''} > 0$, lnormal faulting
If $\sigma_{y''z''} < 0$, reverse faulting

Draw the arrows showing how the faults would slip, and determine whether the slip is normal or reverse