

MECHANICAL ANALYSIS OF THE DIKE PATTERN OF THE SPANISH PEAKS AREA, COLORADO

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ABSTRACT

It is possible to explain the shape of the remarkable dike pattern surrounding the Spanish Peaks, Colorado, by an analysis of stresses if a regional-stress system in which the direction of greatest principal pressure was parallel to the line of symmetry exhibited by the dike pattern superposed on a local-stress system is assumed. This local-stress system is caused by hydrostatic pressure exerted by the intrusive mass of the Spanish Peaks. For simplification of the computations a circular hole is assumed at the position of West Spanish Peak.

The mountain front west of the Spanish Peaks provides an unknown boundary condition, which however can be described with satisfactory results by the assumption of an image source. On the basis of the assumption that the relation between stress field and dike pattern is the one proposed by Anderson (1951, Chap. 3) it is then possible to compute the dike pattern. This computed dike pattern agrees in many respects with the observed dike pattern.

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INTRODUCTION

Just east of the Culebra Range in Colorado a double mountain, the Spanish Peaks, rises from the surrounding plain to an elevation of 13,633 feet. The locality, well known for its system of radial dikes and accompanying sills, was first described by Hills (1900; 1901).

Intrusive bodies surrounded by an aureole of dikes are not rare in the western United States. Weed and Pirsson (1894) had already described such radial patterns in the Highwood Mountains of Montana. Later Pirsson (1905) published his classical study of the Highwood

Mountains. In a petrological study of the dike rocks of the Spanish Peaks area, Knopf (1936, p. 1784) drew attention to the great similarity in chemical composition of the rocks of both regions. Other systems of radiating dikes along the eastern border of the Rocky Mountains are in the Sunlight area of Wyoming (Parsons, 1939) and in the Big Belt Range of Montana (Lyons, 1944). Mention of a radiating system of dikes in the Mitre Peak area of Texas is also made by Ives (1941, p. 347).

The dike pattern of the Spanish Peaks area is made unique by its peculiar shape (Fig. 1). Whereas the other patterns described are

regularly radial or are confused because of several superposed systems, the geometrical pattern of dikes surrounding the Spanish Peaks shows the following three significant characteristics:

remarkable dike system at Spanish Peaks is as follows (Knopf, 1936, p. 1739):

"Possibly, the cause for this swinging around to an east-west direction is that the forces that, near the stocks, produced the radial fissuring,



FIGURE 1.—SYSTEM OF DIKES SURROUNDING THE SPANISH PEAKS
After Knopf, 1936

(1) The pattern of dikes is clearly symmetrical about a line running N. (75°) E. through West Spanish Peak.

(2) Originating for the most part in West Spanish Peak, the dikes tend to curve eastward.

(3) The westward-trending dikes are much shorter than those trending eastward, which can be followed for distances of more than 25 miles.

Most of the dikes of the Spanish Peaks area which, according to Knopf, number probably not less than 500, are vertical; a few dip less than 80°. Their thickness ranges from 2 to 60 feet; 10 feet is the most common. Most of the dikes appear to have originated after the formation of West Peak. A few shorter dikes, originating from East Peak, were probably intruded earlier.

Knopf's explanation for the origin of the

opened up, at a greater distance from the stocks, the latent tension fissures produced during the compression that had folded the sedimentary beds into a broad syncline before the stocks were injected, *i.e.*, the dikes availed themselves of latent tension breaks across the axis of the fold."

From a mechanical viewpoint a more satisfactory explanation can be given by an analysis of the dike pattern in terms of stress. Such an explanation is based on the assumption that an intimate relation exists between stress field and dike pattern. Anderson (1951, Chap. 3) postulated that the formation of dikes is due to a wedging effect of the intruding magma. According to his theory dikes form along planes lying normal to the direction of least principal stress or in other words along planes of greatest and intermediate principal stress.

Anderson's assumptions are derived directly from the work of Griffith (1921; 1924). From

determinations of the tensile strength of brittle materials it was noted that the observed values of the tensile strength were one or two orders of magnitude lower than values expected from theoretical considerations (Orowan, 1949, p. 192). To explain this discrepancy Griffith assumed that the tested brittle material contained numerous small cracks of random orientation. These small cracks he envisaged as small ellipsoids of large eccentricity. Stresses applied to the material cause large stress concentrations at points near the ends of the ellipsoids. The sudden and rapid propagation of the crack occurs then as follows. If the crack is given a small virtual increase in length some energy is required to break the bonds between particles in the material. There is also a small increase in strain energy of the plate. Griffith postulated that, as soon as the total work done by the external forces exceeds the sum of the increases in surface energy and strain energy of the material, the crack will spread. In this manner the observed tensile strength can be related to the length of the crack. Also the orientation of the cracks first to become unstable under various systems of applied loadings can be investigated. In general the orientation of those cracks does not coincide with that of the principal stresses in the plate. Therefore Griffith suggested that his theory could explain the inclined fractures often observed in specimens of brittle material deformed under various states of triaxial stress. In view of the basic assumption made by Griffith that the stress-strain relation satisfies Hooke's law exactly up to the moment of rupture this suggestion seems doubtful. His analysis probably can be applied to the wedging effect of a fluid under pressure, however, because little energy will be dissipated in permanent deformation. It is then assumed that the dike is an ellipse of large eccentricity on the walls of which a hydrostatic pressure p is exerted by the intruding magma. This leads to large but very localized tensile stresses at the tip of the crack (Griffith, 1921; 1924). The largest tensile stresses occur for a crack whose long axis is oriented in the direction of the largest principal stress. Because the dike will originate in this orientation it will during its growth remain along such a maximum stress trajectory. The problem therefore is to

determine the maximum stress trajectories of the stress field.

The stress field in the material of the crust is obtained by superposition of two simpler fields. The first, which we shall call the local field, is caused by the fluid pressure in the central hole, and the second is a regional field which is assumed to vary little over large distances. This is the field in the crust at the moment the dikes were injected.

The symmetry of the dike pattern indicates that the total-stress field must be symmetric in the same manner. The absence of dikes west of the mountain front suggests that this mountain front acted as a more or less rigid boundary, and the divergence of dikes from one central area indicates that the stress in this area can be described by the formulas used to compute the stress around a hole—which for convenience shall be considered circular—in an infinite plate caused by an internal pressure p .

The dike pattern derived on the assumptions that there is no regional-stress field does not resemble the dike pattern around the Spanish Peaks. Thus the existence of a regional field at the time of dike injection is very probable. It seems plausible then to assume a regional-stress field in which the greatest principal stress had the direction of the line of symmetry exhibited by the Spanish Peaks dike system. This agrees with the intuitive notion that the mountain ranges west of the Spanish Peak are related to such a stress field. The superposition of this regional-stress field over the local-stress field created by the dike injection yields a stress pattern which is in striking agreement with the pattern of dikes surrounding the Spanish Peaks.

LOCAL-STRESS FIELD

To simplify the problem of finding the local-stress field, it is necessary to make the following assumptions:

- (1) The "fluid" igneous mass rose through a vertical circular hole to the surface.
- (2) The focal point of the curvilinear dike pattern is at West Spanish Peak.
- (3) The mountain front was straight and acted as an almost rigid boundary.

Treating the problem in two dimensions reduces it to the problem of finding the stresses in a

semi-infinite plate pierced near its boundary by a circular hole, in which a hydrostatic pressure is exerted. The boundary condition imposed by the mountain front is attained by assuming an "image" source, as is commonly

simple reason. Although the "exact" problem can be solved with the appropriate mathematical tools, its greater complexity in no way guarantees greater accuracy, since the exact boundary conditions along the mountain front

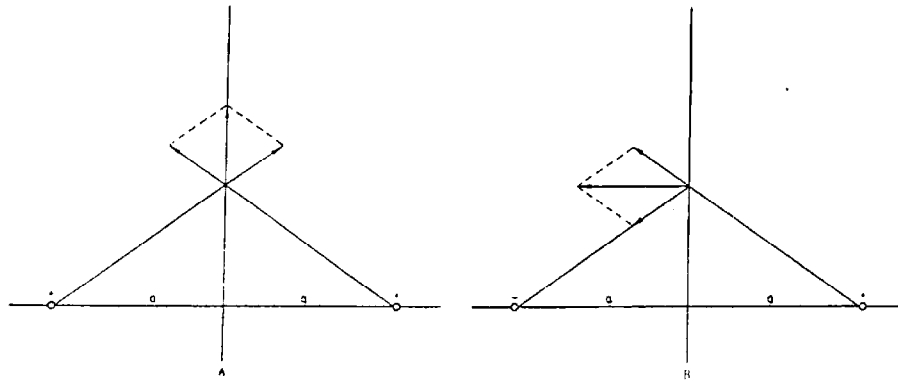


FIGURE 2.—DISPLACEMENTS AT THE MOUNTAIN FRONT
A. For a positive image source
B. For a negative image source

done in potential theory. By "image" source here is meant another circular hole in which either the same or a different hydrostatic pressure is exerted, and which is placed somewhere at the other side of the mountain front. The usefulness of the concept of image sources lies in the fact that the stresses, which act along the mountain front, and which must be given to obtain a solution, are determined by superposition of the stresses caused by the real hole and its images. The stresses along the mountain front are unknown, and, therefore, it is impossible to decide where the image sources must be placed and what their respective pressures are. However, the symmetry of the dike pattern suggests that an "image source" can be placed at a point which is the mirror image of the real hole with respect to the mountain front. This image source, which will be assumed to be of the same magnitude as the real source, may be positive or negative. When the image is positive, *i.e.*, if the fluid within it exerts a hydrostatic pressure, there is a displacement along the front (Fig. 2A). If the source is negative, *i.e.*, the hole tends to close, the mountain front is displaced normal to itself (Fig. 2B). These displacements are relatively small.

The preference to treat the problem in this manner, rather than to solve the analogous problem for an exactly rigid boundary, has a

are unknown. Moreover the "exact" solution leads essentially to the same results.

For a single hole in an infinite plane, the stresses at a point P , which result from hydrostatic pressure in the hole, are functions only of R , the distance from the center of the hole to the point P . The easiest way of solving the problem is to use Airy's stress function ϕ , which satisfies the equation $\nabla^2 \nabla^2 \phi = 0$. From this function ϕ the stress components are derived from the following equations (Timoshenko, 1934, Chap. 3):

$$\begin{aligned}\sigma_R &= \frac{1}{R} \frac{\partial \phi}{\partial R} + \frac{1}{R^2} \frac{\partial^2 \phi}{\partial \theta^2}; \\ \sigma_\theta &= \frac{\partial^2 \phi}{\partial R^2}; \\ \tau &= -\frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial \phi}{\partial \theta} \right),\end{aligned}\tag{1}$$

where σ_R , σ_θ and τ are the normal radial, normal tangential, and shear stress respectively. The stresses which act on a small element of volume are shown in Figure. 3. The stress function ϕ for a hole in an infinite plane is obtained from $\nabla^2 \nabla^2 \phi = 0$ with the condition that the stresses are independent of θ and vanish at infinity; then $\phi = A \ln R$, where A is a constant. If in two holes the same hydrostatic pressure is exerted, and the origin of the co-ordinate

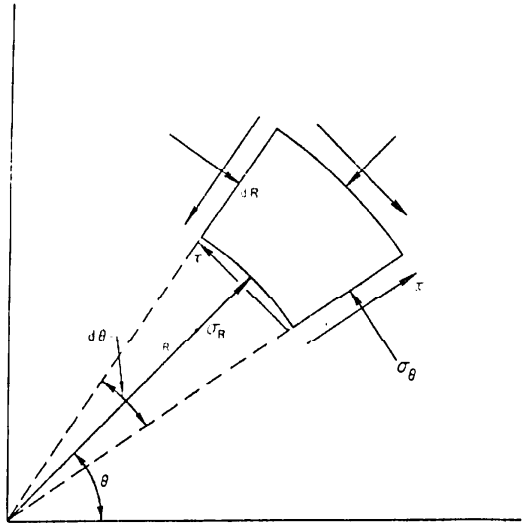


FIGURE 3.—STRESSES ACTING ON A SMALL ELEMENT OF VOLUME

where for simplification of the formulac the following substitution is made:

$$R^2 + a^2 = 2\xi^2.$$

In case $\phi = A_2 \ln (r_1/r_2)$, which applies if the pressure in both holes is equal but opposite in sign, the stress components are given by

$$\sigma_R = -\sigma_\theta = \frac{A_2 a R \cos \theta}{(\xi^4 - a^2 R^2 \cos^2 \theta)^2} [\xi^4 - 2a^2 \xi^2 + a^4 \cos^2 \theta], \quad (4)$$

$$\tau = A_2 \frac{a R \sin \theta}{(\xi^4 - a^2 R^2 \cos^2 \theta)^2} [\xi^4 - a^4 \cos^2 \theta].$$

To draw the stress pattern, the angle α , which the direction of one of the principal stresses makes with the radial direction, must be known

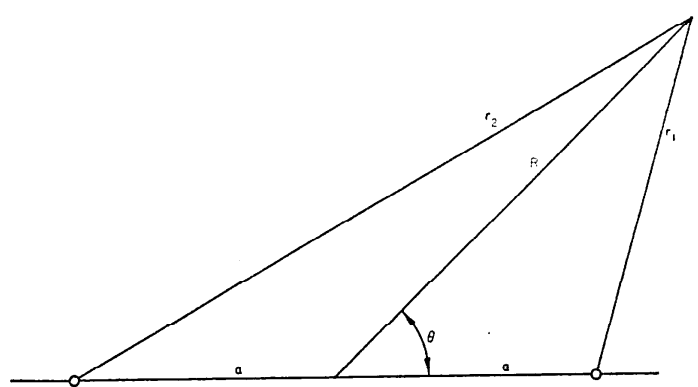


FIGURE 4.—CO-ORDINATE NOTATION USED IN STRESS ANALYSIS

system is taken halfway between the two holes (Fig. 4), the function ϕ is given by

$$\begin{aligned} \phi &= A_1 \ln (r_1 r_2); \\ r_1^2 &= R^2 + a^2 - 2aR \cos \theta; \\ r_2^2 &= R^2 + a^2 + 2aR \cos \theta. \end{aligned} \quad (2)$$

Since $\phi = A_1 \ln r$ satisfies $\nabla^2 \phi = 0$ and $\sigma_R + \sigma_\theta = \nabla^2 \phi$, then

$$\sigma_R + \sigma_\theta = 0.$$

Performing the differentiations indicated by (1), the stress components in a point (R, θ) are given by

$$\begin{aligned} \sigma_R = -\sigma_\theta &= \frac{A_1}{(\xi^4 - a^2 R^2 \cos^2 \theta)^2} \\ &[\xi^6 - \xi^4 a^2 \sin^2 \theta - \xi^2 a^2 R^2 \cos^2 \theta - a^4 R^2 \cos^2 \theta \sin^2 \theta], \\ \tau &= -\frac{A_1 a^2 \sin \theta \cos \theta}{(\xi^4 - a^2 R^2 \cos^2 \theta)^2} \\ &[\xi^4 - 2\xi^2 R^2 + a^2 R^2 \cos^2 \theta], \end{aligned} \quad (3)$$

as a function of the variables. This angle α (Fig. 5) is given by

$$\tan 2\alpha = \frac{2\tau}{\sigma_R - \sigma_\theta} = \frac{\tau}{\sigma_R}. \quad (5)$$

Substituting (3) into (5):

$$\tan 2\alpha = -\frac{a^2 \sin \theta \cos \theta [\xi^4 - 2\xi^2 R^2 + a^2 R^2 \cos^2 \theta]}{[\xi^6 - \xi^4 a^2 \sin^2 \theta - a^2 R^2 \xi^2 \cos^2 \theta - a^4 R^2 \cos^2 \theta \sin^2 \theta]}.$$

It is obvious that as R goes to infinity $\tan 2\alpha$ goes to zero. Hence, the stress trajectories at infinity are parallel to a pencil of rays through the origin, *i.e.*, these rays are asymptotic to the stress trajectories. This is quite as should be expected, for at infinity the two holes act as one.

Singular points in the stress pattern are obtained from indeterminate values of $\tan 2\alpha$

or in other words for values of R and θ which make both σ_R and τ zero. There are only four such points

$$\begin{matrix} R = a & & R = a \\ \theta = 0, \pi & \text{and} & \theta = \pm\pi/2. \end{matrix}$$

The first two correspond with both sources and are, therefore, of little interest.

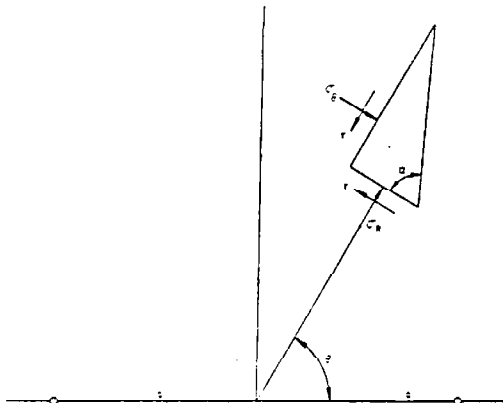


FIGURE 5.—DEFINITION OF THE ANGLE α

The stress pattern can now be drawn by computing values of α for different values of R and θ . In points of the circle $R = a$, the directions of the two principal stresses point toward both sources.

The resulting stress pattern is shown in Figure 6. The magnitude of the two principal stresses σ_p at any point is given by

$$\sigma_p = \pm \sqrt{\sigma_R^2 + \tau^2} = A_1 \frac{\sqrt{\xi^4 - a^2 R^2 \sin^2 \theta}}{\xi^4 - a^2 R^2 \cos^2 \theta} \quad (7)$$

In case of an "image" source of negative sign $\tan 2\alpha$ is given by

$$\tan 2\alpha = \frac{\tan \theta \times (\xi^4 - a^4 \cos^2 \theta)}{\xi^4 - 2\xi^2 a^2 + a^4 \cos^2 \theta} \quad (8)$$

This pattern also possesses the property that the directions of the principal stresses in the circle $R = a$ point toward both sources. It can be shown that the point $R = 0$ is, apart from both sources, the only singular point in the stress pattern. The resulting stress pattern is shown in Figure 7. The magnitude of the principal stress is given by

$$\sigma_p = \pm \frac{A_2 a R}{\xi^4 - a^2 R^2 \cos^2 \theta} \quad (9)$$

In both cases the value of σ_p at singular points is zero, which is immediately apparent from the fact that σ_R and τ are zero. This means that the material in these points is in an unstressed state. But it can be verified that the

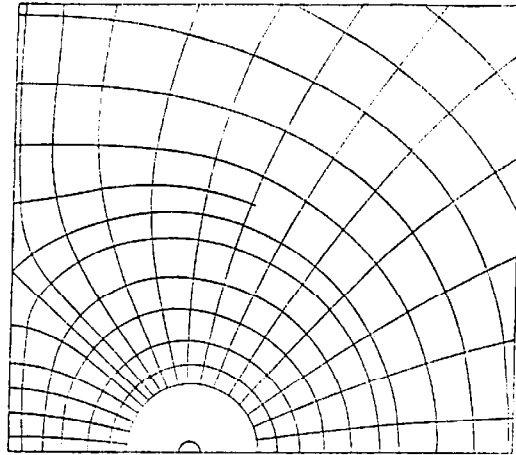


FIGURE 6.—PATTERN OF PRINCIPAL-STRESS TRAJECTORIES CAUSED BY TWO SOURCES OF EQUAL SIGN

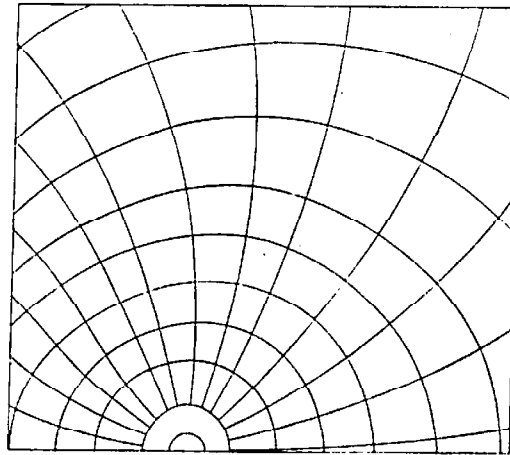


FIGURE 7.—PATTERN OF PRINCIPAL-STRESS TRAJECTORIES CAUSED BY TWO SOURCES OF OPPOSITE SIGN

displacements reach a maximal value in those points. Figures 6 and 7 show that the sign of σ_p changes along a trajectory passing through a singular point.

If the stress components of both cases multiplied by different factors are added, the stress pattern resulting from the case of two sources of different "strength," situated symmetrically with respect to the mountain front, is obtained. However, stress patterns obtained in this manner do not resemble the dike pattern of the Spanish Peaks area.

SUPERPOSITION OF THE REGIONAL-STRESS FIELD

As pointed out in the Introduction, the most probable reason for this discrepancy is that at the time the dikes were injected a regional-stress field existed. The symmetry of the pattern immediately suggests that this regional-stress field was also symmetrical; in particular the eastward curving of the dikes suggests that its direction of greatest principal stress was parallel to the line of symmetry of the pattern. It is not unlikely that this pattern was related to the northward-trending mountain range west of the Peaks. Therefore, it is assumed that this regional-stress pattern can be described by

$$\phi = \frac{B}{2}x^2 + \frac{C}{2}y^2,$$

where

$$\begin{aligned} \sigma_x &= \frac{\partial^2 \phi}{\partial y^2} = C, & \sigma_y &= \frac{\partial^2 \phi}{\partial x^2} = B, \\ \tau_{xy} &= -\frac{\partial^2 \phi}{\partial x \partial y} = 0, \end{aligned}$$

or expressed in polar co-ordinates:

$$\begin{aligned} \phi &= \frac{B}{2}R^2 \cos^2 \theta + \frac{C}{2}R^2 \sin^2 \theta, \\ \sigma_R &= B \sin^2 \theta + C \cos^2 \theta, \\ \sigma_\theta &= B \cos^2 \theta + C \sin^2 \theta, \\ \tau &= \frac{B - C}{2} \sin 2\theta. \end{aligned} \tag{10}$$

If expressions (3) and (10) are added the stress pattern caused by the two superposed stress patterns can be determined in the same manner as before. A stress pattern closely resembling the stress pattern of the dikes radiating from the Spanish Peaks will result.

Computations were carried out for the positive case $C = 2B$; $B = A_1/a^2$. In the superposed system the principal stress in the x direction (normal to the mountain front) is twice as great as the stress in the y direction, and its magnitude is equal to the magnitude of the principal local stress in the origin in the case of two symmetric holes with equal hydrostatic pressure, as may be seen when $R = 0$ is substituted in (7).

The stress components and $\tan 2\alpha$ are then given by

$$\begin{aligned} \sigma_R &= \frac{A}{(\xi^4 - a^2 R^2 \cos^2 \theta)^2} \\ &[\xi^6 - \xi^4 a^2 \sin^2 \theta - \xi^2 a^2 R^2 \cos^2 \theta - a^4 R^2 \cos^2 \theta \sin^2 \theta] \\ &\quad + \frac{A}{a^2} \sin^2 \theta + \frac{2A}{a^2} \cos^2 \theta. \\ \sigma_\theta &= \frac{-A}{(\xi^4 a^2 R^2 \cos^2 \theta)^2} \\ &[\xi^6 - \xi^4 a^2 \sin^2 \theta - \xi^2 a^2 R^2 \cos^2 \theta - a^4 R^2 \cos^2 \theta \sin^2 \theta] \\ &\quad + \frac{A}{a^2} \cos^2 \theta + \frac{2A}{a^2} \sin^2 \theta. \\ \tau &= -\frac{A a^2 \cos \theta \sin \theta}{(\xi^4 - a^2 R^2 \cos^2 \theta)} [\xi^4 + a^2 R^2 \cos^2 \theta - 2\xi^2 R^2] \\ &\quad - \frac{A}{2a^2} \sin 2\theta. \end{aligned}$$

$$\tan 2\alpha = \frac{-2a^4 \cos \theta \sin \theta [\xi^4 + a^2 R^2 \cos^2 \theta - 2\xi^2 R^2] - \sin 2\theta [\xi^4 - a^2 R^2 \cos^2 \theta]^2}{2a^2 [\xi^6 - \xi^4 a^2 \sin^2 \theta - \xi^2 a^2 R^2 \cos^2 \theta - a^4 R^2 \cos^2 \theta \sin^2 \theta] + \cos 2\theta [\xi^4 - a^2 R^2 \cos^2 \theta]^2}$$

As R approaches infinity $\tan 2\alpha = -\tan 2\theta$, or the stress trajectories coincide at infinity with the stress trajectories of the superposed system. The resulting stress pattern is shown in Figure 8. In all of its important aspects, this pattern resembles that of the dikes radiating from the Spanish Peaks.

If the holes have opposite signs, expressions for the stresses are obtained in the same manner as above by addition of (4) and (10). For $\tan 2\alpha$ we obtain

$$\tan 2\alpha = \frac{2a^3 R \sin \theta [\xi^4 - a^4 \cos^2 \theta] - \sin 2\theta [\xi^4 - a^2 R^2 \cos^2 \theta]^2}{2a^3 R \cos \theta [\xi^4 - 2a^2 \xi^2 + a^4 \cos^2 \theta] + \cos 2\theta [\xi^4 - a^2 R^2 \cos^2 \theta]^2}$$

The corresponding stress pattern is shown in Figure 9. Figures 8 and 9 show a remarkable resemblance. That the difference between the patterns is greatest near the mountain front is not surprising, as the boundary conditions along that front are different, whereas at infinity they are the same. The question of which case provides the best fit to the Spanish Peaks dike system is not very important.

As already pointed out, the boundary conditions along the mountain front are uncertain, and, therefore, neither of the two cases will be quite correct. The map published by Knopf (1936, Fig. 1) shows that most of the westward-

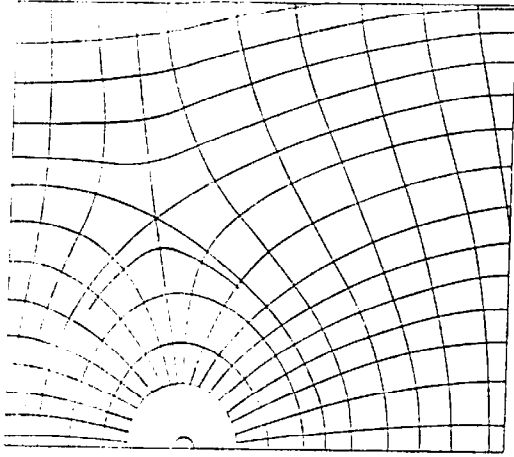


FIGURE 8.—PATTERN OF PRINCIPAL-STRESS TRAJECTORIES CAUSED BY TWO SOURCES OF EQUAL SIGN AND SUPERPOSED REGIONAL-STRESS SYSTEM

trending dikes end at an acute angle to the mountain front, which is slightly bent inward. This may indicate that the assumption of an image of opposite sign is the better one. It is important to note that if the ratio of the constants A_1 , A_2 , B , C is changed a better fit between computed and observed pattern may be obtained. So a large value of B with respect to A_1 , A_2 , and C gives a stress pattern in which the stress trajectories diverging from the Spanish Peaks and asymptotic to eastward-trending trajectories of the regional stress system are compressed into a narrow band. If the value of B is decreased, this band can be broadened. If the values of A_1 and A_2 are increased and B and C are kept constant, the influence of the local stress system is emphasized. Therefore, if the right ratios of A_1 , A_2 , B , and C are selected, it is possible to obtain a better fit between computed and observed pattern than is shown in Figures 6, 7, 8, and 9.

Of much interest are the dikes which are normal to the mountain front and do not originate in the center of West Spanish Peak. Some of these dikes are crossed by, and some cross, dikes converging to the Spanish Peaks. Thus not all dikes are formed contempora-

neously. It seems probable, however, that both are the result of the same igneous mechanism.

Inspection of the other dike systems mentioned in the Introduction shows that most are regularly radial. Only the map of dikes sur-

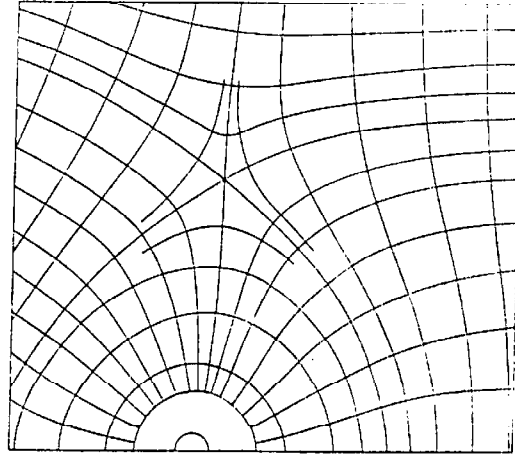


FIGURE 9.—PATTERN OF PRINCIPAL-STRESS TRAJECTORIES CAUSED BY TWO SOURCES OF OPPOSITE SIGN AND SUPERPOSED REGIONAL-STRESS SYSTEM

rounding Sunlight Volcano (Parsons, 1939, p. 14) shows a pattern with very slight curvature. This indicates that, in these cases, the boundary condition imposed by the mountain front was absent.

DISPLACEMENTS

The order of magnitude of the elastic displacements is easily found if no regional stress is present. If U_R is the radial displacement, E Young's modulus, and ν Poisson's ratio we have:

$$\frac{\partial U_R}{\partial R} = -\frac{1}{E}(\sigma_R - \nu\sigma_\theta) = \frac{1 + \nu}{E} \frac{\partial^2 \phi}{\partial R^2};$$

or

$$U_R = \frac{1 + \nu}{E} \frac{\partial \phi}{\partial R} + f(\theta); \quad (11)$$

or

$$U_R = \frac{2(1 + \nu)}{E} A_1 R \left\{ \frac{R^2 + a^2 - 2a^2 \cos^2 \theta}{(R^2 + a^2)^2 - 4a^2 R^2 \cos^2 \theta} \right\} + f(\theta).$$

Quantity U_R must be zero, for $R = 0$, hence $f(\theta) \equiv 0$. The value of U_R along the mountain

front is obtained if $\theta = \pi/2$ is substituted in (11):

$$U_R = \frac{2(1 + \nu)}{E} \cdot \frac{A_1 R}{a^2 + R^2}$$

which reaches its greatest absolute value if $R = a$. Therefore:

$$U_R(\text{max}) = \frac{A_1}{2\mu a};$$

where μ is the coefficient of rigidity for surface rocks. To obtain an estimate of this maximal displacement, probable values of the quantities involved must be substituted. The value of μ for rocks under surface conditions is about 3.5×10^{11} dynes/cm². Hence if A/a^2 represents a stress of about 10^9 dynes/cm², which is undoubtedly high, and as a , the distance from West Spanish Peak to the mountain range is about 1.4×10^6 cm, A is approximately 2×10^{21} dynes. Thus the magnitude of U_R is found to be

$$\frac{2 \times 10^{21}}{7 \times 10^{11} \times 1.4 \times 10^6} = 2 \times 10^3 \text{ cm.}$$

However, it is not very likely that A had this very great order of magnitude. Stresses of the order of 10^5 dynes/cm² are more likely to produce overthrusting and rupturing. Even if the value of μ was taken too large, then still the fact that the value of A is rather high makes the value of 2×10^3 cm a likely one. However, when an additional stress is present, all the displacements are different. The displacements caused by the superposed stress system alone are given by

$$U_x = \frac{1}{E} (2 - \nu) Bx,$$

$$U_y = \frac{1}{E} (1 - 2\nu) By.$$

(A surface element in the origin of the coordinate system is considered fixed.) These

displacements are greater with increase in distance from the origin. Therefore, the total displacements far from the sources are due only to the superposed stress field. To what extent the elastic shortening of the area is compensated by the dilation caused by the injected matter of the dikes is a matter of conjecture. The data, on which such a calculation must be based, are too inaccurately known.

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