GG611
Structural Geology Section

Steve Martel
POST 805
smartel@hawaii.edu

Lecture 4

Rheology and Mechanics

Stresses Control How Rock Fractures

http://hvo.wr.usgs.gov/kilauea/update/images.html

Outline

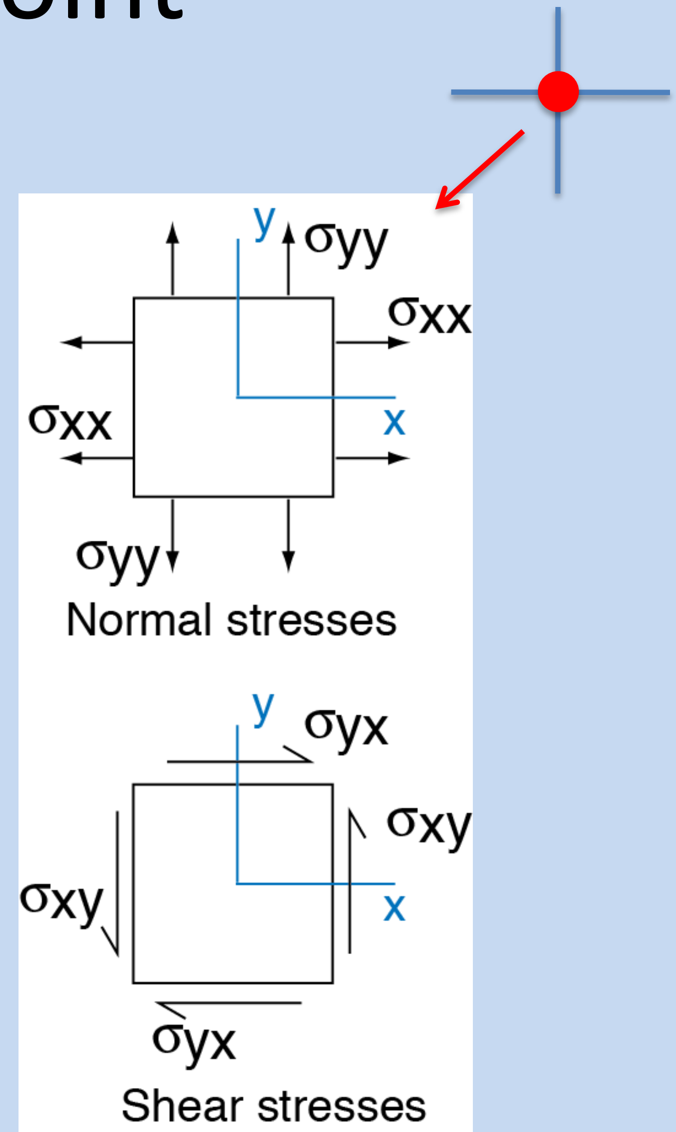
I. Stress at a point
II. Strain at a point
III. Rheology
IV. Mechanics of fractures and folds
V. Appendix: Eigenvectors, eigenvalues, and principal stresses

Stress at a Point

- Stresses refer to balanced internal "forces (per unit area)". They differ from force vectors, which, if unbalanced, cause accelerations
- "On -in convention": The stress component σ_{ij} acts ***on*** the plane normal to the i-direction and acts ***in*** the j-direction

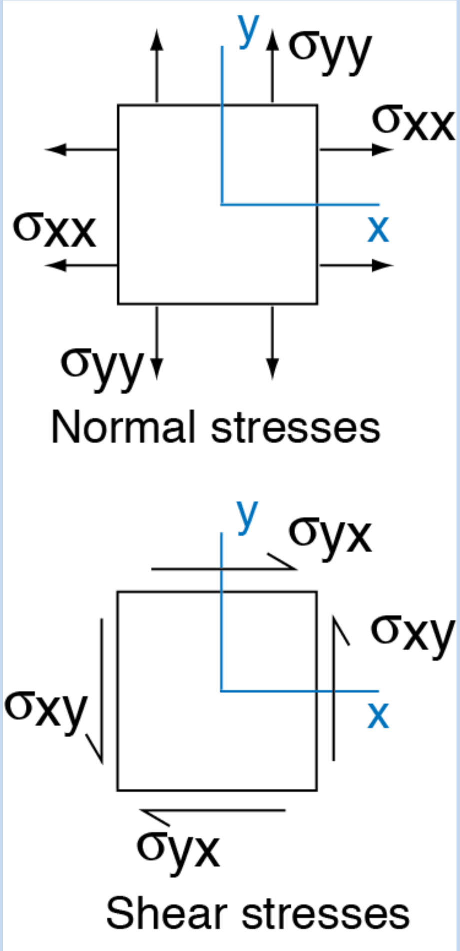
  1 Normal stresses: i=j

  2 Shear stresses: i≠j



Stress at a Point

- Dimensions of stress: force/unit area
- Convention for stresses
  - Tension is positive
  - Compression is negative
  - Follows from on-in convention
  - Consistent with most mechanics books
  - Counter to most geology books

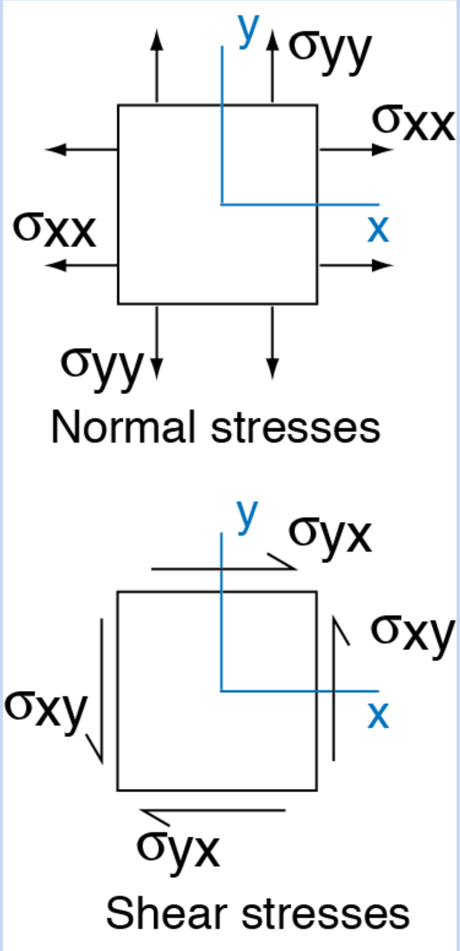


Stress at a Point

- $$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$$ 2-D
  4 components

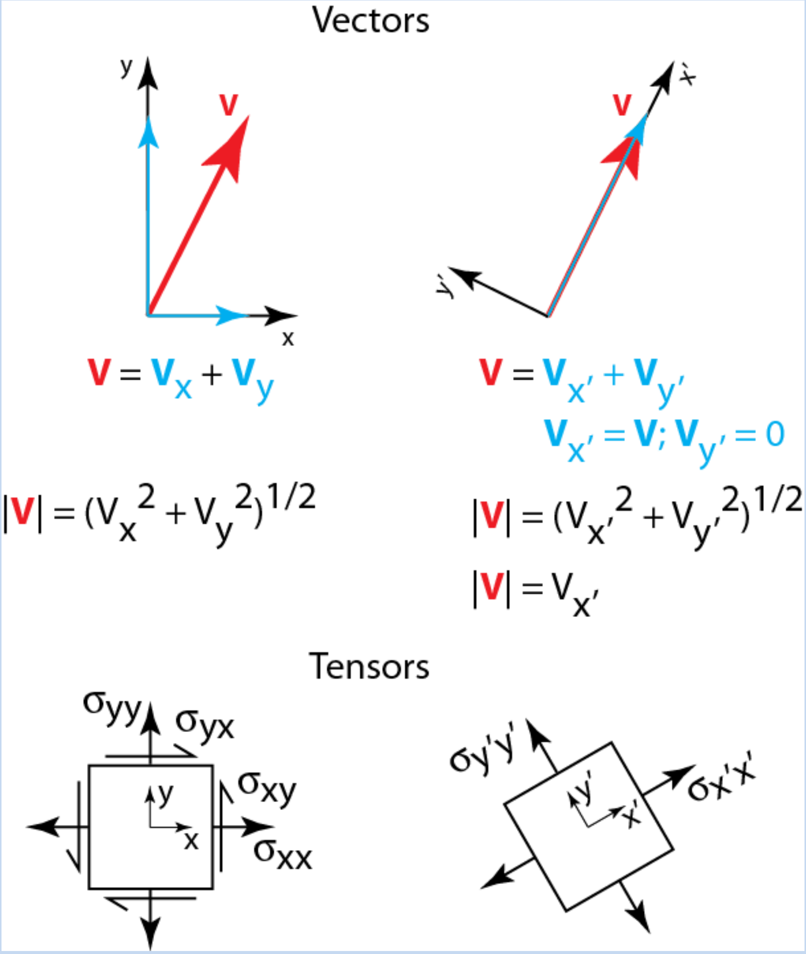
  $$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$ 3-D
  9 components

- For rotational equilibrium, $\sigma_{xy} = \sigma_{yx}$, $\sigma_{xz} = \sigma_{zx}$, $\sigma_{yz} = \sigma_{zy}$; stress matrix is symmetric
- In nature, the state of stress can (and usually does) vary from point to point



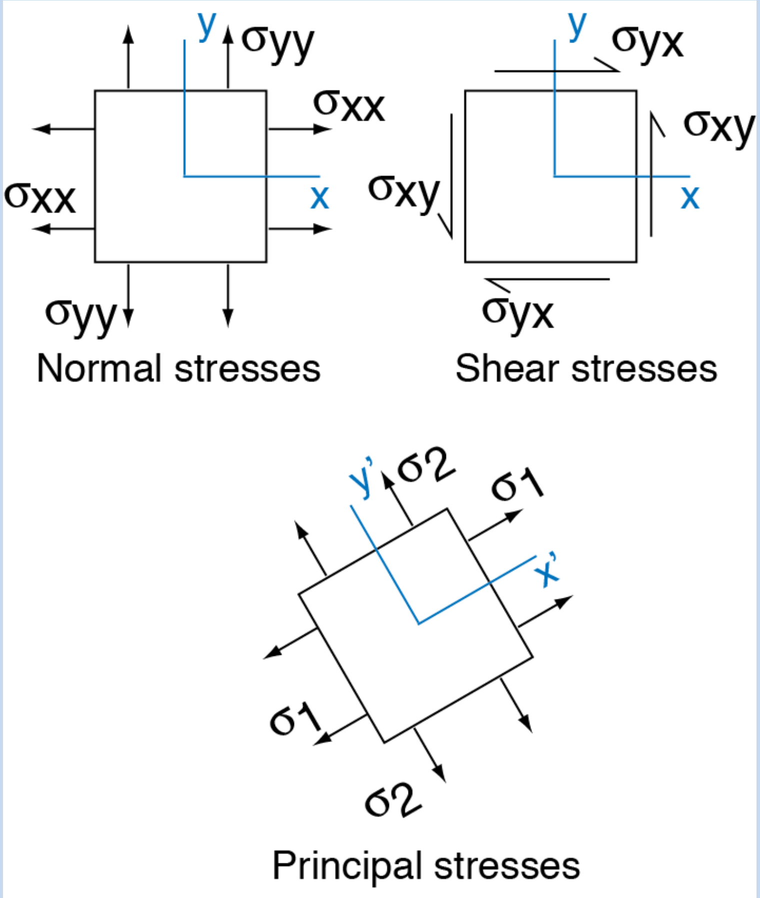
Stress at a Point

- Analogy with vectors
  - The components of a vector vary with the reference frame, even though the vector does not
  - For certain reference frame orientations, some vector/tensor components are zero
  - The non-zero components are meaningful and illuminating in a reference frame where some components are zero



Principal Stresses

- Have magnitudes and orientations
- Principal stresses act on planes which feel no shear stress
- Principal stresses are normal stresses
- Principal stresses act on perpendicular planes owing to symmetry of stress tensor
- The maximum, intermediate, and minimum principal stresses are usually designated σ_1, σ_2, and σ_3, respectively
- Designated by a single subscript

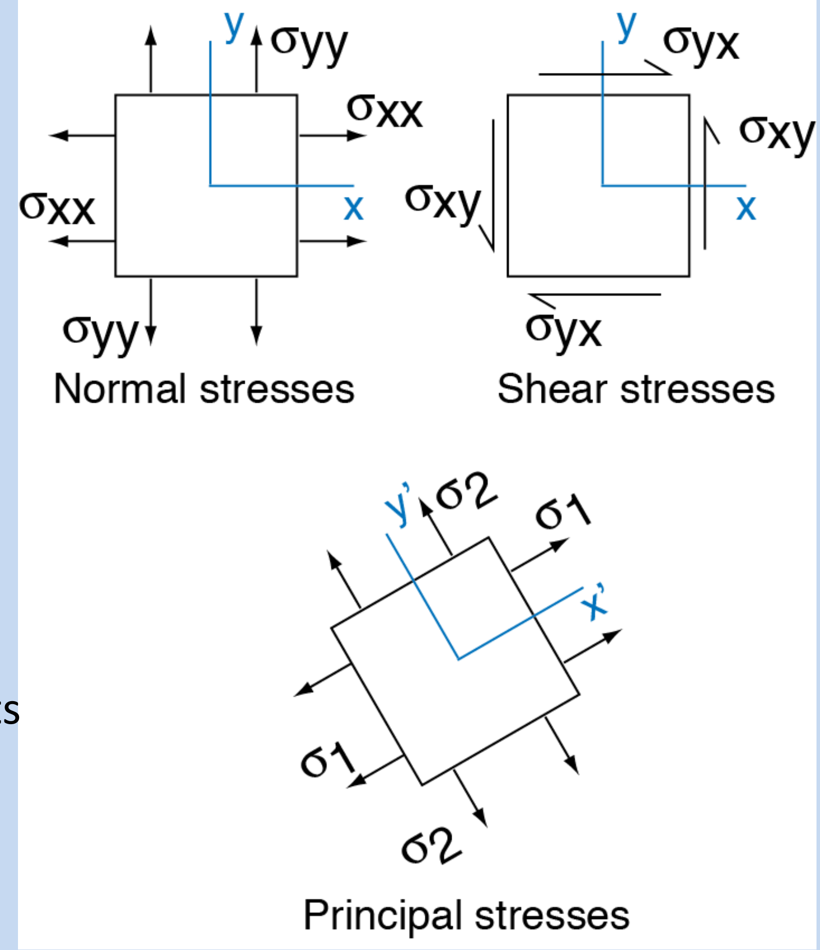


Principal Stresses

F **Principal stresses represent the stress state most simply**

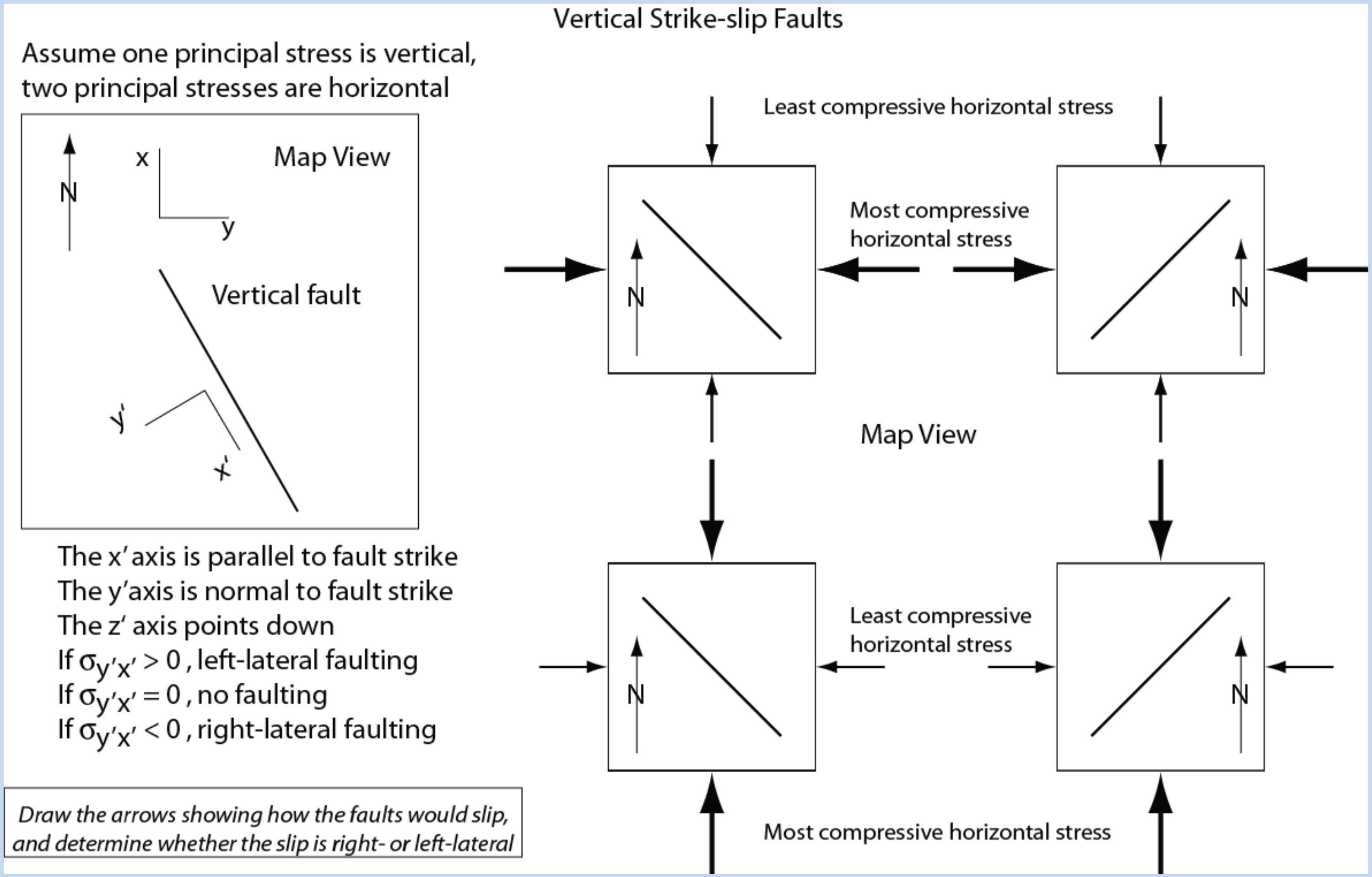
G $$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$ 2-D, 2 components

H $$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$ 3-D, 3 components



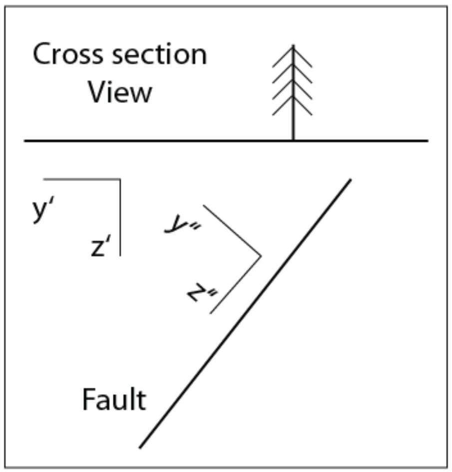
* If $\sigma_1 = \sigma_2 = \sigma_3$, the state of stress is called isotropic. This occurs beneath a still body of water.

Application: Vertical Strike-slip Faults



Application: Dip-slip Faults

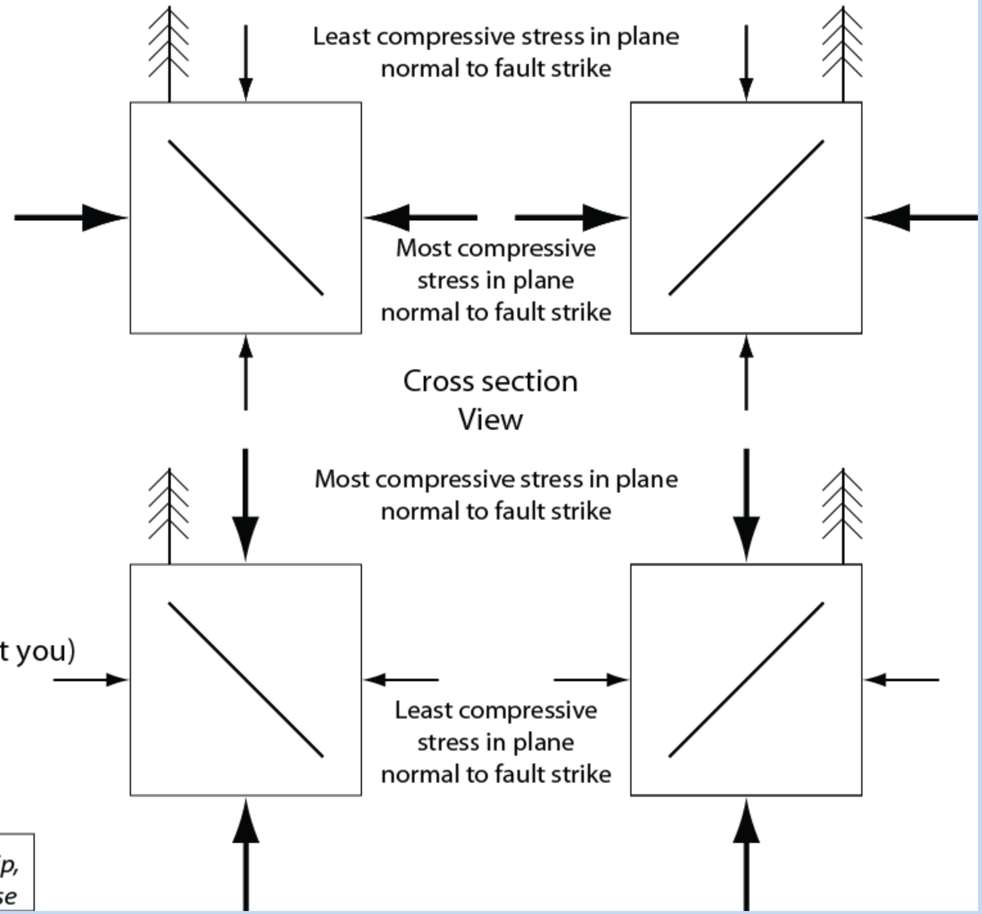
Assume one principal stress is vertical, two principal stresses are horizontal, and the horizontal principal stresses are parallel and normal to fault strike.



The x'' axis is parallel to fault strike (at you)
The y'' axis is normal to the fault
The z'' azis points down-dip
If $\sigma_{y''z''} > 0$, lnormal faulting
If $\sigma_{y''z''} < 0$, reverse faulting

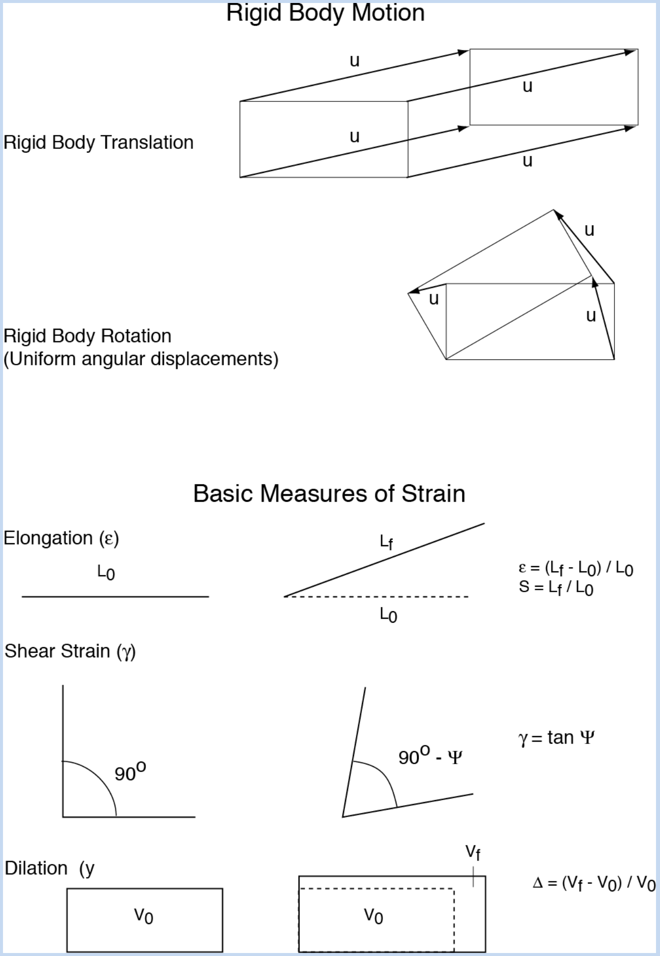
Draw the arrows showing how the faults would slip, and determine whether the slip is normal or reverse

Non-vertical Dip-slip Faults

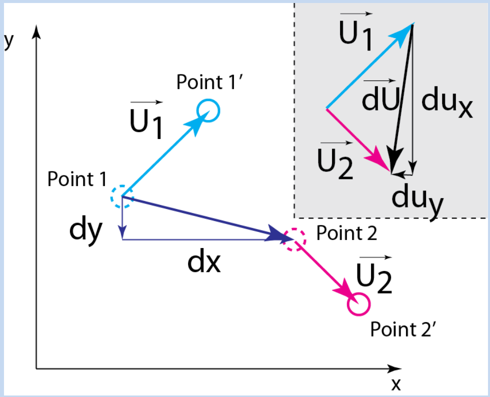


Strain at a point: Basic Concepts

- Normal strain (ε): change in relative line length
- Shear strain (γ): change in angle between originally perpendicular lines
- Volumetric strain (Δ): change in relative volume
- Based on rates of change of displacement as a function of position
- Strains are <u>dimensionless</u>



Finite strain at a point



- Chain rule relates difference in initial positions (dx and dy) of neighboring points to difference in displacements (dx' and dy')

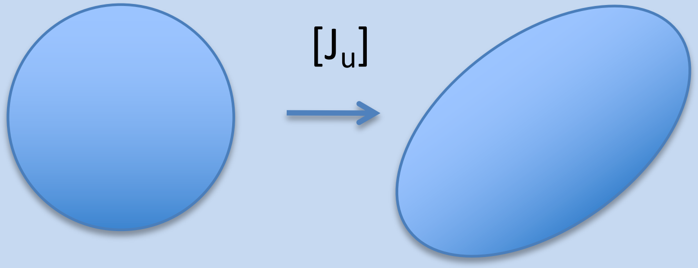
$$du_x = \frac{\partial u_x}{\partial x}dx + \frac{\partial u_x}{\partial y}dy$$

$$du_y = \frac{\partial u_y}{\partial x}dx + \frac{\partial u_y}{\partial y}dy$$

- At a point, displacement derivatives are constants
- Matrix relating difference in displacement [dU] to difference in initial position [dX] contains constants

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} \Rightarrow [dU] = [J_u][dX]$$

- Unit circles (spheres) deform to ellipses (ellipsoids)



Infinitesimal strain at a point

- In infinitesimal strain, displacement derivatives are small relative to 1
- The infinitesimal strain matrix contains normal strains (on main diagonal) and shear strains (off-diagonal terms)
- The infinitesimal strain matrix is symmetric
- Infinitesimal principal strains are perpendicular

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix}$$

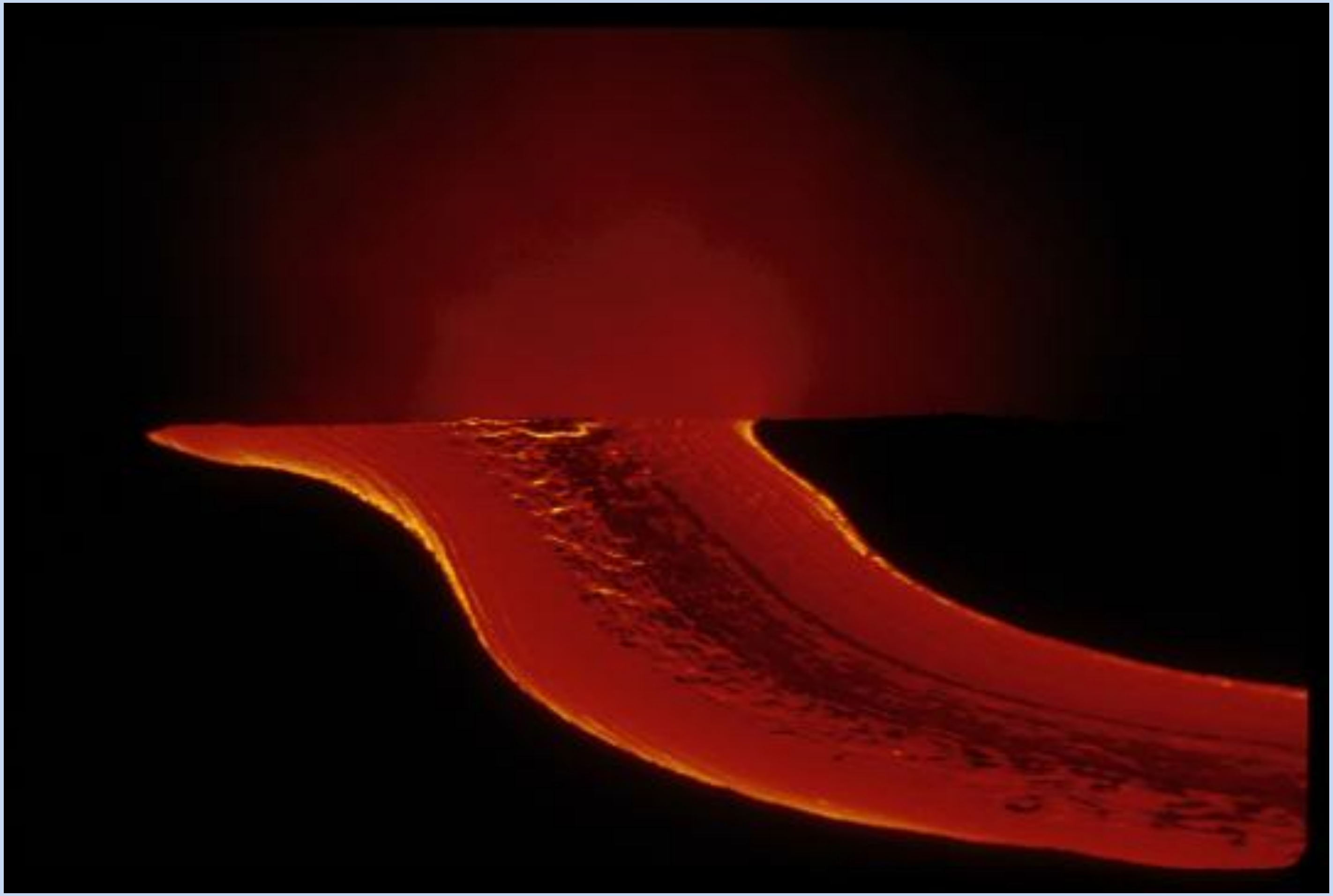
$$= \begin{bmatrix} \dfrac{\partial u_x}{\partial x} & \dfrac{1}{2}\left(\dfrac{\partial u_x}{\partial y} + \dfrac{\partial u_y}{\partial x}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u_x}{\partial y} + \dfrac{\partial u_y}{\partial x}\right) & \dfrac{\partial u_y}{\partial y} \end{bmatrix}$$

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix}$$

Rheology

- The relationship between the flow or deformation of a material and the loads causing the flow or deformation
- Typically relates stress to strain, or stress to strain rate
- In reality, rheology is a complicated functions of pressure, temperature, fluid content, etc.
- We generally use simple rheologic models

Rheology: Viscous (fluid) behavior
Shear strain rate is proportional to shear stress



http://manoa.hawaii.edu/graduate/content/slide-lava

Rheology: Ductile (plastic) behavior
No strain until stress reaches a critical level

http://hvo.wr.usgs.gov/kilauea/update/images.html

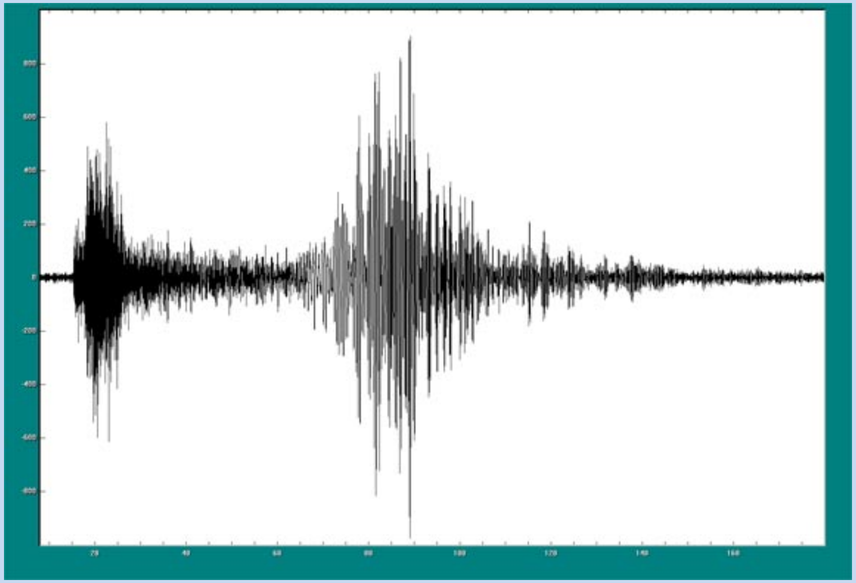
http://www.hilo.hawaii.edu/~csav/gallery/scientists/LavaHammerL.jpg

Rheology: Brittle behavior (fracture)
Deformation is discontinuous

Rheology: Linear elastic behavior

- Deformation reverses when stress is relieved
- Stress and infinitesimal strain are linearly related
- Principal strains and principal stresses are parallel in isotropic materials

$$[\sigma] = [C][\varepsilon]$$



https://thegeosphere.pbworks.com/w/page/24663884/Sumatra

http://www.earth.ox.ac.uk/__data/assets/image/0006/3021/seismic_hammer.jpg

Mechanics of Fractures and Folds

- Displacement and stresses over a region can be obtained by solving boundary value problems
- Requirements
  - Body geometry
  - Boundary conditions (e.g., stress components or displacements acting *on* a boundary surface)
  - Rheology
  - Governing equation(s) for equilibrium and compatibility
  - General solution
  - Specific solution that honors boundary conditions

Geologic Problem: Radiating Dikes

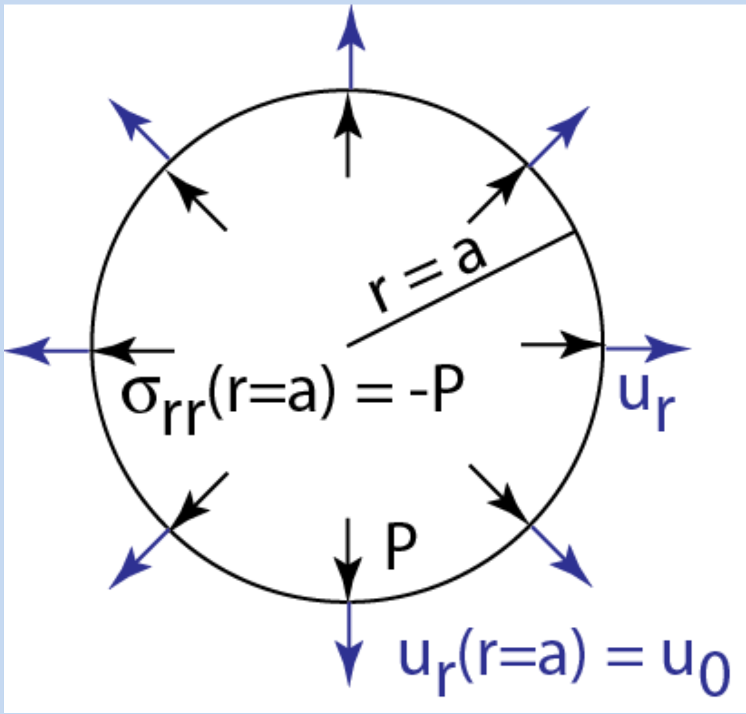
Shiprock, New Mexico

Aerial view showing radial dikes

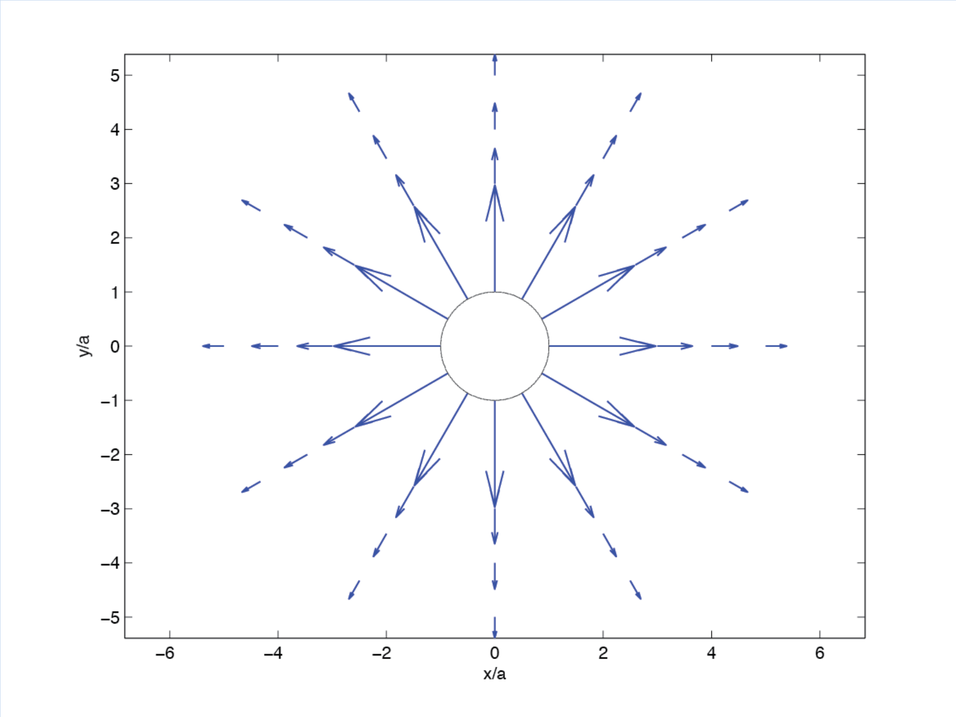
Both images from
http://en.wikipedia.org/wiki/Shiprock#Images

Displacement Field Around a Pressurized Hole in an Elastic Plate

Geometry and boundary conditions



Displacement field

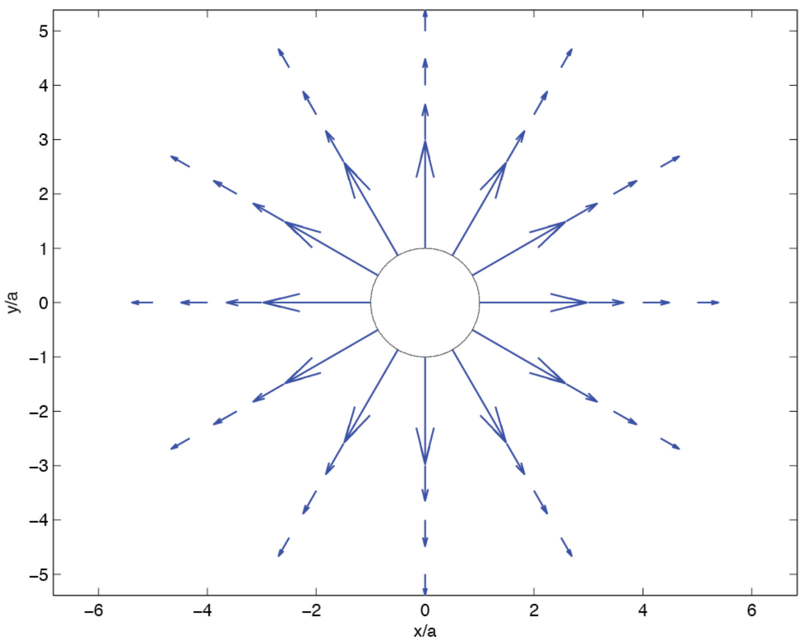


For $u_0 > 0$

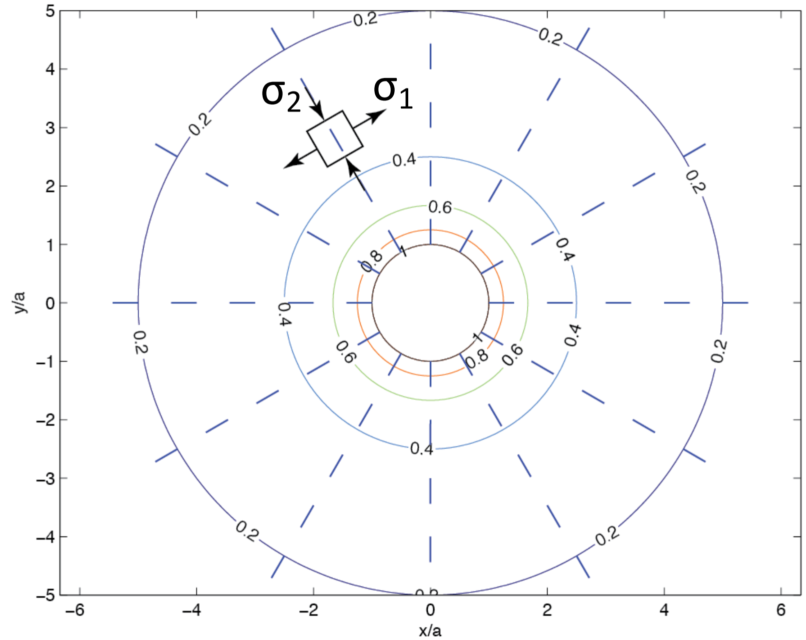
$$(u_r/u_0) = (a/r)$$

Displacement and Stress Fields Around a Pressurized Hole

Displacement field



Stress field



For $u_0 > 0: (u_r/u_0) = (a/r)$

For $P < 0: (\sigma_1/P) = -(a/r)^2 \quad (\sigma_2/P) = (a/r)^2$

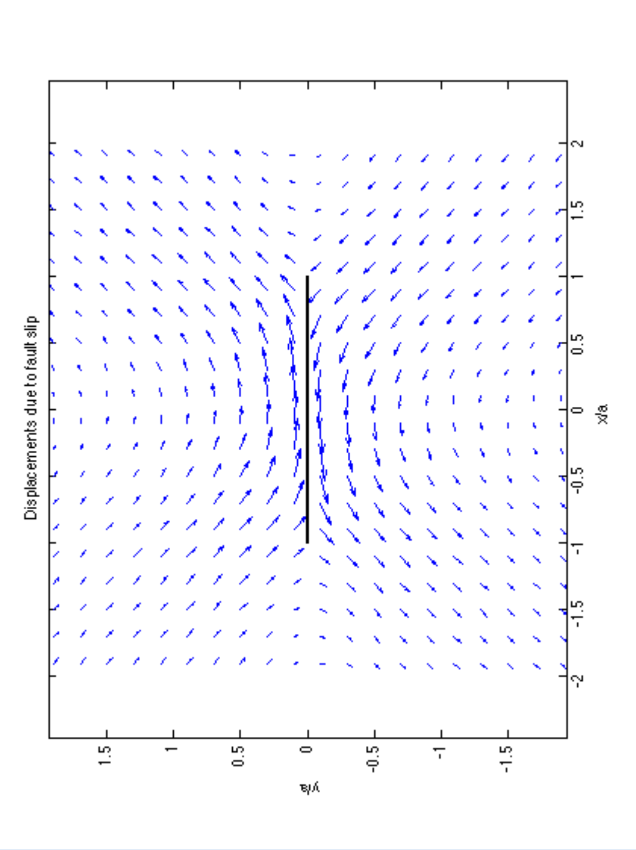
Dikes Open in Direction of Most Tensile Stress, Propagate in Plane Normal to Most Tensile Stress

http://hvo.wr.usgs.gov/kilauea/update/images.html

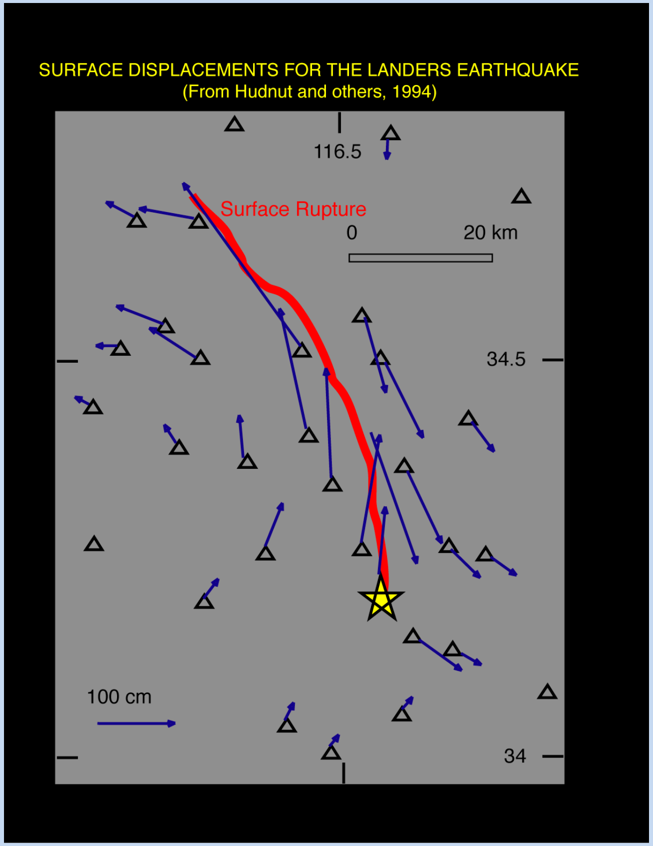
Modeled vs. Measured Displacement Fields Around a Fault

Model for Frictionless, 2D Fault in an Elastic Plate

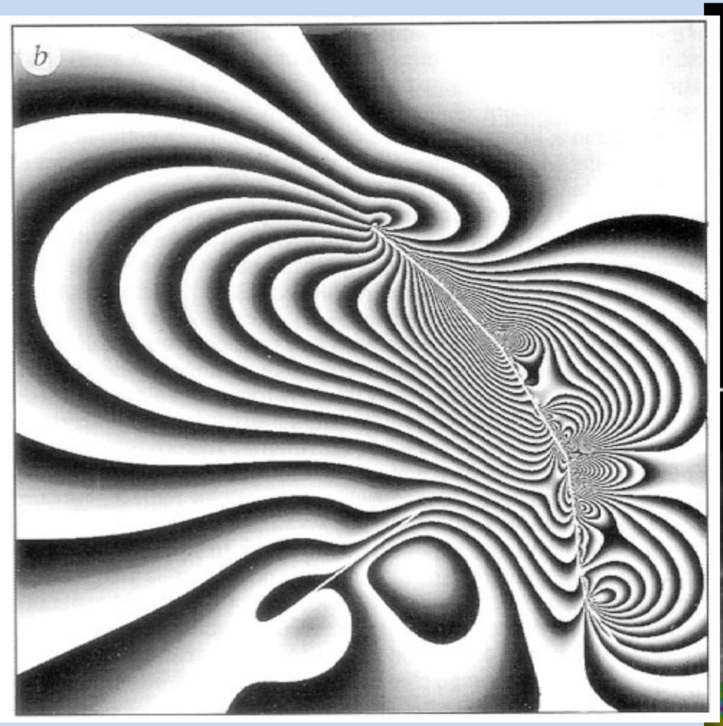
Model Displacement field



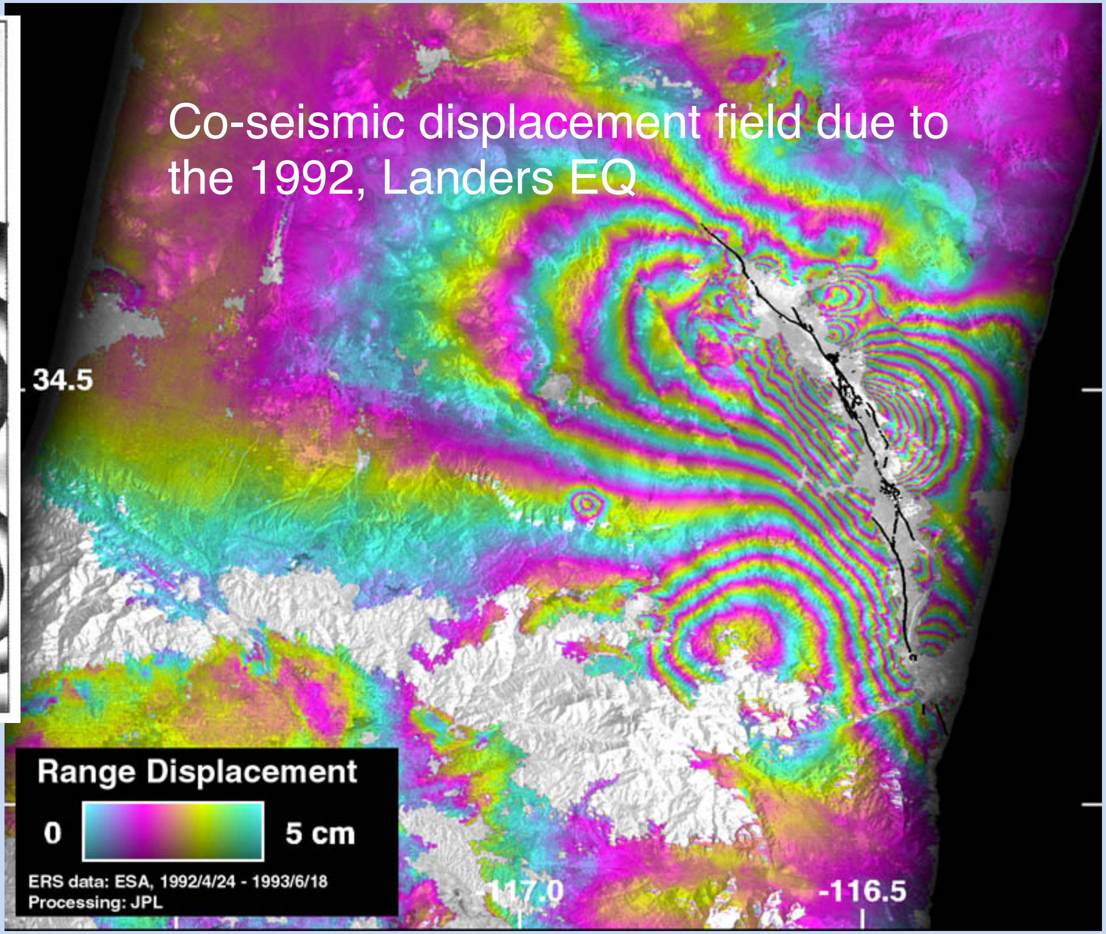
GPS Displacements, Landers 1992



Modeled vs. Measured Co-seismic Displacement, Landers Earthquake



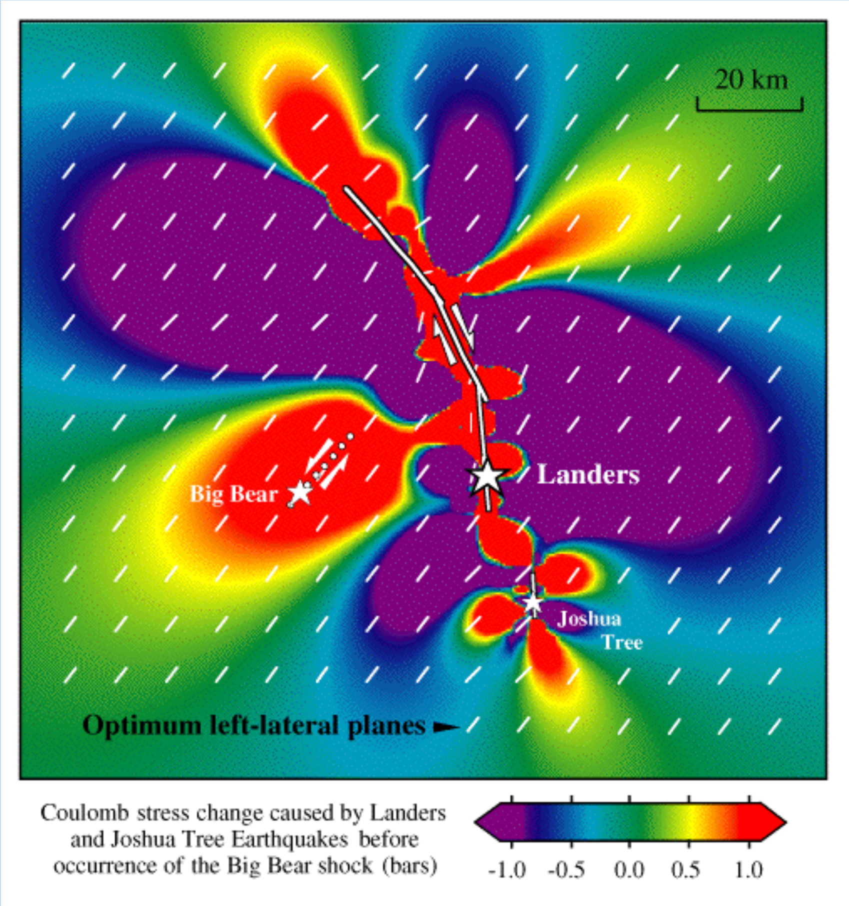
Co-seismic deformation can be modeled using elastic dislocation theory



(based on Massonnet et al., 1993)

Predicted "Coulomb" Stress Changes Caused by 1992 Landers Earthquake, California

- From King et al., 1994 (Fig. 11)
- Coulomb stress change caused by the Landers rupture. The left-lateral ML=6.5 Big Bear rupture occurred along dotted line 3 hr 26 min after the Landers main shock. The Coulomb stress increase at the future Big Bear epicenter is 2.2-2.9 bars.

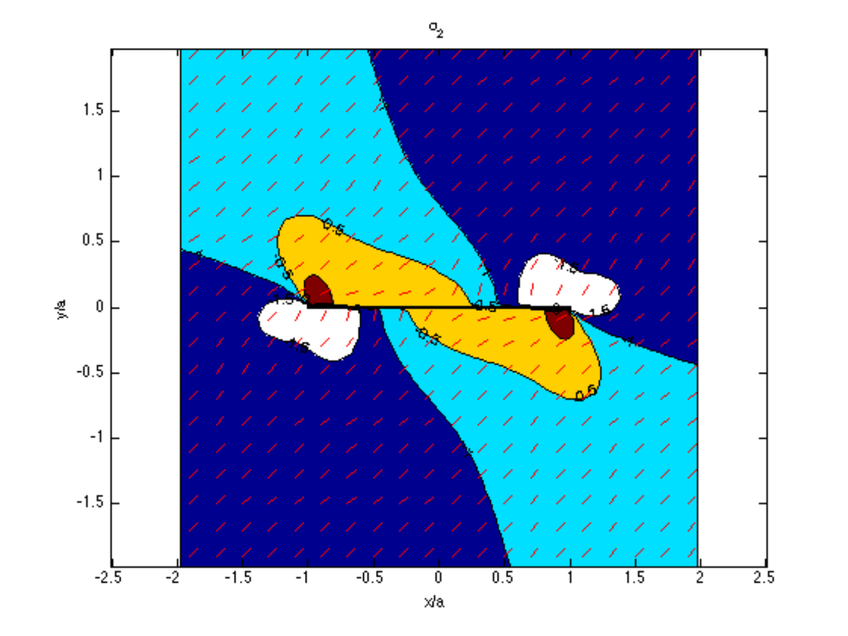


http://earthquake.usgs.gov/research/modeling/papers/landers.php

Stress Fields Around a Frictionless, 2D Model Fault in an Elastic Plate vs. Observations

Model stress field: Most tensile stress & stress trajectories

Tail cracks at end of left-lateral strike-slip fault



Note location and orientation of “tail cracks”

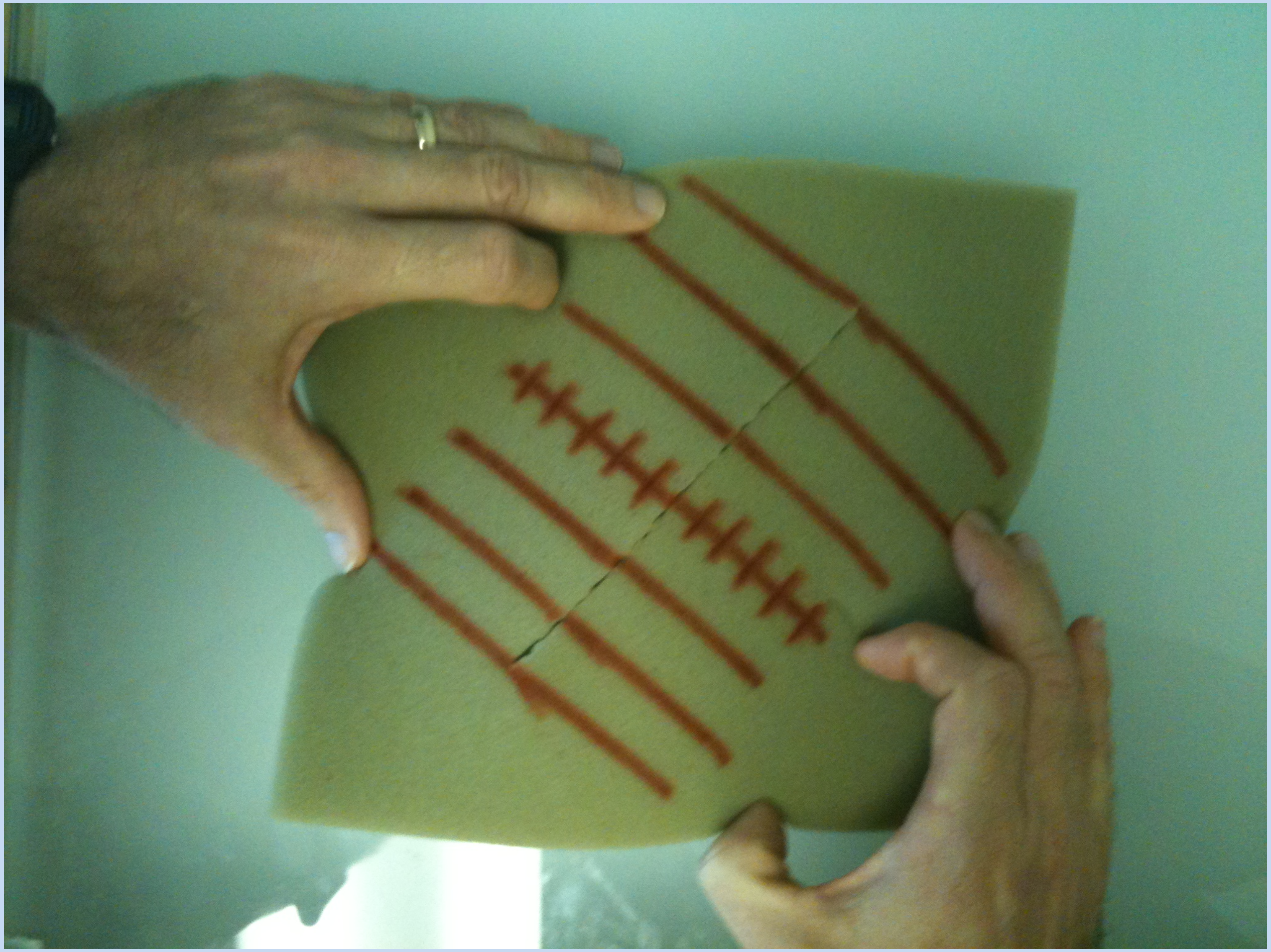
Folding Along a Fault

From Grasemann et al., 2005

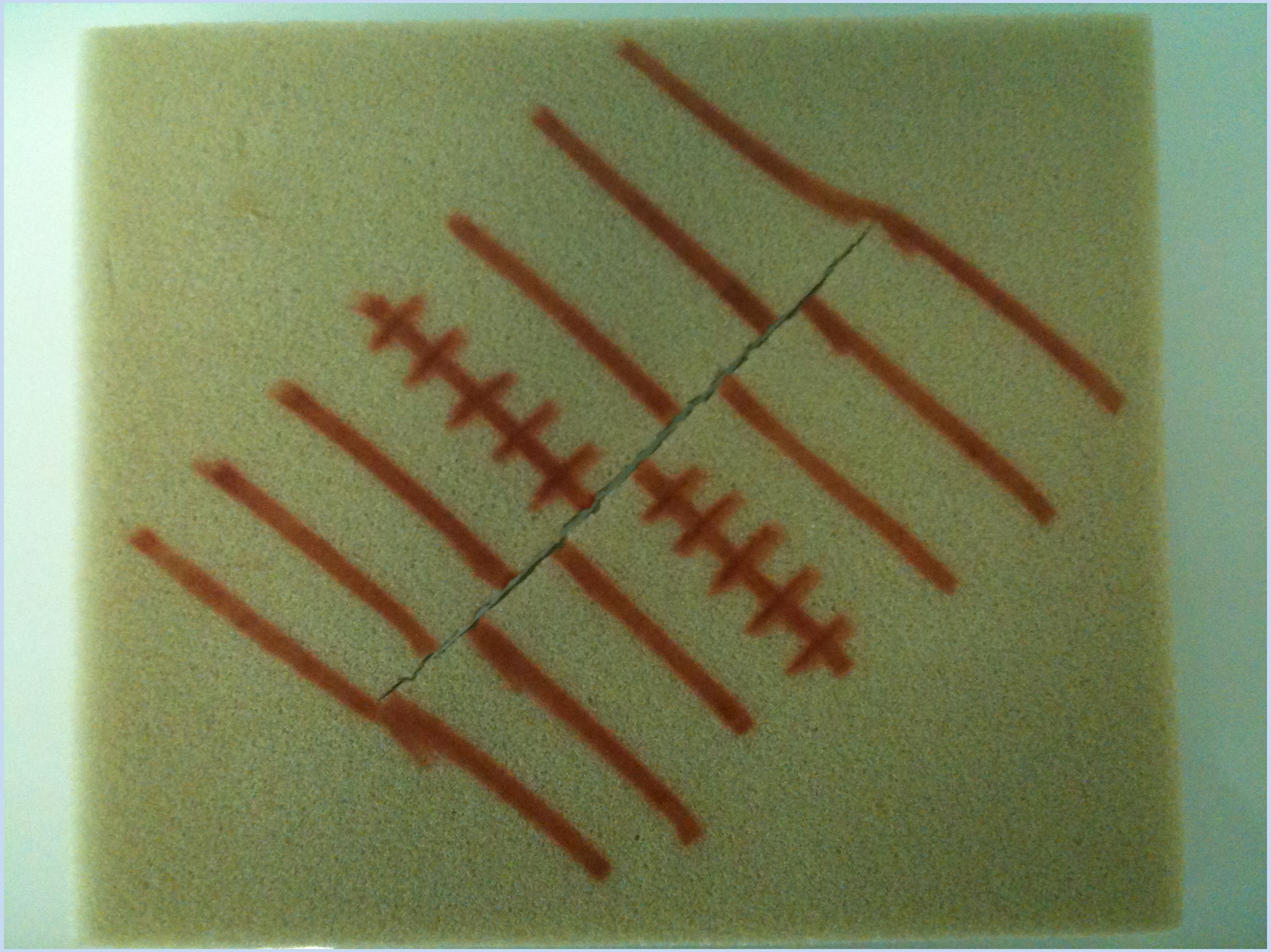
Folding Along a Fault, Koae Fault System

From Martel and Langley, 2006

"Fault" in Foam Rubber, Before Slip

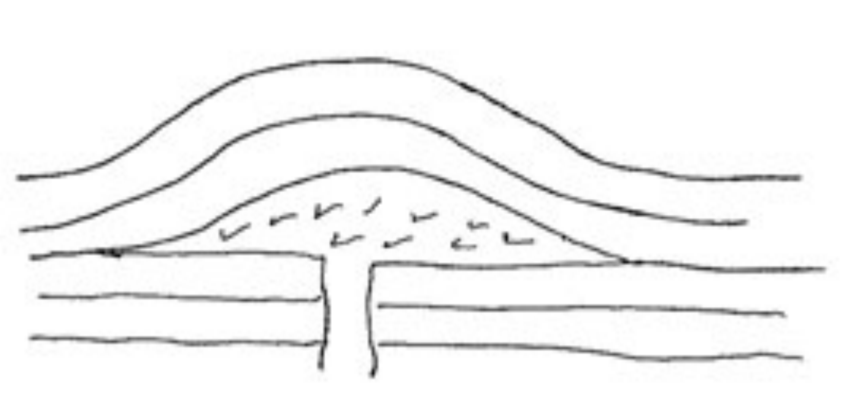


"Fault" in Foam Rubber, After Slip



Other Mechanisms for Folding

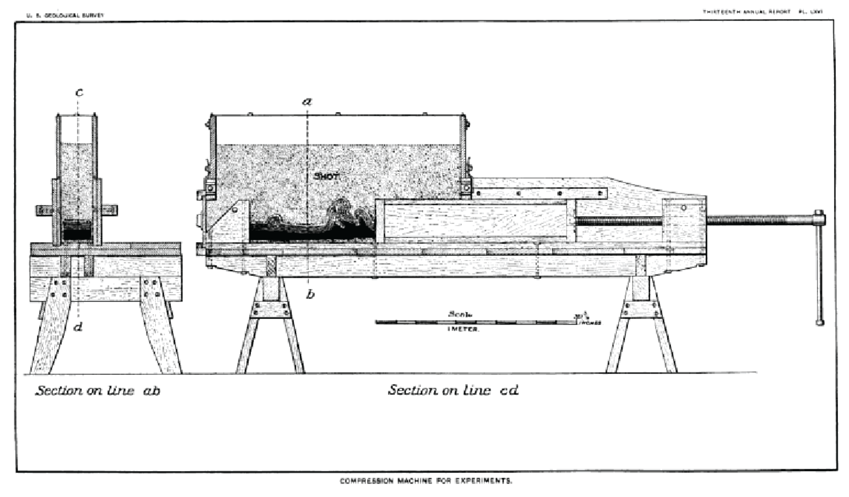
Flexure over intrusions



GK Gilbert's first sketch of a laccolith

http://pangea.stanford.edu/~annegger/images/colorado%20plateau/laccolith_sketch.jpg

From lateral shortening



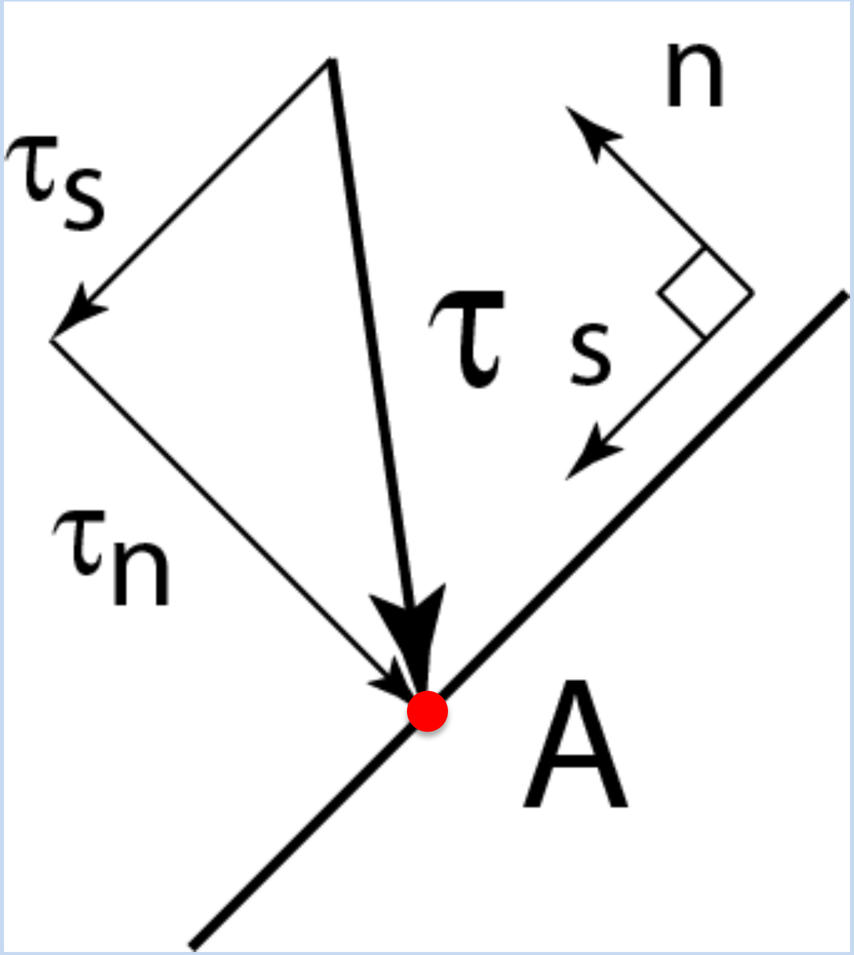
Experimental device of Bailey Willis

From Willis, 1894

Appendix: Eigenvectors, eigenvalues and principal stresses

Traction Vector on a Plane

- Stress vector (traction)
  - $\vec{\tau} = \lim_{A \to 0} \vec{F} / A$
  - Traction vectors can be added as vectors
  - A traction vector can be resolved into normal (τ_n) and shear (τ_s) components
    - A normal traction (τ_n) acts perpendicular to a plane
    - A shear traction (τ_s) acts parallel to a plane
  - Local reference frame
    - n-axis is normal to plane
    - s-axis is parallel to plane

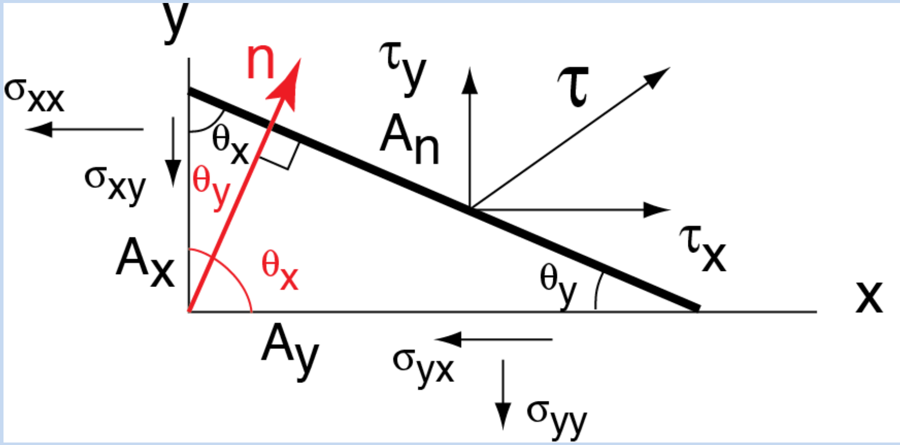


Cauchy's Formula

- Transforms stress state at a point to the traction acting on a plane with normal $\vec{\mathbf{n}}$
- Transforms normal vector $\vec{\mathbf{n}}$ to the traction vector $\vec{\boldsymbol{\tau}}$
- $\tau_j \quad = \quad n_i \quad \sigma_{ij}$

τ_j	n_i	σ_{ij}
Traction component that acts ***in*** the j-direction	Dimensionless weighting factor (cosine between the n- and i- directions;)	Stress component that acts on a plane with its normal in the j-direction, and that acts ***in*** the j-direction

- Expansion (2D)
  - $\tau_x = n_x\sigma_{xx} + n_y\sigma_{yx}$
  - $\tau_y = n_y\sigma_{xy} + n_y\sigma_{yy}$



Cauchy's Formula

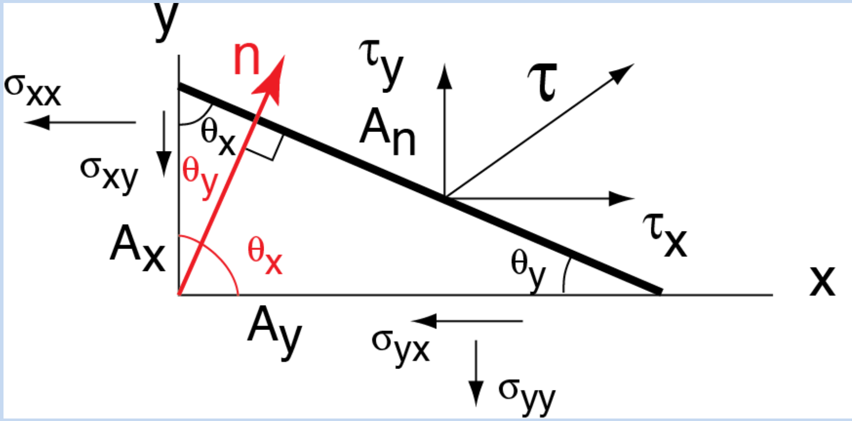
E Derivation

Contributions to τ_x

Based on a force balance

Note that all contributions must act in x-direction

1 $$\tau_x = w^{(1)}\sigma_{xx} + w^{(2)}\sigma_{yx}$$



2 $$\frac{F_x}{A_n} = \left(\frac{A_x}{A_n}\right)\frac{F_x}{A_x} + \left(\frac{A_y}{A_n}\right)\frac{F_x}{A_y}$$

3 $$\tau_x = n_x\sigma_{xx} + n_y\sigma_{yx}$$

Similarly

4 $$\tau_y = n_x\sigma_{xy} + n_y\sigma_{yy}$$

$n_x = \cos\theta_{nx} = \cos\theta_x$

$n_y = \cos\theta_{ny} = \cos\theta_y$

Principal Stresses

III Eigenvectors and eigenvalues

A — τ has a magnitude and direction

$$\begin{bmatrix} \tau_x \\ \tau_y \end{bmatrix} = \left|\vec{\tau}\right| \begin{bmatrix} n_x \\ n_y \end{bmatrix} \qquad Let\ \lambda = \left|\vec{\tau}\right|$$

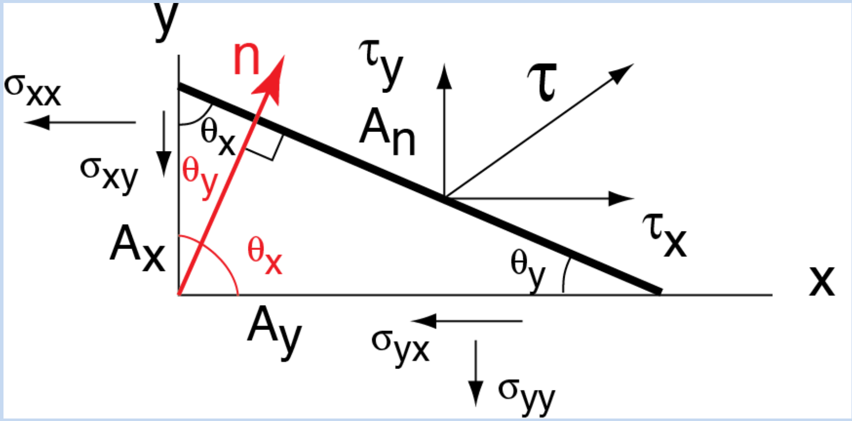
B — τ from Cauchy's formula

$$\begin{bmatrix} \tau_x \\ \tau_y \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{yx} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix}$$

C

$$\begin{bmatrix} \sigma_{xx} & \sigma_{yx} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix} = \lambda \begin{bmatrix} n_x \\ n_y \end{bmatrix}$$

$\vec{n}$ is unit normal to plane A_n



The form of (C) is [A][X]=λ[X], and [σ] is symmetric

Principal Stresses

1 $[\sigma][X]=\tau[X]$

2 This is an eigenvalue problem (e.g., $[A][X]=\lambda[X]$)

A $[\sigma]$ is a stress tensor (represented as a square matrix)

B τ is a scalar

C $[X]$ is a vector

D $[X]$, $[\sigma][X]$, and $\tau[X]$ all point in the same direction

3 Solving for τ yields the principal stress magnitudes

(Most tensile σ_1, Intermediate σ_2, least tensile σ_3)

4 Solving for $[X]$ yields the principal stress directions

Principal stresses are normal stresses and mutually perpendicular ($[\sigma]$ is symmetric)