GG612
Structural Geology Section

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Lecture 3

Mechanics

Isostasy

Stresses Control How Rock Fractures

http://hvo.wr.usgs.gov/kilauea/update/images.html

Stress Vector

I Stress vector (traction)

A $\vec{\tau} = \lim_{A \to 0} \vec{F} / A$

B Traction vectors can be added as vectors

C A traction vector can be resolved into normal (τ_n) and shear (τ_s) components

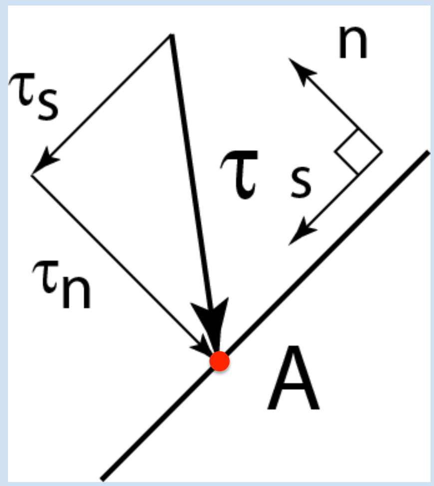
1 A normal traction (τ_n) acts perpendicular to a plane

2 A shear traction (τ_s) acts parallel to a plane

D Local reference frame

1 The n-axis is normal to the plane

2 The s-axis is parallel to the plane



Stress at a Point

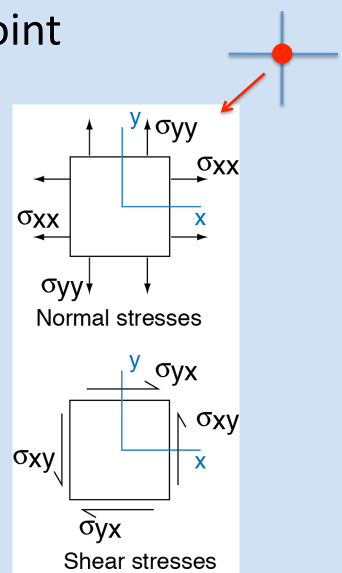
II Stress at a point (cont.)

A Stresses refer to balanced internal "forces (per unit area)". They differ from force vectors, which, if unbalanced, cause accelerations

B "On -in convention": The stress component σ_{ij} acts on the plane normal to the i-direction and acts in the j-direction

1 Normal stresses: i=j

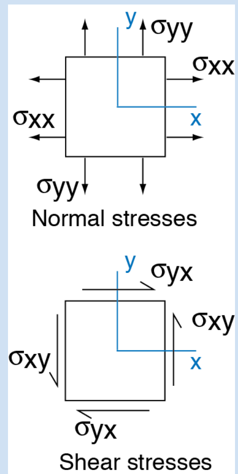
2 Shear stresses: i≠j



Stress at a Point

II Stress at a point

- C Dimensions of stress: force/unit area
- D Convention for stresses
  1. Tension is positive
  2. Compression is negative
  3. Follows from on-in convention
  4. Consistent with most mechanics books
  5. Counter to most geology books



Stress at a Point

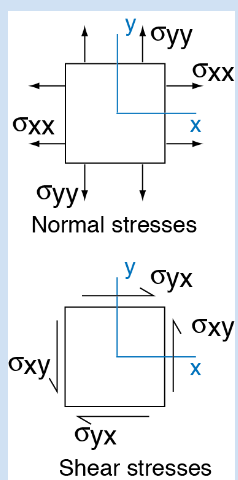
II Stress at a point

C $\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$ 2-D 4 components

D $\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$ 3-D 9 components

E For rotational equilibrium, $\sigma_{xy} = \sigma_{yx}$, $\sigma_{xz} = \sigma_{zx}$, $\sigma_{yz} = \sigma_{zy}$

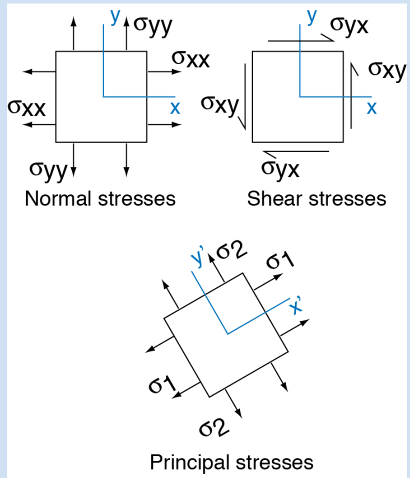
F In nature, the state of stress can (and usually does) vary from point to point



Principal Stresses

III Principal Stresses (these have magnitudes and orientations)

- A Principal stresses act on planes which feel no shear stress
- B The principal stresses are normal stresses.
- C Principal stresses act on perpendicular planes
- D The maximum, intermediate, and minimum principal stresses are usually designated σ_1, σ_2, and σ_3, respectively.
- E Principal stresses have a single subscript.



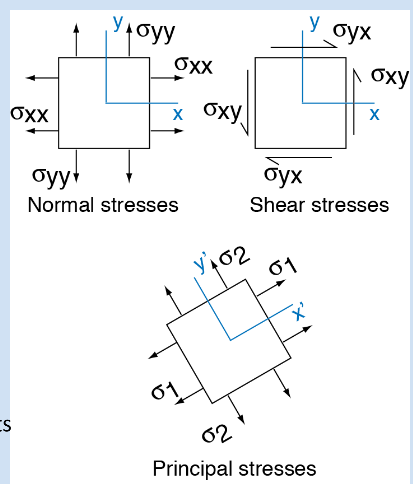
Principal Stresses

III Principal Stresses (cont.)

- F <u>Principal stresses represent the stress state most simply</u>

G $\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$ 2-D 2 components

H $\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$ 3-D 3 components



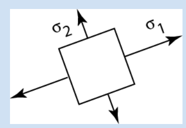
* If $\sigma_1 = \sigma_2 = \sigma_3$, the state of stress is called isotropic. This occurs beneath a still body of water.

Principal Stresses

1. $[\sigma][X]=\tau[X]$
2. This is an eigenvalue problem (e.g., $[A][X]=\lambda[X]$)
   - A $[\sigma]$ is a stress tensor (represented as a square matrix)
   - B τ is a scalar
   - C $[X]$ is a vector
   - D $[X]$, $[\sigma][X]$, and $\tau[X]$ all point in the same direction
3. Solving for τ yields the principal stress magnitudes

   (Most tensile σ_1, Intermediate σ_2, least tensile σ_3)
4. Solving for $[X]$ yields the principal stress directions

   Principal stresses are normal stresses and are mutually perpendicular



Geologic Problem: Radiating Dikes

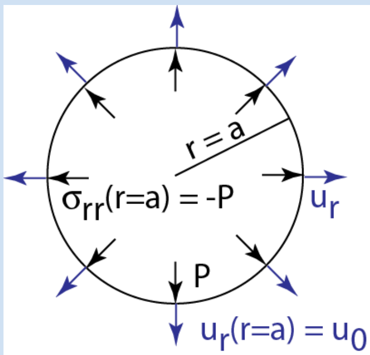
Shiprock, New Mexico

Aerial view showing radial dikes

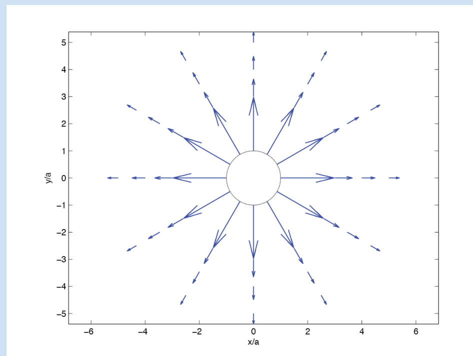
Both images from http://en.wikipedia.org/wiki/Shiprock#Images

Displacement Field Around a Pressurized Hole in an Elastic Plate

Geometry and boundary conditions



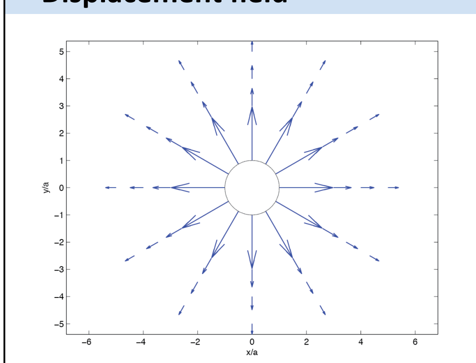
Displacement field



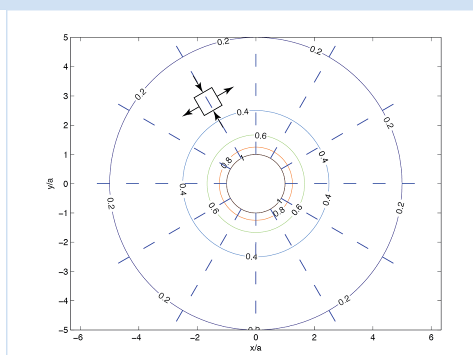
$$\left(u_r / u_{r,hole}\right) = \left(a/r\right)$$

Displacement and Stress Fields Around a Pressurized Hole

Displacement field



Stress field



$$\left(u_r / u_{r,hole}\right) = \left(a/r\right)$$

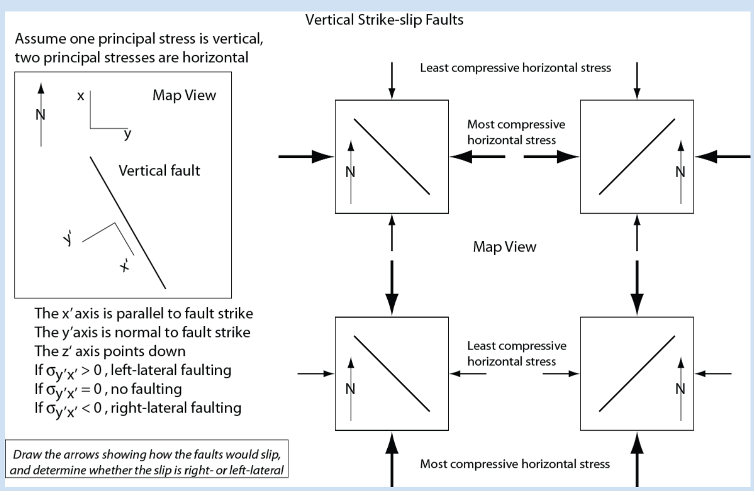
$$\left(\sigma_1 / P\right) = \left(a/r\right)^2$$

$$\left(\sigma_2 / P\right) = -\left(a/r\right)^2$$

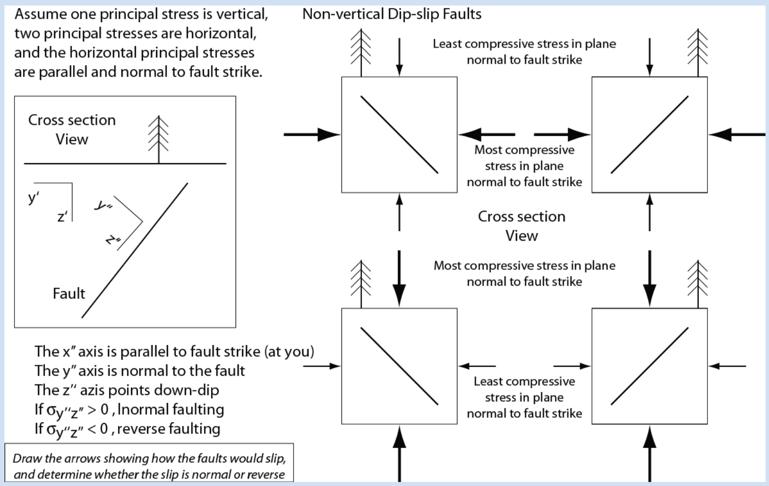
Dikes Open in Direction of Most Tensile Stress, Propagate in Plane Normal to Most Tensile

http://hvo.wr.usgs.gov/kilauea/update/images.html

Fault Mechanics: Vertical Strike-slip Faults



Fault Mechanics: Dip-slip Faults



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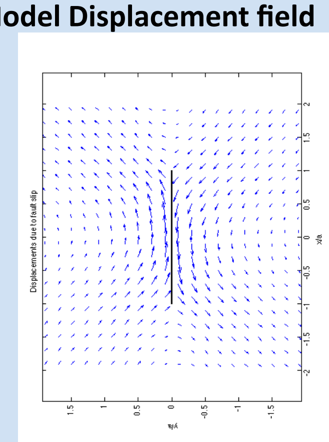
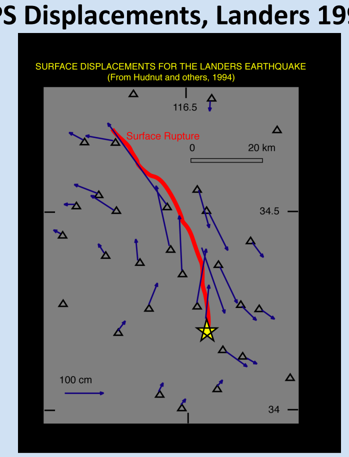
Modeled vs. Measured Displacement Fields Around a Fault

Model for Frictionless, 2D Fault in an Elastic Plate

Model Displacement field



GPS Displacements, Landers 1992



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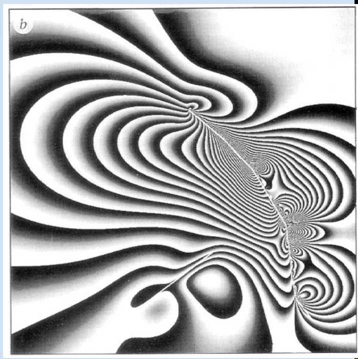
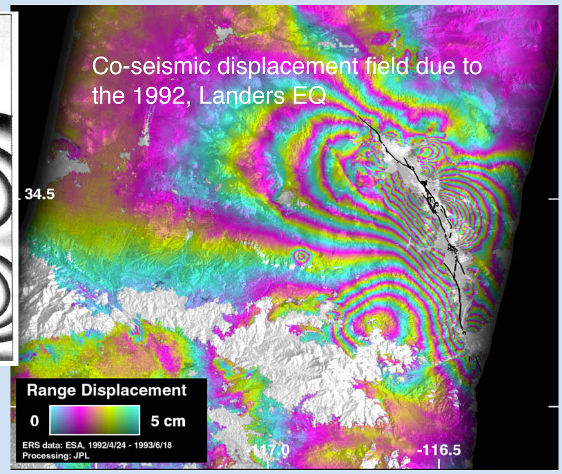
Modeled vs. Measured Co-seismic Displacement, Landers Earthquake





Co-seismic deformation can be modeled using elastic dislocation theory

(based on Massonnet et al., 1993)

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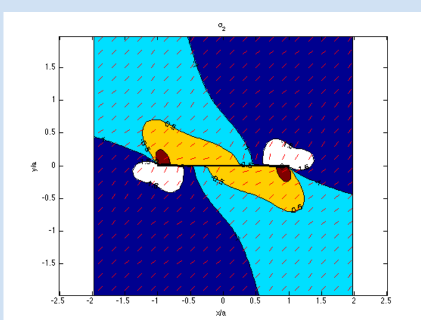
Stress Fields Around a Frictionless, 2D Model Fault in an Elastic Plate vs. Observations

Model stress field: Most tensile stress & stress trajectories

Tail cracks at end of left-lateral strike-slip fault



Note location and orientation of "tail cracks"

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Isostasy

- Refers to gravitational equilibrium
- Provides a physical rationale for the existence of mountains
- Based on force balance and buoyancy concepts

$$P = \int_0^h \rho(h)g(h)\,dh$$

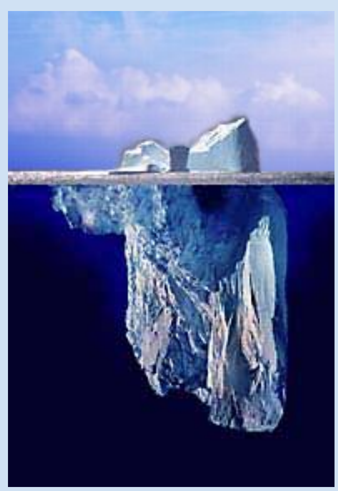
P = pressure (convention: compression is positive here)
ρ = density
g = gravitational acceleration

For constant *ρ* and constant g,
P = *ρ*gh

http://en.wikipedia.org/wiki/File:Iceberg.jpg

Isostasy

- Assumes a “compensation depth” at which pressures beneath two prisms are equal and the material beneath behaves like a static fluid, where $P_1 = P_2$
- Flexural strength of crust not considered
- Gravity measurements yield crustal thickness and density variations
- Complemented by seismic techniques



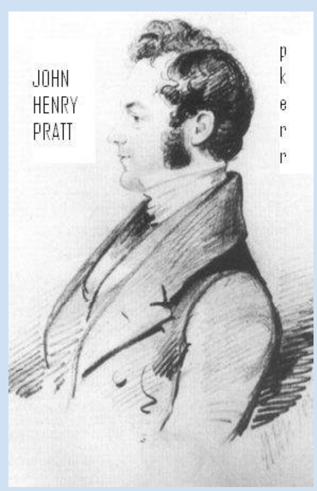
http://en.wikipedia.org/wiki/File:Iceberg.jpg

Isostasy
History

- Roots go back to da Vinci
- Term coined by Clarence Edward Dutton (USGS)
- Post-1800 interest triggered by surveying errors in India
- Two main models: Pratt, Airy

John Henry Pratt
(6/4/1809-12/28/1871)

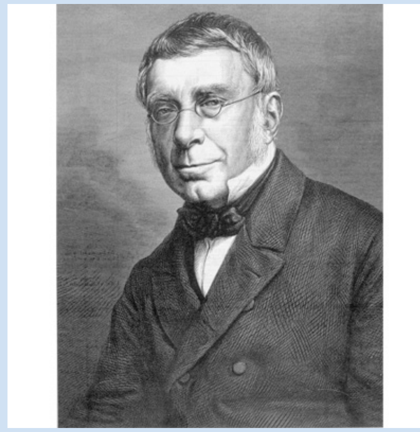
- Pratt, J.H., 1855, On the attraction of the Himalaya Mountains, and of the elevated regions beyond them, upon the Plumb-line in India. Philosophical Transactions of the Royal Society of London, v. 145, p. 53-100.
- British clergyman and mathematician
- Archdeacon of India



http://sphotos.ak.fbcdn.net/photos-ak-snc1/v2100/67/88/730660017/n730660017_5593665_6871.jpg

Sir George Biddell Airy (7/27/1801 - 1/2/1892

- Airy, G.B., 1855, On the computation of the Effect of the Attraction of Mountain-masses, as disturbing the Apparent Astronomical Latitude of Stations in Geodetic Surveys. Philosophical Transactions of the Royal Society of London. v. 145, p.101-104.
- British Royal Astronomer from 1835-1881
- Determined the mean density of the Earth from pendulum experiments in mines
- Contributor to elasticity theory (telescope deformation)
- Opponent of Charles Babbage from 1842 to ??



http://www.computerhistory.org/babbage/georgeairy/img/5-2-1.jpg

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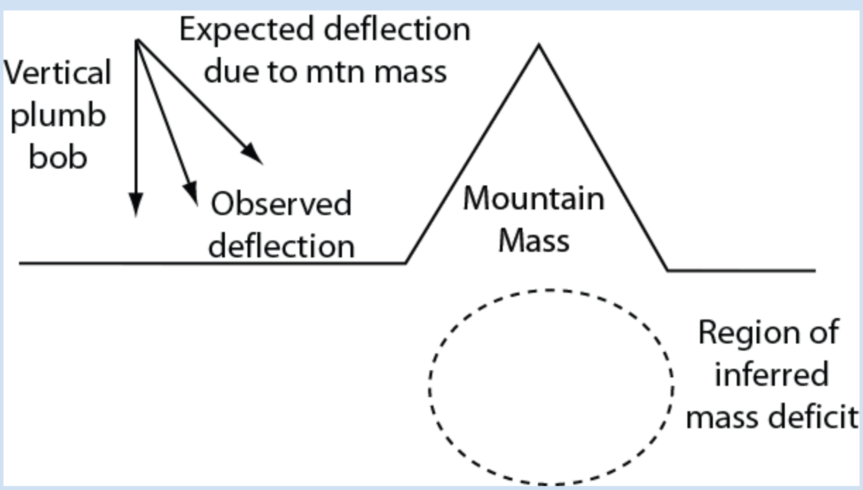
Isostasy and Gravity Measurements



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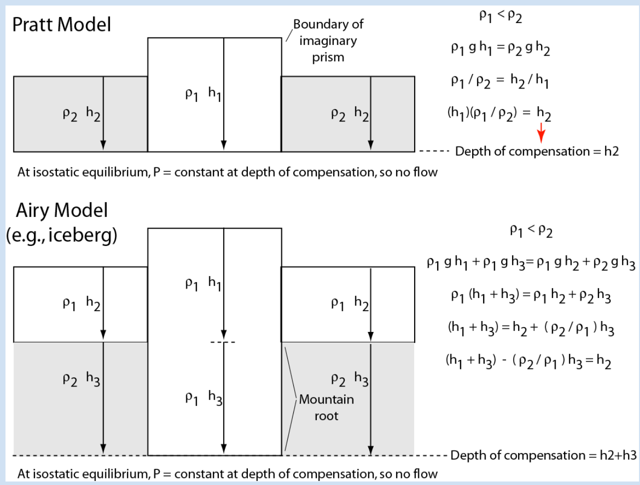
Comparison of Isostatic Models



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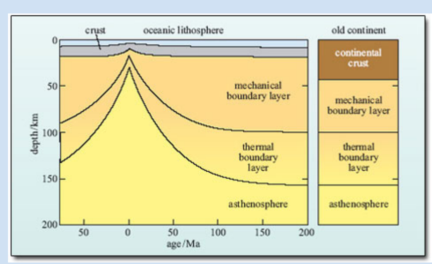
Thermal Isostasy (e.g., Turcotte and Schubert, 2002)

- Oceanic crust thickens and increases in density as it cools with time
- Oceanic crust thickens and increases in density with distance from ridge
- Depth to seafloor increases with distance from ridge



http://openlearn.open.ac.uk/file.php/2717/!via/oucontent/course/414/s279_1_014i.jpg

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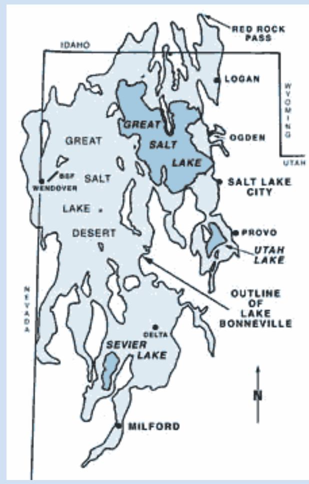
Isostatic Rebound: Lake Bonneville



http://geology.utah.gov/online/pi-39/images/pi39-01.gif

Shorelines of Lake Bonneville Tilt Away from Lake

http://k43.pbase.com/g6/93/584893/2/79634985.GXiIakLZ.jpg