## SUBSIDENCE IN THREE DIMENSIONS: CENTER OF DILATION (MOGI SOURCE) (43)

I Main Topics

A Center of dilation (contraction) in a half-space

B Case histories

C References

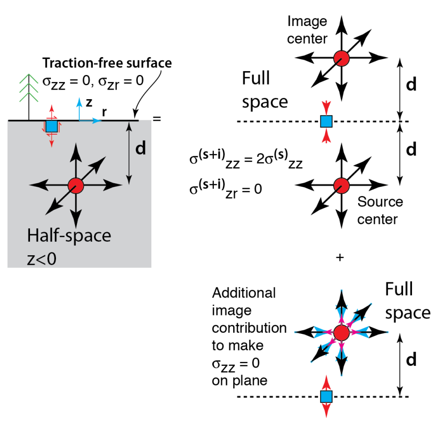
* Patterned after Segall, 2010



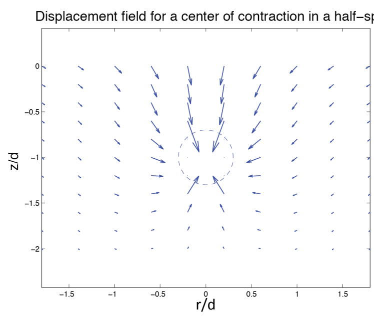
## II Center of dilation in half-space (Mogi source)

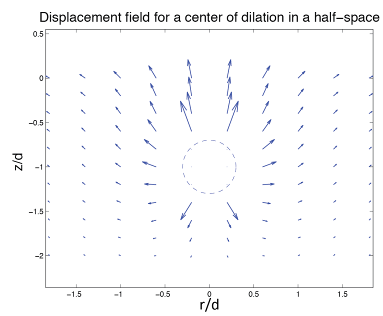
A Construction of solution by sources and images

1. Half-space: $z < 0$
2. Source center in a full space, _by itself_, has local radial displacements and stress singularity, but induces stresses along midplane (dashed) between source and image
3. Image center has strength of source, but is located above midplane
4. Source and image together double $\sigma_{zz}$ along midplane; annul $\sigma_{zr}$
5. Additional image contribution annuls $\sigma_{zz}$ along midplane between source and image



# B Sub-surface displacement field (z < 0) for centers of contraction and dilation



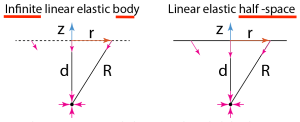

1. Displacements (and hence strains) near center similar to those for a center in an infinite body
2. Displacements at the surface radiate towards or from center at depth (just like for a center in an infinite body) but are larger by a factor of 4(1-ν) [Davies, 2003]



# C Displacement field at the surface (z = 0) for a center of dilation

Center of contraction
Infinite linear elastic body

Center of contraction
Linear elastic half -space




Displacements are radial (towards center) everywhere, including near center

Purely radial displacements (towards center) along surface and near center

Displacements at the surface of half-space radiate towards or from center at depth (just like for a center in an infinite body) but are larger by a factor of 4(1-ν) [Davies, 2003]

$$u_R^{\infty} = \frac{\Delta V}{4\pi}\frac{R}{R^3}$$

At plane a distance $d$ above center

$$u_{R(z=0)}^{\infty} = \frac{\Delta V}{4\pi}\frac{\left(r^2+d^2\right)^{1/2}}{R^3}$$

$$u_{r(z=0)}^{\infty} = \frac{\Delta V}{4\pi}\frac{r}{R^3}$$

$$u_{z(z=0)}^{\infty} = \frac{\Delta V}{4\pi}\frac{d}{R^3}$$

*At surface* $(z=0)$

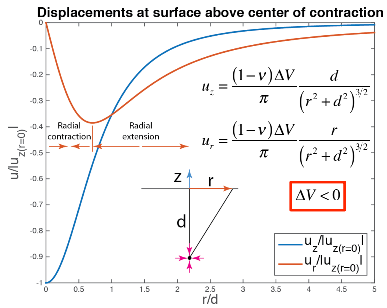
$$u_{R(z=0)}^{hs} = (1-\nu)\frac{\Delta V}{\pi}\frac{R}{R^3}$$

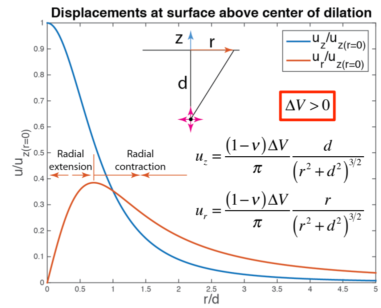
$$u_{R(z=0)}^{hs} = (1-\nu)\frac{\Delta V}{\pi}\frac{\left(r^2+d^2\right)^{1/2}}{R^3} = \left[4(1-\nu)\right]u_{R(z=0)}^{\infty}$$

$$u_{r(z=0)}^{hs} = (1-\nu)\frac{\Delta V}{\pi}\frac{r}{R^3} = \left[4(1-\nu)\right]u_{r(z=0)}^{\infty}$$

$$u_{z(z=0)}^{hs} = (1-\nu)\frac{\Delta V}{\pi}\frac{d}{R^3} = \left[4(1-\nu)\right]u_{z(z=0)}^{\infty}$$

# D Displacements at the surface (z = 0) for centers of contraction and dilation



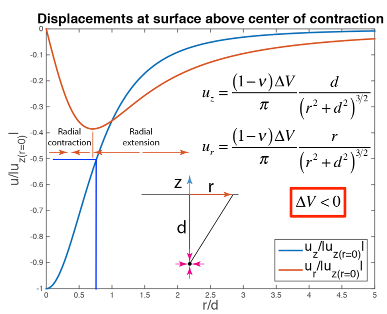



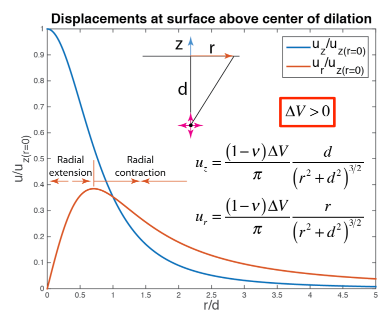

| Radial strain $\varepsilon_{rr}$ at z = 0 | Center of contraction | Center of dilation |
|---|---|---|
| r/d < 0.71 | Contraction | Extension |
| r/d > 0.71 | Extension | Contraction |

Vertical displacement magnitude decrease by 50% at $r_{1/2} \approx 0.77d$



# E Displacements at the surface (z = 0) for centers of contraction and dilation




Vertical displacement magnitude decreases by 50% at $r_{1/2} \approx 0.77d$
So, $d \approx r_{1/2}/0.77$
The maximum radial displacement occurs at $r^* \approx 0.71d$
So, alternatively, $d \approx r^*/0.71$
From $d$ and ground displacements at different positions, $\Delta V$ can be calculated

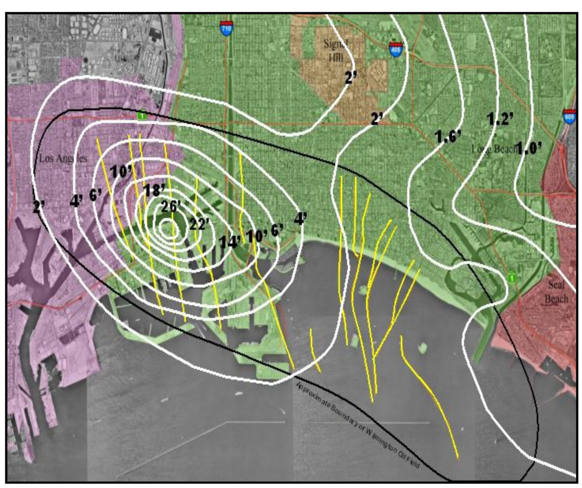
# III Case histories

1 Wilmington oil field, Long Beach, CA

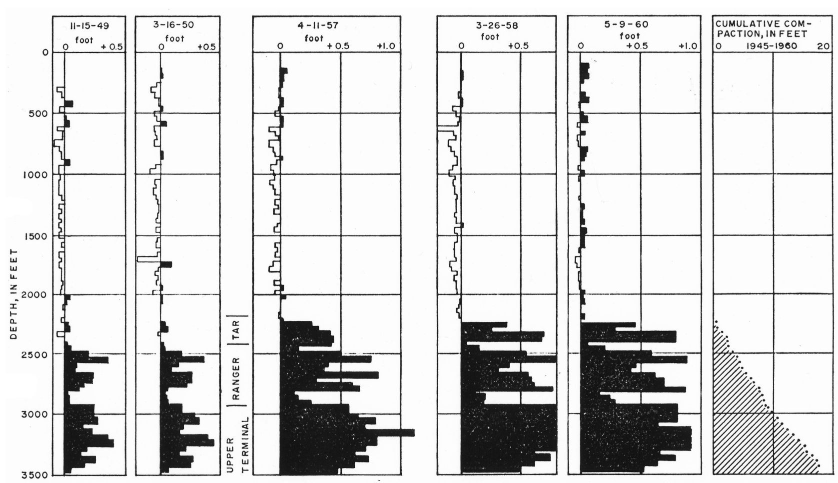
2 Darwin volcano, Galapagos Islands



# 1 Surface displacements, Wilmington oil field, Long Beach, CA

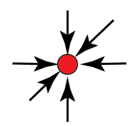


## A Vertical extension above producing horizons

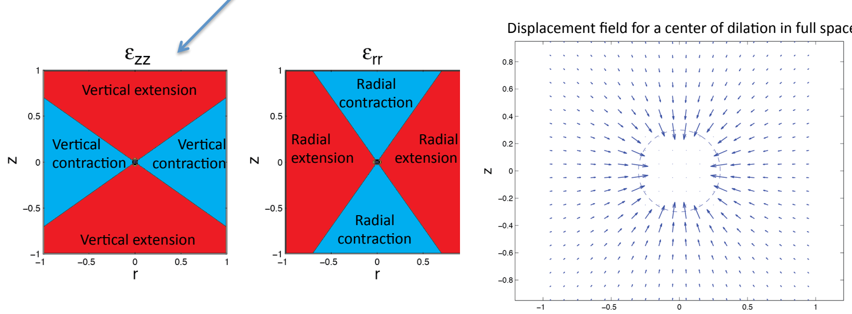





## B Normal strains and displacements in a vertical plane through a center of contraction

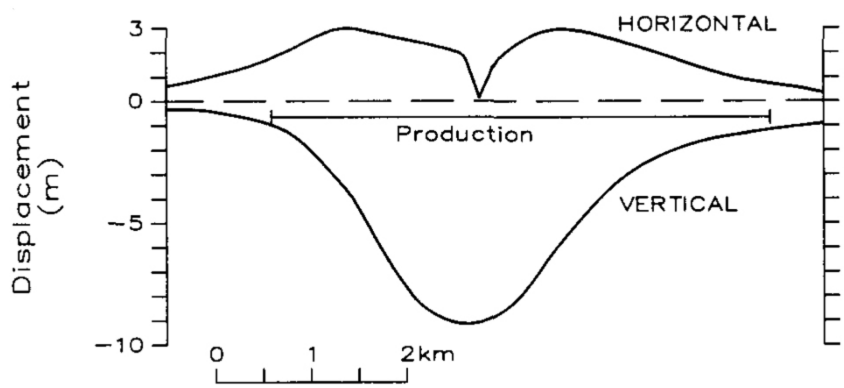


Vertical extension predicted above producing horizons




Displacements not shown within dashed circle for diagrammatic reasons

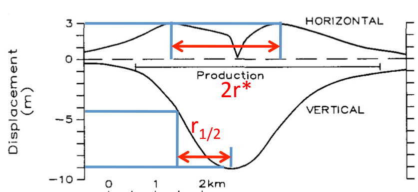
# C Surface displacements from the Wilmington oil field, Long Beach, CA



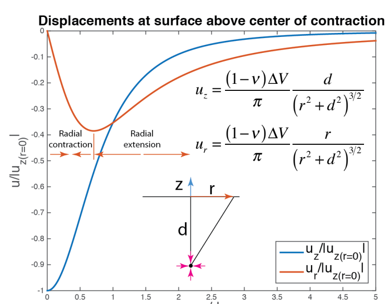



# D Wilmington oil field data and model predictions

2r* ≈ 2.3 km   r* ≈ 1.15 km



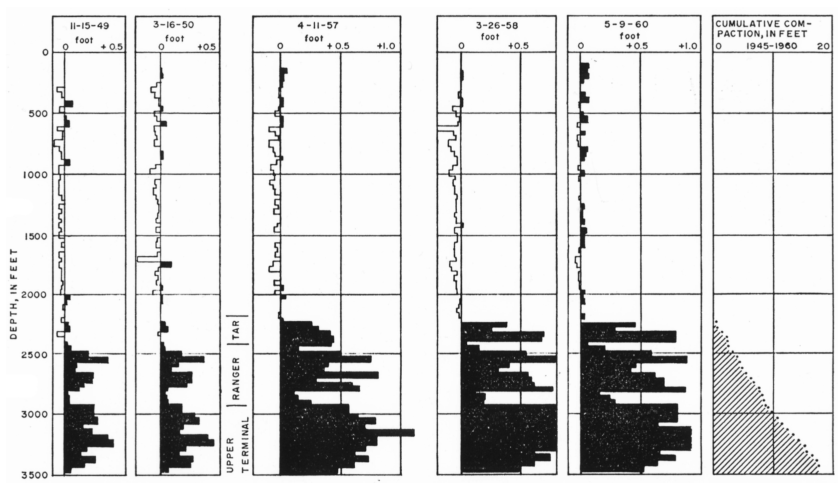

$r_{1/2}$ = 1.0 km




$d \approx r_{1/2}/0.77$ = 1.0 km/0.77 = 1.3 km

$d \approx r^*/0.71$ = 1.15 km/0.71 = 1.5 km

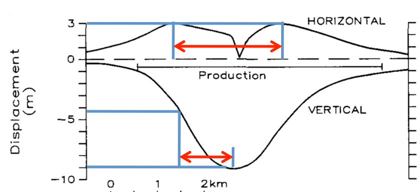
# E Producing horizons centered at ~ 1km depth



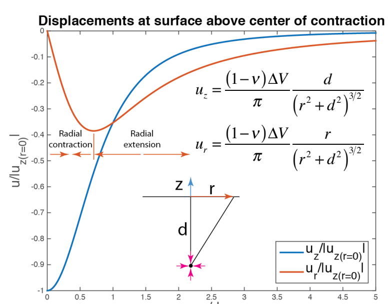



# F Wilmington oil field data and model predictions for volume loss

$2r^* \approx 2.3$ km $r^* \approx 1.15$ km




$r_{1/2} = 1.0$ km




$$u_z = \frac{(1-\nu)\Delta V}{\pi}\frac{d}{\left(r^2+d^2\right)^{3/2}}$$

$$u_r = \frac{(1-\nu)\Delta V}{\pi}\frac{r}{\left(r^2+d^2\right)^{3/2}}$$

$d \approx r_{1/2}/0.77 = 1.0$ km$/0.77 = 1.3$ km

$d \approx r^*/0.71 = 1.15$ km$/0.71 = 1.5$ km

$$\Delta V = \frac{u_z\pi}{(1-\nu)}\frac{\left[r^2+d^2\right]^{3/2}}{d} = \frac{(-9m)\pi}{(1-0.25)}\frac{\left((0m)^2+(1300m)^2\right)^{3/2}}{1300m} = -6.4x10^7 m^3$$

$$\Delta V = \frac{u_r\pi}{(1-\nu)}\frac{\left[r^2+d^2\right]^{3/2}}{r} = \frac{(-3m)\pi}{(1-0.25)}\frac{\left((1150m)^2+(1500m)^2\right)^{3/2}}{1500m} = -7.4x10^7 m^3$$

# G Produced volume at Wilmington oil field vs. model predictions

From November 1936 to July 1, 1969 the oil field produced approximately 2.0 x $10^8$ cubic meters (Mayuga and Allen)

$$\Delta V = \frac{u_z \pi}{(1-\nu)} \frac{\left[r^2+d^2\right]^{3/2}}{d} = \frac{(-9m)\pi}{(1-0.25)} \frac{\left((0m)^2+(1300m)^2\right)^{3/2}}{1300m} = -6.4x10^7 m^3$$

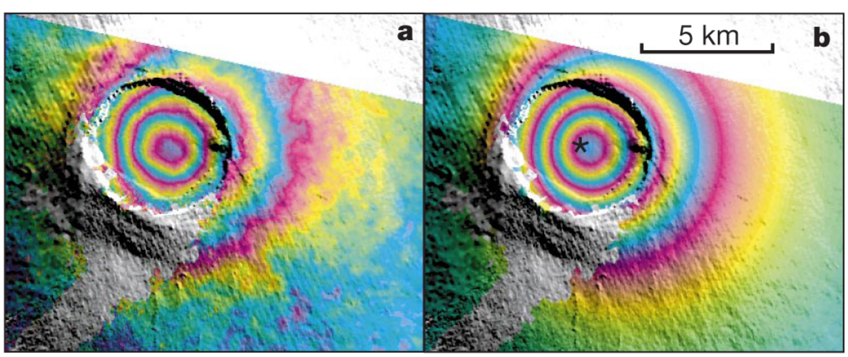
$$\Delta V = \frac{u_r \pi}{(1-\nu)} \frac{\left[r^2+d^2\right]^{3/2}}{r} = \frac{(-3m)\pi}{(1-0.25)} \frac{\left((1150m)^2+(1500m)^2\right)^{3/2}}{1500m} = -7.4x10^7 m^3$$

Fluid volume produced is ~3 times the modeled solid volume loss

Possible parallel: in the San Joaquin Valley, the pumped volume is ~3 times the subsidence volume



# 2 Darwin volcano: ground surface displacements vs. model predictions




SAR interferogram of Darwin volcano, Galapagos Islands. (a) Data from 1992 to 1998. (b) Predicted interferogram based on a center of dilation located at a depth of 3 km beneath the star. Each color cycle represents 5 cm of displacement along the line of sight. From Amelung et al., 2000.

# References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