CONSOLIDATION MECHANICS REVIEW (41)

A Review of key points

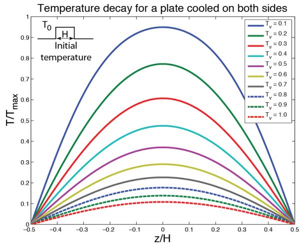
B Worked examples

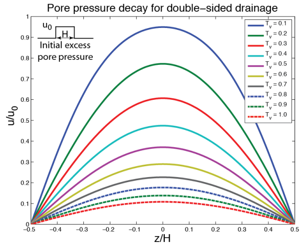
II Review of key points

A Heat flow analogous to flow of fluid during consolidation

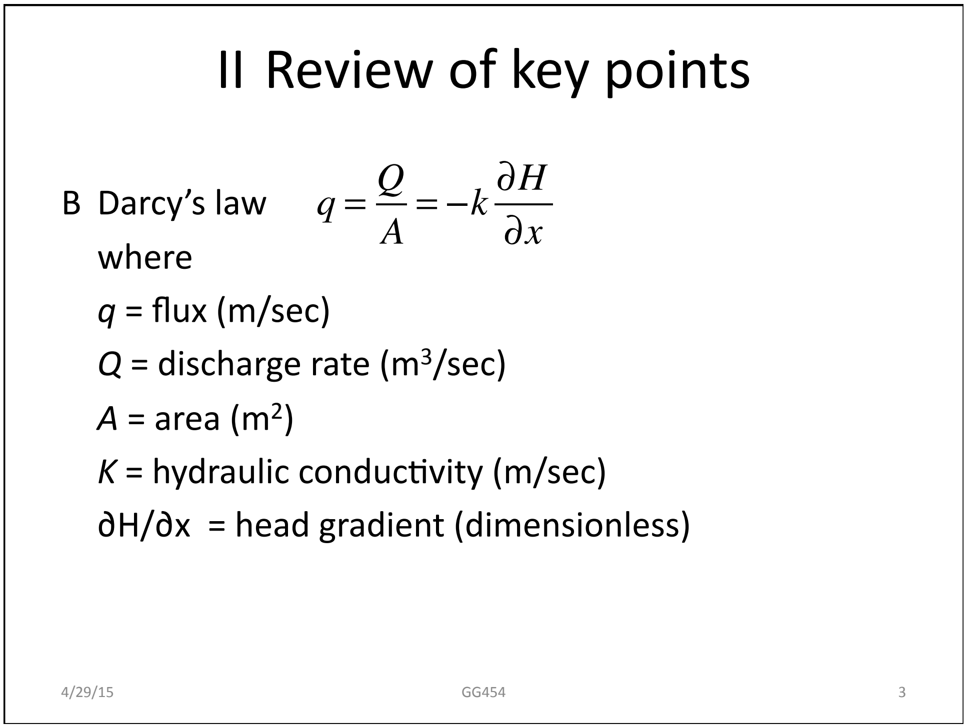
$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2} \qquad \frac{\partial u_{excess}}{\partial t} = C_v \frac{\partial^2 u_{excess}}{\partial z^2}$$

Heat flow	Fluid flow
T Temperature	u_{excess} Excess pore pressure
t Time	t Time
α Thermal diffusivity	C_v Coefficient of consolidation
z position	z position


Temperature decay for a plate cooled on both sides


Pore pressure decay for double−sided drainage

Here $T_v = t^* = t/(H^2/C_v)$



II Review of key points

B Darcy's law $q = \frac{Q}{A} = -k\frac{\partial H}{\partial x}$

where

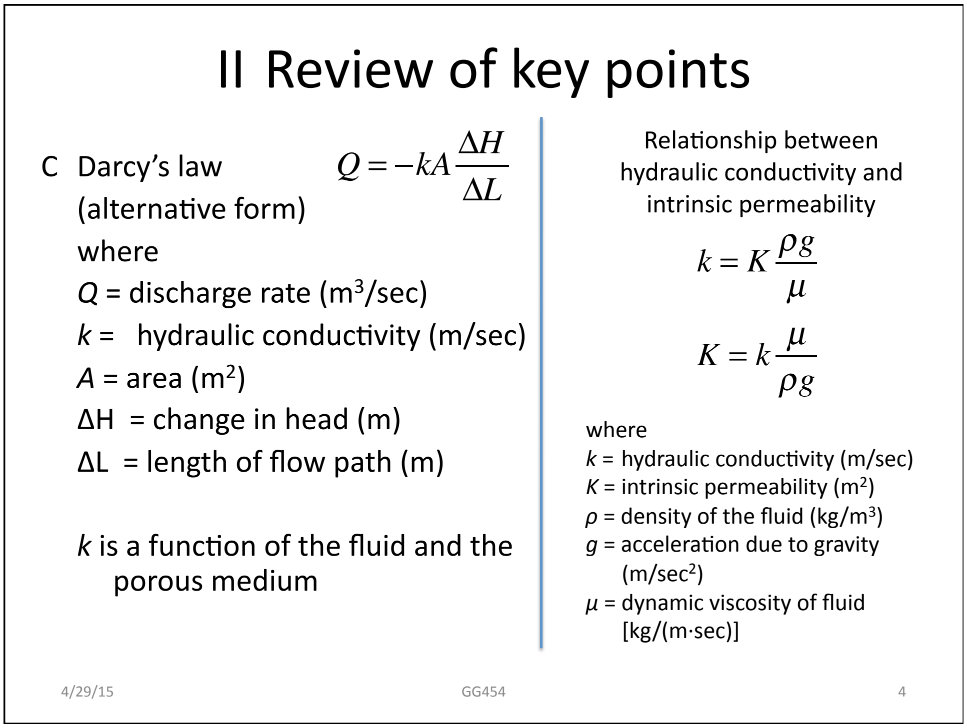
q = flux (m/sec)

Q = discharge rate (m^3/sec)

A = area (m^2)

K = hydraulic conductivity (m/sec)

$\partial H/\partial x$ = head gradient (dimensionless)



II Review of key points

C Darcy's law (alternative form) $Q = -kA\frac{\Delta H}{\Delta L}$

where

Q = discharge rate (m^3/sec)

k = hydraulic conductivity (m/sec)

A = area (m^2)

ΔH = change in head (m)

ΔL = length of flow path (m)

k is a function of the fluid and the porous medium

Relationship between hydraulic conductivity and intrinsic permeability

$$k = K\frac{\rho g}{\mu}$$

$$K = k\frac{\mu}{\rho g}$$

where

k = hydraulic conductivity (m/sec)

K = intrinsic permeability (m^2)

ρ = density of the fluid (kg/m^3)

g = acceleration due to gravity (m/sec^2)

μ = dynamic viscosity of fluid [kg/(m·sec)]

II Review of key points

- B Controls on 1-D consolidation time (t_c) of soils and sediments
  - 1 The time for consolidation should scale with the volume of water squeezed out of the soil
  - 2 The volume of expelled water is the product of three terms
    - a Effective stress change (Δσ’)
    - B Compressibility of the solid skeleton (i.e., m_v, the coefficient of volume change)
    - C Volume of the consolidating layer (or height H of the layer)

$$t_c \propto \Delta\sigma' m_v H$$

II Review of key points

- B Controls on 1-D consolidation time (t_c) of soils and sediments (cont.)
  - 3 The time [for consolidation] should be inversely proportional to how fast the water can flow through the consolidating layer
  - 4 From Darcy’s law: the velocity of flow is scales with the product of the permeability (k) and the hydraulic gradient
  - 5 The hydraulic gradient matches the excess fluid pressure lost within the layer divided by the distance the pore fluid flows (H)
  - 6 The excess fluid pressure loss scales with Δσ’

$$t_c \propto \frac{1}{k\Delta\sigma'/H} = \frac{H}{k\Delta\sigma'}$$

II Review of key points

B Controls on 1-D consolidation time (t_c) of soils and sediments (cont.)

7 t_c increases with increasing compressibility (i.e., with increasing m_v)

8 t_c Increases rapidly with increasing volume (height) of the soil/sediment mass (H)

9 t_c Decreases with increasing permeability (k)

10 t_c is ***independent*** of the magnitude of the effective stress change, assuming m_v is a constant

$$t_c \propto \frac{\Delta\sigma' m_v H}{k\Delta\sigma'/H} = \frac{m_v H^2}{k}$$

II Review of key points

C Dimensional analysis illuminates consolidation time

1 1-D consolidation equation

$$\frac{\partial u_e}{\partial t} = C_v \frac{\partial^2 u_e}{\partial x^2}$$

$$\frac{pressure}{time} = [C_v]\frac{pressure}{length^2} \Rightarrow [C_v] = \frac{length^2}{time}$$

where

C_v = coefficient of consolidation

$$= \frac{k}{\rho_{water} g m_v}$$

m_v = coefficient of volume change

II Review of key points

2 The dimensionless time used in <u>dimensionless</u> expressions for consolidation is found in the same way as in lecture 38

Heat flow	Consolidation
$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$	$\frac{\partial u_e}{\partial t} = C_v \frac{\partial^2 u_e}{\partial x^2}$
$t_c \approx \frac{H^2}{\alpha}$	$t_c \approx \frac{H^2}{C_v}$
$t^* = \frac{t}{t_c} = \frac{t}{\left(\frac{H^2}{\alpha}\right)}$	$t^* = \frac{t}{t_c} = \frac{t}{\left(\frac{H^2}{C_v}\right)}$

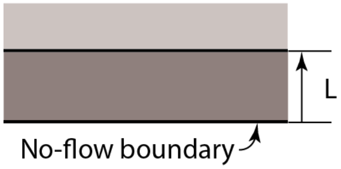
II Review of key points

D The key length scale (L) is the flow path length

1. For double-sided drainage, L = half the layer thickness
2. For single-sided drainage, L = the whole layer thickness

Double-sided drainage

Single-sided drainage



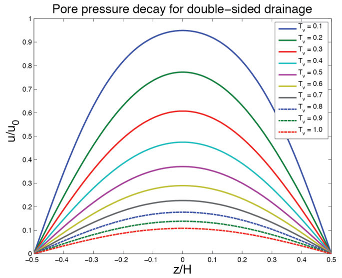
III Calculating consolidation

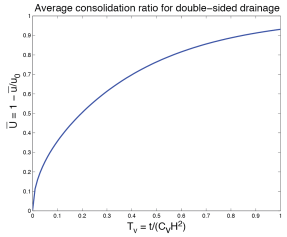
$$U(t)=\frac{\Delta H(t)}{\Delta H(t\to\infty)}=\frac{\Delta u_{excess}}{\Delta u_{excess(\max)}}=1-\frac{u_{excess}}{u_0}$$

E Consolidation ratio for double-sided drainage of a uniform excess pressure pulse* [for analytical solution see Terzaghi (1943) or Lambe and Whitman (1969)]

1 For $T_v = t/(C_v H^2) > 0.1$
  - a Excess pore pressure distribution (u) is approximately parabolic
  - b Ū (ave. consolidation ratio)
    - i $\bar{U}=1-\frac{\bar{u}}{u_0}$
    - ii $\bar{U}=1-\frac{\text{area beneath } u \text{ -curve}}{\text{area beneath boxcar function}}$
    - iii $\bar{U}\approx 1-\frac{2}{3}\frac{u_{\max}}{u_0}$

2 ~92% consolidation at T_v=1





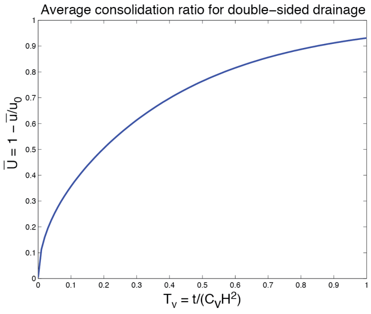
II Review of key points

E Average consolidation ratio (Ū)

1 Ū(t*=1) ≈ 92%

2 Ū(t*=3) ≈ 99%

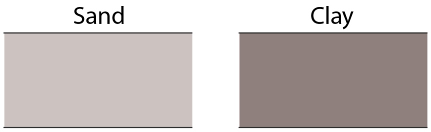
3 About 92% of the ultimate primary consolidation occurs by the dimensionless time t* = 1



$$t^*=\frac{t}{\left(\frac{x_{\max}{}^2}{C_v}\right)}$$

III Examples

A Consider layers of clay and sand, each 10’ thick, and suppose that the coefficient of volume change (“compressibility”) of the sand is 1/5 that of the clay, and the permeability of the sand is 10,000 times that of the clay.



$k_{sand} = 10^4 k_{clay}$
$m_{v\ sand} = 0.2\ m_{v\ clay}$

III Examples

1 What is the ratio of the consolidation times of the sand and clay?

$$\frac{t_{clay}}{t_{sand}} = \frac{H_{clay}{}^2 m_v(clay)/k_{clay}}{H_{sand}{}^2 m_v(sand)/k_{sand}} = \frac{m_v(clay)/k_{clay}}{m_v(sand)/k_{sand}} = \frac{5/1}{1/10{,}000} = 50{,}000$$

2 If a 10’-thick layer of clay reaches 90% consolidation in 10 years, how long would it take for a clay layer 40’ thick to reach that level of consolidation?

$$\frac{t_{40'}}{t_{10'}} = \frac{m_{v40'} H_{40'}{}^2 / k_{40'}}{m_{v10'} H_{10'}{}^2 / k_{10'}} = \frac{H_{40'}{}^2}{H_{10'}{}^2} = \frac{40^2}{10^2} = 16$$

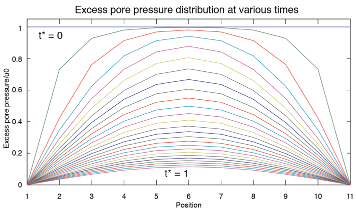
16 x 10 years = 160 years

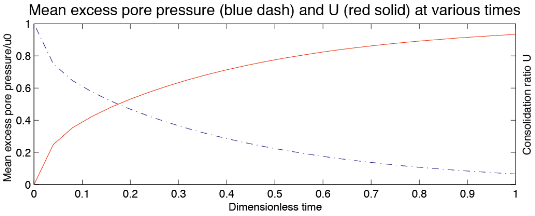
III Examples

B How long will a layer of clay take to reach 90% consolidation if the initial excess pore pressure distribution is constant across the layer and the layer is drained from its top and bottom? One dimensionless time unit (t*) is given by = 1.

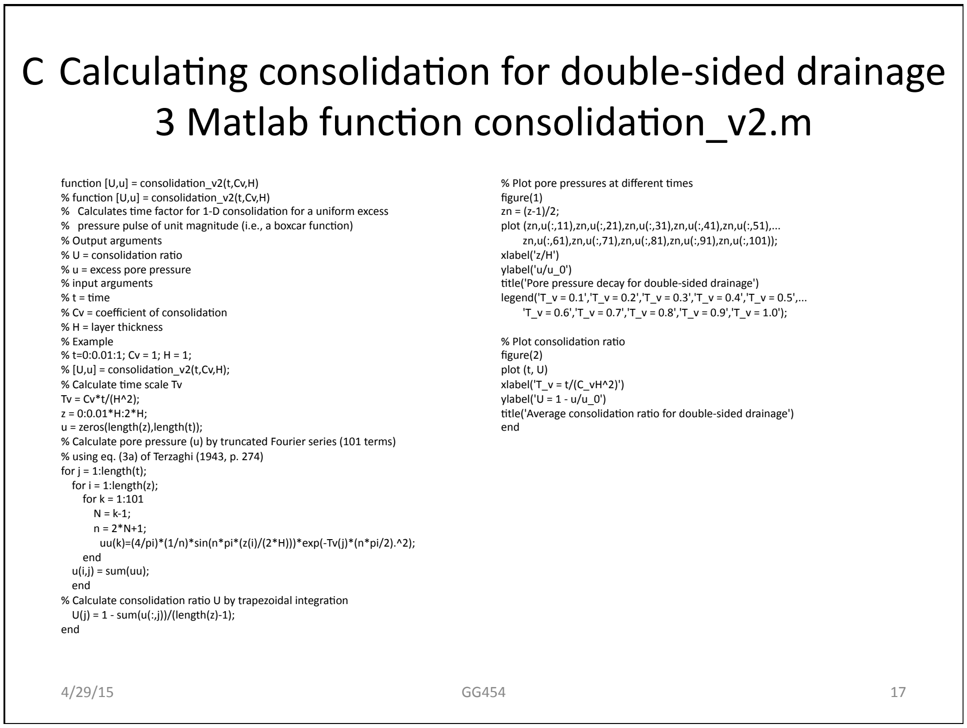
C Calculating consolidation for double-sided drainage

4 Matlab script transient_heat4.m


Excess pore pressure distribution at various times


Mean excess pore pressure (blue dash) and U (red solid) at various times

So after one dimensionless time step [i.e. $t^* = t/(H^2/C_v) = 1$], about 92% (i.e., roughly 90%) of the ultimate consolidation will have occurred

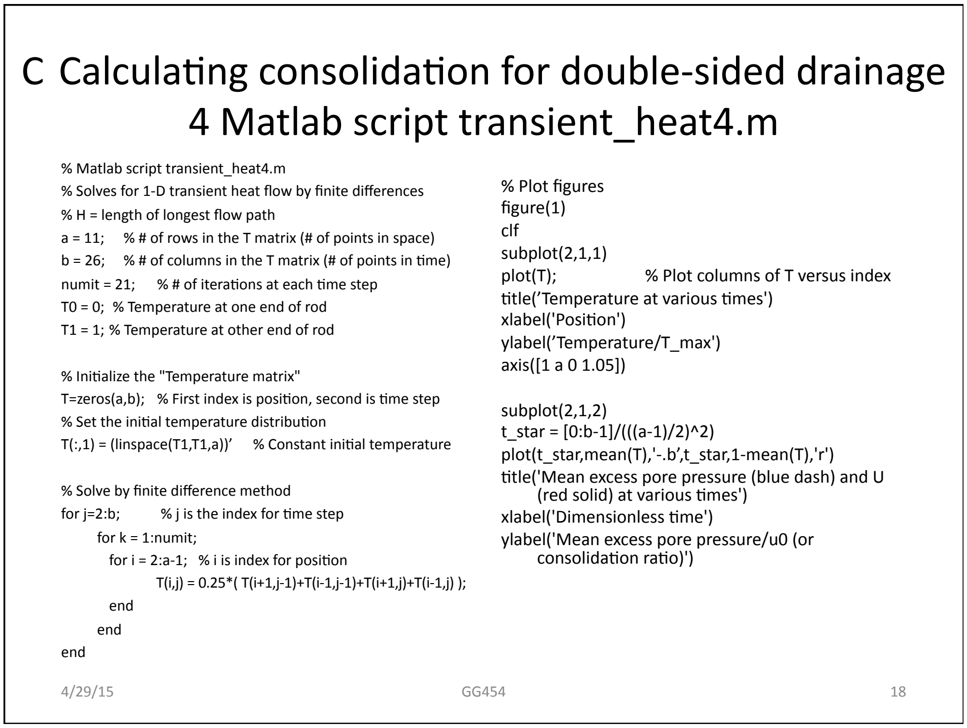


C Calculating consolidation for double-sided drainage
3 Matlab function consolidation_v2.m

```
function [U,u] = consolidation_v2(t,Cv,H)
% function [U,u] = consolidation_v2(t,Cv,H)
%   Calculates time factor for 1-D consolidation for a uniform excess
%   pressure pulse of unit magnitude (i.e., a boxcar function)
% Output arguments
% U = consolidation ratio
% u = excess pore pressure
% input arguments
% t = time
% Cv = coefficient of consolidation
% H = layer thickness
% Example
% t=0:0.01:1; Cv = 1; H = 1;
% [U,u] = consolidation_v2(t,Cv,H);
% Calculate time scale Tv
Tv = Cv*t/(H^2);
z = 0:0.01*H:2*H;
u = zeros(length(z),length(t));
% Calculate pore pressure (u) by truncated Fourier series (101 terms)
% using eq. (3a) of Terzaghi (1943, p. 274)
for j = 1:length(t);
   for i = 1:length(z);
      for k = 1:101
         N = k-1;
         n = 2*N+1;
           uu(k)=(4/pi)*(1/n)*sin(n*pi*(z(i)/(2*H)))*exp(-Tv(j)*(n*pi/2).^2);
      end
   u(i,j) = sum(uu);
   end
% Calculate consolidation ratio U by trapezoidal integration
   U(j) = 1 - sum(u(:,j))/(length(z)-1);
end
```

```
% Plot pore pressures at different times
figure(1)
zn = (z-1)/2;
plot (zn,u(:,11),zn,u(:,21),zn,u(:,31),zn,u(:,41),zn,u(:,51),...
      zn,u(:,61),zn,u(:,71),zn,u(:,81),zn,u(:,91),zn,u(:,101));
xlabel('z/H')
ylabel('u/u_0')
title('Pore pressure decay for double-sided drainage')
legend('T_v = 0.1','T_v = 0.2','T_v = 0.3','T_v = 0.4','T_v = 0.5',...
      'T_v = 0.6','T_v = 0.7','T_v = 0.8','T_v = 0.9','T_v = 1.0');

% Plot consolidation ratio
figure(2)
plot (t, U)
xlabel('T_v = t/(C_vH^2)')
ylabel('U = 1 - u/u_0')
title('Average consolidation ratio for double-sided drainage')
end
```



C Calculating consolidation for double-sided drainage
4 Matlab script transient_heat4.m

```
% Matlab script transient_heat4.m
% Solves for 1-D transient heat flow by finite differences
% H = length of longest flow path
a = 11;    % # of rows in the T matrix (# of points in space)
b = 26;    % # of columns in the T matrix (# of points in time)
numit = 21;    % # of iterations at each time step
T0 = 0;  % Temperature at one end of rod
T1 = 1; % Temperature at other end of rod

% Initialize the "Temperature matrix"
T=zeros(a,b);   % First index is position, second is time step
% Set the initial temperature distribution
T(:,1) = (linspace(T1,T1,a))’    % Constant initial temperature

% Solve by finite difference method
for j=2:b;         % j is the index for time step
      for k = 1:numit;
        for i = 2:a-1;   % i is index for position
                T(i,j) = 0.25*( T(i+1,j-1)+T(i-1,j-1)+T(i+1,j)+T(i-1,j) );
        end
      end
end
```

```
% Plot figures
figure(1)
clf
subplot(2,1,1)
plot(T);                % Plot columns of T versus index
title(’Temperature at various times')
xlabel('Position')
ylabel(’Temperature/T_max')
axis([1 a 0 1.05])

subplot(2,1,2)
t_star = [0:b-1]/(((a-1)/2)^2)
plot(t_star,mean(T),'-.b’,t_star,1-mean(T),'r')
title('Mean excess pore pressure (blue dash) and U
      (red solid) at various times')
xlabel('Dimensionless time')
ylabel('Mean excess pore pressure/u0 (or
      consolidation ratio)')
```

References

- Lambe, T.W., and Whitman, R.V., 1969, Soil Mechanics: Wiley, New York, 553 p.