EFFECTIVE STRESS AND MOHR-COULOMB FAILURE (26)

I Main Topics

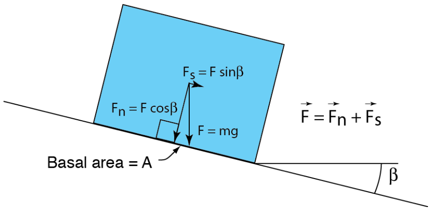
- A Driving and resisting stresses at the base of an inclined block on an infinite plane
- B Factor of safety
- C Effective stress
- D Mohr-Coulomb shear failure
- E Factors promoting shear failure

II Driving and resisting stresses at the based of a dry inclined block on an infinite plane

A Key assumptions*

1 Sliding surface is planar
2 Sliding surface and block are infinitely long
3 Stress is uniform along base of block
4 No stress concentrations
5 Normal stress parallel to surface does not affect normal stress perpendicular to surface

B Driving stress: $\sigma_s = F_s/A$
Slope-parallel component of block weight / block base area



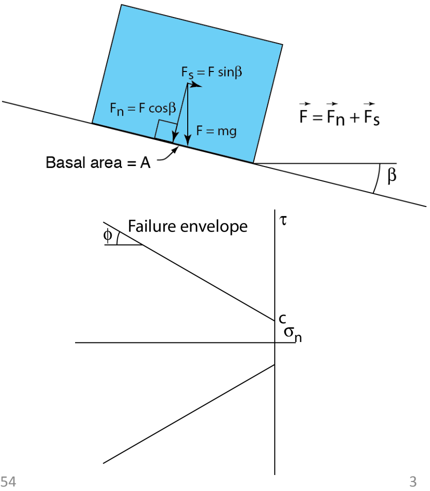
* These assumptions are not strictly valid. More realistic assumptions have important non-intuitive consequences (Martel, 2004, 2006)

II Driving and resisting stresses at the based of a dry inclined block on an infinite plane

C Resisting stress (τ)

$\tau = c + (F_n \tan \phi)/A$

- a c = cohesion
  - i shear strength if $\sigma_n = 0$
  - ii For dry sand, $c \approx 0$
- b $F_n = mg \cos(\beta)$
- c $\tan \phi$ = coefficient of friction = μ
- d ϕ = angle of internal friction



III Factor of safety (F.S.)

A $$F.S. = \frac{F_{resisting}}{F_{driving}} = \frac{F_{resisting}/A}{F_{driving}/A} = \frac{\sigma_{resisting}}{\sigma_{driving}}$$

B Interpretation of F.S. < 1

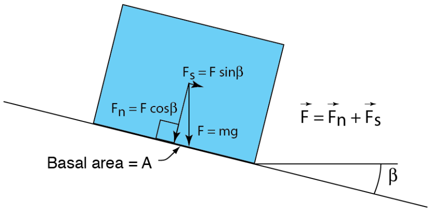
1. Driving force exceeds available resisting force
2. Failure predicted

C Interpretation of F.S. > 1

1. Available resisting force exceeds driving force
2. Failure not predicted

D For dry cohesionless (sandy) soils

1. $$F.S. = \frac{F \cos\beta \tan\phi}{F \sin\beta} = \frac{\tan\phi}{\tan\beta}$$
2. Interpretation: if ß > ϕ slope is unstable (prone to sliding)



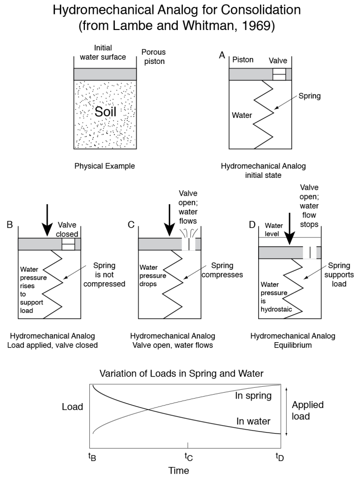
IV Effective stress

A For soils: σ' = σ_{total} - P

1 σ' = Effective stress = load supported by solid framework

2 σ_{total} = total normal stress porous material is subjected to

3 P = pore pressure of pore fluid = load supported by fluid


Hydromechanical Analog for Consolidation
(from Lambe and Whitman, 1969)

IV Effective stress

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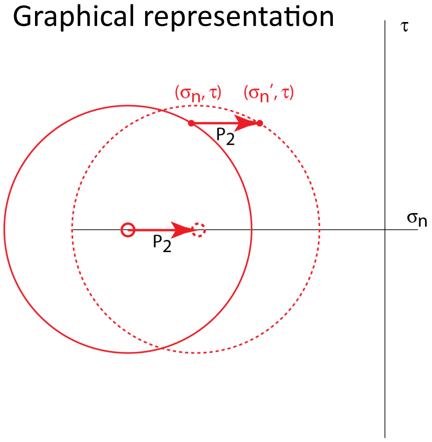
Graphical representation



IV Effective stress

B General case (For soil and rock): $\sigma' = \sigma_{total} - \alpha P$ (Nur and Byerlee, 1971, JGR, v. 76, p. 6414-6419)

1 $\alpha = 1 - (K_{agg}/K_s)$

a K = Bulk modulus = Δpressure/(ΔV/V)

b K_{agg} = bulk modulus of dry aggregate

c K_s = bulk modulus of the solid component of the aggregate

2 For soils, $K_{agg} << K_s$ so $\alpha \approx 1$

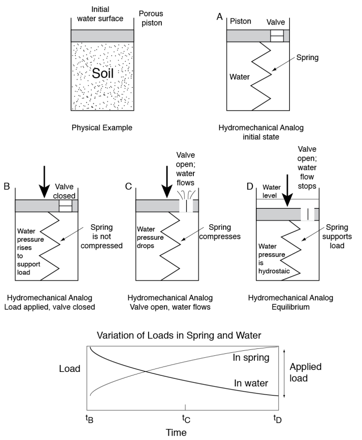
3 For some rocks $\alpha \neq 1$ $(0 < \alpha < 1)$

IV Effective stress

C Pore pressure does not "lubricate" the failure surface; the pore pressures acts to float the material overlying the failure surface

D Until the pore fluid has time to flow, increased loads on a saturated "soil" are supported by an increase in the pore pressure.

Hydromechanical Analog for Consolidation
(from Lambe and Whitman, 1969)



IV Effective stress: Example 1

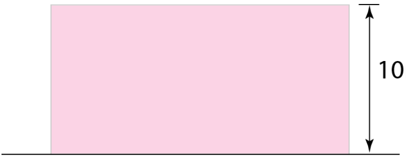
Suppose a block of impermeable, zero porosity quartz of density 2.67 g/cm^3 rests on a horizontal surface. What is the total normal stress and effective stress at the base of a 10m tall block of pure quartz?

$\sigma_n = (\rho_{quartz})(g)(h)$

$\sigma_n = (2.67 \times 10^3\ kg/m^3)(9.81\ m/sec^2)(10m)$

$\sigma_n = 2.62 \times 10^5$ Pa

$\sigma_n' = \sigma_n$ (no pore pressure at base)



IV Effective stress: Example 2

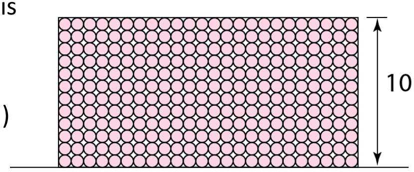
Now suppose a dry block of quartz sandstone rests on a horizontal surface. Further assume that the porosity of the sandstone is 20%. What is the total normal stress at the base of a 10m tall block of this sandstone ?

$\sigma_n = (\rho_{sandstone})(g)(h)$

$\sigma_n = (2.14 \times 10^3\ kg/m^3)(9.81\ m/sec^2)(10m)$

$\sigma_n = 2.10 \times 10^5$ Pa

$\sigma_n' = \sigma_n$ (no pore pressure at base)



$\rho_{sandstone} = 0.8*2.67\ kg/m^3 = 2.14\ kg/m^3$

IV Effective stress: Example 3

Now suppose a saturated block of quartz sandstone rests on a horizontal surface. Further assume that the porosity of the sandstone is 20%. What is the total normal stress and total effective stress at the base of a 10m tall block of this sandstone?

$\sigma_n = (\rho_{sat})(g)(h)$

$\sigma_n = (2.34 \times 10^3\ kg/m^3)(9.81\ m/sec^2)(10m)$

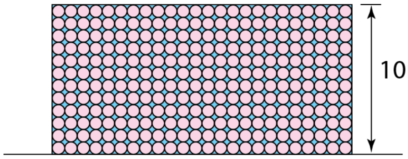
$\sigma_n = 2.30 \times 10^5$ Pa

$\sigma_n' = \sigma_n - P = (\rho_{sat})(g)(h) - (\rho_{water})(g)(h)$

$\sigma_n' = \sigma_n - P = 2.30 \times 10^5$ Pa $- 0.98 \times 10^5$ Pa

$\sigma_n' = 1.32 \times 10^5$ Pa

So saturating the block increases the total stress but decreases the effective stress



$\rho_{sat} = 0.8*2.67\ kg/m^3 + 0.2*1.00\ kg/m^3$
$= 2.34\ kg/m^3$

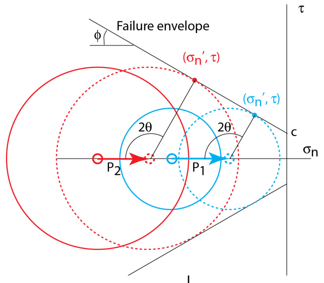
$\rho_{sat} \neq \rho_{sandstone} + \rho_{water}$ $(2.30 \neq 2.13 + 1.00)$

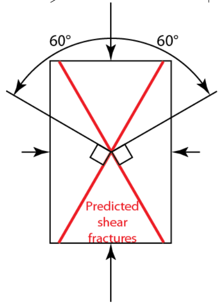
V Mohr-Coulomb shear failure

A $\tau_{failure} = c + \sigma'_{failure} \tan\phi$, where σ' = effective normal stress

- $(\phi + 90°)/2 = \theta$
  If $\phi = 30°$, then $\theta = 60°$

B Shear failure predicted on planes with normals at ±60° to the most compressive stress for isotropic materials

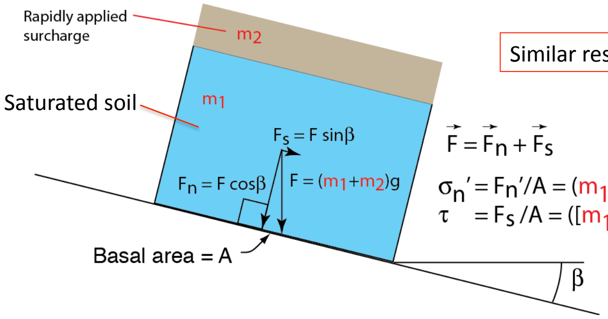




V Mohr-Coulomb shear failure

C Surcharge rapidly applied to a saturated soil

1 Over short times, saturated soil won't drain
2 Weight of surcharge borne by pore pressure
3 Effective normal stress at base unchanged
4 Shear driving traction does increase



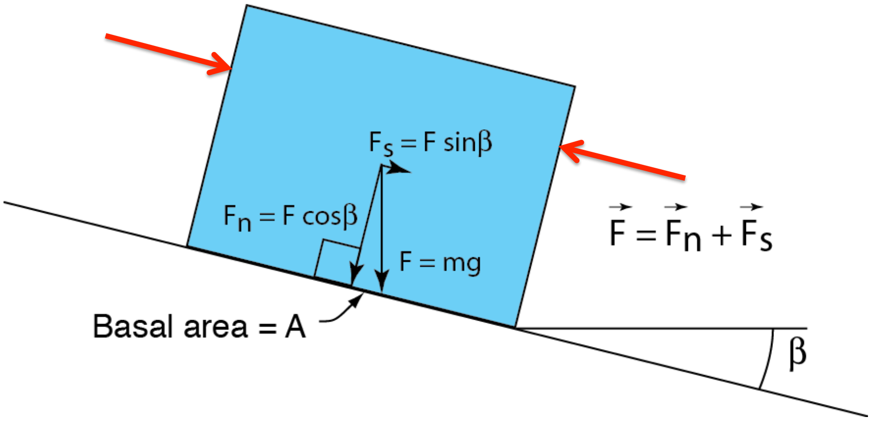
Similar result if m_2 is water

$\vec{F} = \vec{F}_n + \vec{F}_s$

$\sigma_n' = F_n'/A = (m_1 g)\cos\beta/A$

$\tau = F_s/A = ([m_1 + m_2]g)\cos\beta/A$

VI Factors promoting shear failure



$\vec{F} = \vec{F}_n + \vec{F}_s$

A Increase in Pore Pressure (Decrease in effective stress)

1 Rain
2 Reservoir level rise

Hiroshima, August, 2014

http://www.ibtimes.co.uk/landslide-slams-into-hiroshima
-western-japan-after-months-rain-overnight-1461862

Vaiont dam, Italy

http://en.wikipedia.org/wiki/Vajont_Dam

B Increase in shear driving stress

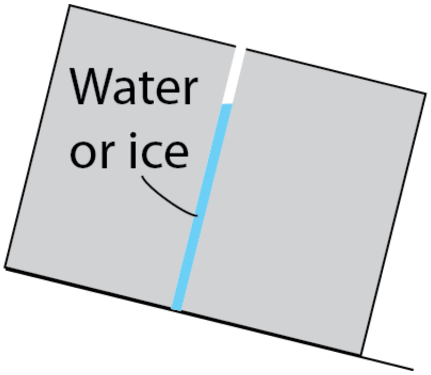
1 Surcharge
    a Construction of fill
    b Accumulation of talus
    c Stockpiles of ore, waste
    d Weight of buildings
    e Weight of precipitation
    f Weight of water from leaking pipes, lawn watering, pools, etc.

Surcharge to compact underlying soil

http://dharma-rain.org/wp-content/uploads/2013/10/Surcharge.jpg

B Increase in shear driving stress

2 Increase in Lateral Pressure
- a Water in cracks
- b Swelling (ice, clays)



B Increase in shear driving stress

3 Transient Earth Stresses
- a Earthquakes
- b Vibrations from machinery, traffic, blasting, etc.
- c Volcanic processes

Rockfalls on Mt. Baldwin
Triggered by 1980 Mammoth Lakes earthquakes

From Wieczorek, 2002

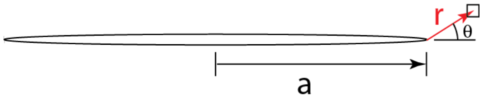
C Weak layers and weak surfaces

- Inherently weak geologic materials (clay, shale, organic material, schists, soft tuffs, talc, serpentine
- Discontinuities (Bedding planes, faults, joints)
- Strata inclined towards free face ("dip slopes")

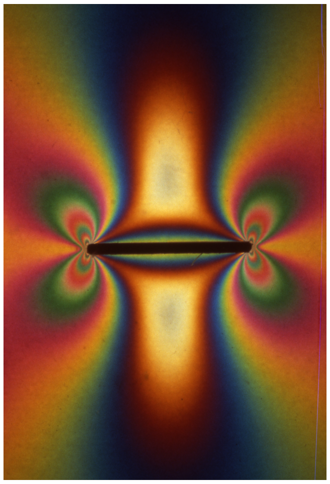
D Reduction in Shear Strength

1 Physical effects
Growth of cracks

$$\sigma_{ij(near-tip)} = \Delta\sigma_m \sqrt{\frac{a}{2r}} f_{ij}(\theta)$$



As a crack grows, the near-tip stresses increase
As cracks form, more stress concentrations arise

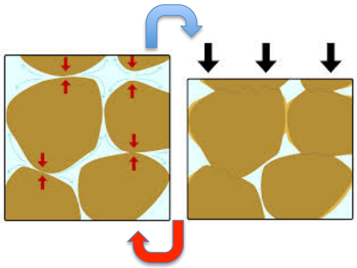


http://webpages.uidaho.edu/~simkat/course_materials/geol542/photoelast.jpg

D Reduction in Shear Strength

2 Chemical effects
  - a Weathering of rocks
  - b Chemical exchange or leaching (e.g. quick clays)
  - c Dissolution of cement
  - d Dehydration/hydration of clay minerals (shrink/swell)

http://en.wikipedia.org/wiki/Tafoni



http://en.wikipedia.org/wiki/Sedimentary_rock

E Removal of Buttressing Support

1 Erosion of toe areas by streams, surf, glacial erosion

2 Mass wasting of supporting lower slope

3 Quarries, canals, road cuts, tunnels

https://dr282zn36sxxg.cloudfront.net/datastreams/f-d%3A919d179372d843bf01af2b7c65f121cf55e7827f6ba7f853c425689e%2BIMAGE_THUMB_POSTCARD%2BIMAGE_THUMB_POSTCARD.1

E Removal of Buttressing Support

4 Rapid drawdown/loss of reservoirs

Teton Dam and reservoir, after failure

http://www.usbr.gov/pn/about/Teton.html

Landslide after loss of reservoir

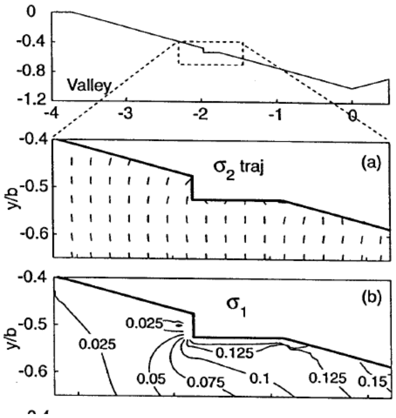
http://www.usbr.gov/pmts/sediment/projects/TetonRiver/TetonRiver.htm

F Other factors

1 Stress concentrations

Lower plot at right shows a predicted tensile stress concentration near the corner of the cut. Middle plot shows predicted orientations of opening cracks (if cracks open).

Predicted stresses in a valley slope

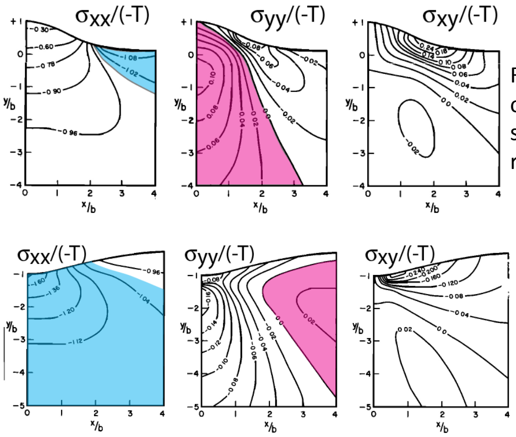


From Muller and Martel, 2000

F Other factors

2 Horizontal compressive stresses parallel to curved slopes
  - a Reduces near-surface compressive stress perpendicular to convex slopes
  - b Increases near-surface compressive stress perpendicular to concave slopes

Plots to right show how topography perturbs a uniform regional tectonic stress T. Horizontal stresses become more compressive than regional compression in blue areas. Vertical compressive stresses diminish in magenta areas.



Right half of a symmetric ridge

Right half of a symmetric valley

From Savage and Swolfs, 1986