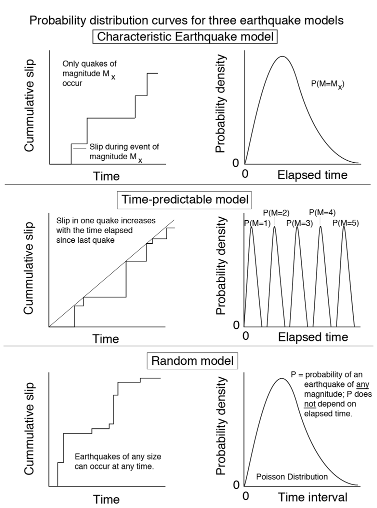
RECURRENCE INTERVALS AND PROBABILITY (18)

I Main Topics
- A Recognition, Characterization, <u>Risk Evaluation</u>, Risk Assessment
- B Recurrence intervals
- C Simple empirical earthquake recurrence models
- D Seismic gaps
- E Probability density functions
- F Exercise

II Recognition, Characterization, <u>Risk Evaluation</u>, Risk Assessment

- A Probabilistic estimates of event likelihood, and cost estimates, are used to evaluate risk
- B Steps 1 and 2 must be done in order to get to step 3 (and then 4)
- C Outcomes depend on model
- D Applicable to many situations



III Recurrence intervals

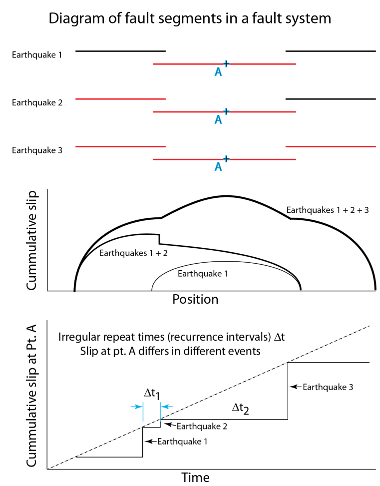
A Used to evaluate when an earthquake might occur

B Recurrence interval = time between consecutive earthquakes (of a given magnitude)

C Can be determined by geologic means

1 Dating individual events (e.g. data from trench study)

2 Average recurrence int. = Average slip per event/ average slip rate



IV Simple empirical earthquake recurrence models

A Characteristic model

1 Same rupture length, slip distribution, and seismic moment

2 Recurrence interval can vary through time

B Constant slip rate model

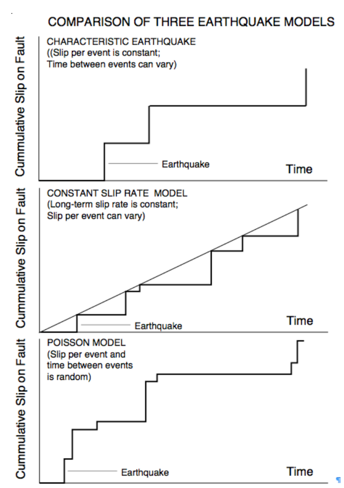
1 Slip rate across fault is constant

2 Recurrence interval depends on slip during earthquake

C Random (Poisson) model

1 Historical record too short to separate patterns from "noise"

2 Earthquakes considered as random events in time



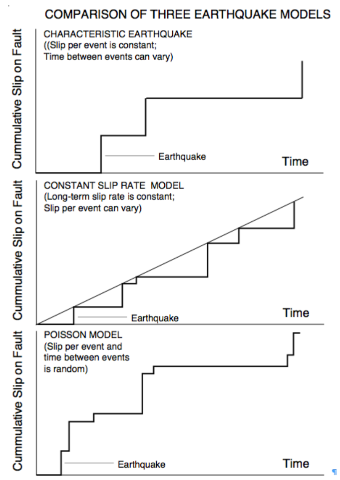
IV Simple empirical earthquake recurrence models

D Sources of error

1 Resolving dates of events

2 Missing events in the record

E These models are not physical models



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V Seismic gaps

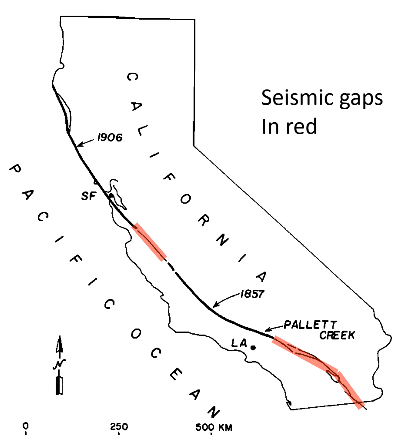
A Used to evaluate where an earthquake is likely to occur

B Along an active fault, the probability of an earthquake increases with time elapsed since the last event

C Physical basis

1 Shear tractions at gap have had time to build

2 Slip on adjacent segments loads the "dormant" segment



From Sieh (1978)

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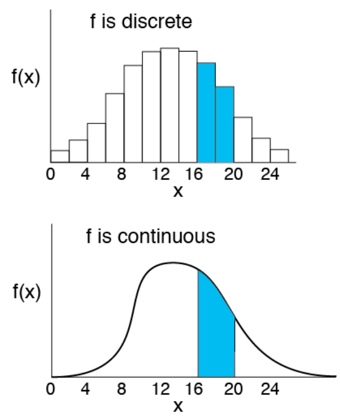
VI Probability density functions

A Probability density functions [f(x)]
1. Used to describe the relative likelihood of an occurrence
2. Probability (a<X<b) = P(a<X<b): probability of an outcome between a and b = area under f(x) from a to b
3. Examples
   a. Discrete example: Probability of a basketball player scoring 16-20 points in a game
   b. Continuous example: Probability of an earthquake of M_w = 7.5 in next 16-20 years

Examples of probability density functions



VI Probability density functions

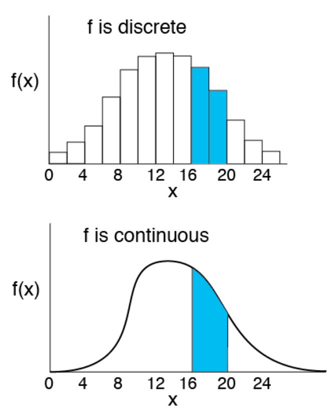
4 $P(-\infty < x < +\infty) = \int_{-\infty}^{\infty} f(x)\,dx = 1 = 100\%$

5 For <u>continuous</u> distributions

a $P(a < x < b) = \int_a^b f(x)\,dx$

b $P(x = a) = \int_a^a f(x)\,dx = 0$

6 For discrete distributions
P(*x*=*a*)≠0 if *x* is in allowable range



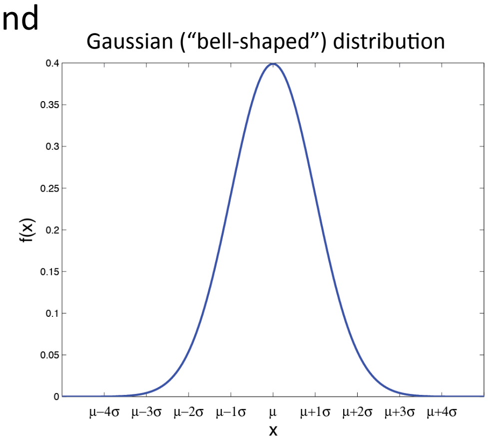
B The normal (Gaussian) distribution

1 Described by mean μ and standard deviation σ

$$g(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\mu=\frac{\sum_{i=1}^{n}x_i}{n}$$

$$\sigma=\sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n-1}}$$



B The normal (Gaussian) distribution

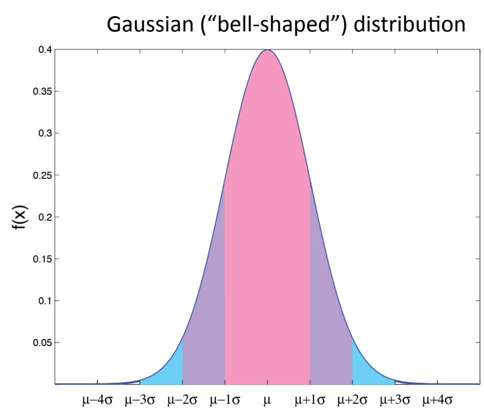
2 P(μ-1σ<x<μ+1σ) ≈ 2/3

3 P(μ-2σ<x<μ+2σ) ≈ 95%

4 P(μ-3σ<x<μ+3σ) ≈ 99%

5 P(μ<x<∞) = 50%

6 P(-∞<x<μ) = 50%



C The Poisson distribution

1 Describes probability of x discrete events in a time interval Δt , where the expected (mean) number of events is μ

2 Events are independent (one does not affect another)

3 $$P(X=x)=\frac{\mu^x e^{-\mu}}{x!}=\frac{(\Delta t/\overline{R})^x e^{-(\Delta t/\overline{R})}}{x!}$$

where

$1/\overline{R}$ = mean recurrence rate

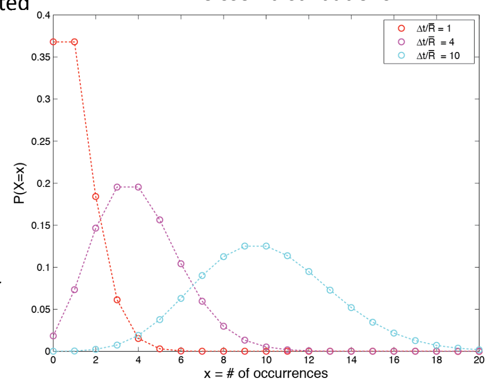
Δt = time interval

x = # of discrete events in interval Δt

μ = expected # of events = $\Delta t/\overline{R}$

$$\overline{R}=\frac{\sum_{i=1}^{n} R_i}{n}$$

Poisson distributions



The maximum probability occurs for

$$x=\Delta t/\overline{R} \text{ and } x=\left(\Delta t/\overline{R}\right)-1$$

VII Exercise: probability of a large earthquake in southern California

<u>Question</u>: What is probability of earthquake in next 30 years?

<u>Data:</u> Time of the last 12 large earthquakes at Pallet Creek (from Sieh, 1984; uncertainties are omitted)

- 1857, 1720, 1550, 1350, 1080, 1015, 935, 845, 735, 590, 350, 260

1 Calculate the average (mean) recurrence interval

- Mean Recurrence Interval = (1857-260)years/11 intervals
  = 1597 yr/11 = 145 years

2 Calculate the recurrence intervals between earthquakes

- 137, 170, 200, 270, 65, 80, 90, 110, 145, 240, 90 years

VII Exercise: probability of a large earthquake in southern California

3 Calculate the standard deviation of the 11 recurrence intervals:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(R_i - \overline{R}\right)^2}{n-1}}$$

where σ is the standard deviation, R_i is the recurrence time between a given pair of events, $\overline{R}$ is the mean recurrence interval, and *n* is the number of recurrence intervals (<u>not</u> the # of quakes).

- σ = 68 years

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VII Exercise: probability of a large earthquake in southern California

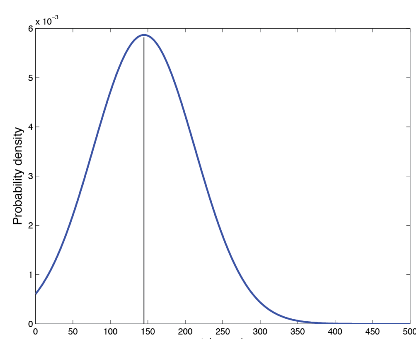
4 <u>Normal distribution model</u>

- Plot shows normal distribution for $\overline{R}$ = 145 years, and σ = 68 years.
- Function is truncated at t = 0

$$\int_{-\infty}^{\infty} f(x) = 1$$

- Will adjust for truncation*

$$\int_{0}^{\infty} f(x) = 0.9835$$



$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-\left(t-\overline{R}\right)}{2\sigma^2}$$

From statistics tables: 145 yrs/68 yrs = 2.1324 sd. dev.
https://www.stat.tamu.edu/~lzhou/stat302/standardnormaltable.pdf

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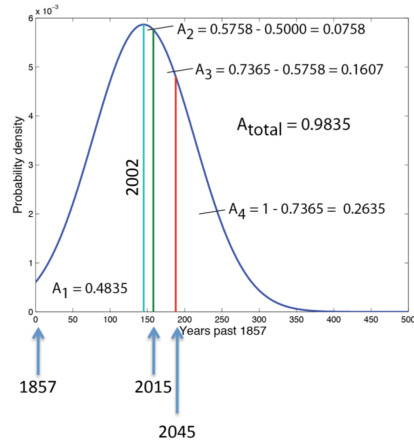
VII Exercise: probability of a large earthquake in southern California

5 Calculation of probability of earthquake from 2015-2045 *made in 1857* assuming a *modified* normal distribution model

Year	Year from $1857+\bar{R}$	Std. Dev. from $1857+\bar{R}$	Area from $1857+\bar{R}$
1857	145	145/68= 2.1324	A_1 = 0.4835
2002	0	0	0
2015	13	13/68 = 0.1912	A_2 = 0.0758
2045	43	43/68 = 0.6324	$A_2 + A_3$ = 0.2365



$P = A_3/A_{total}$ = 0.1607/0.9835 = 16%

VII Exercise: probability of a large earthquake in southern California

6 Calculation of probability of earthquake from 2015-2045 *made in 2015* assuming a *modified* normal distribution model

Year	Year from $1857+\bar{R}$	Std. Dev. from $1857+\bar{R}$	Area from $1857+\bar{R}$
1857	145	145/68= 2.1324	A_1 = 0.4835
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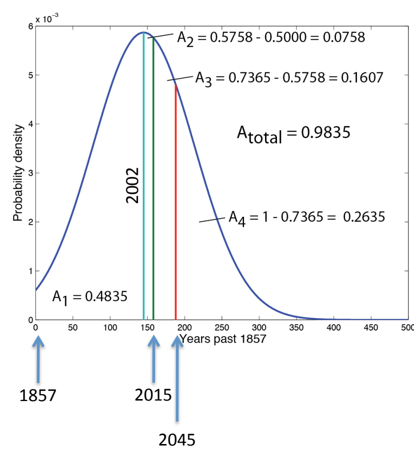


$P = A_3/(A_3 + A_4)$ = 0.1607/0.4242 = 38%

VII Exercise: probability of a large earthquake in southern California

7 Calculation of probability of earthquake from 2015-2045 *made in 1857 or 2015* assuming a Poisson distribution model

- Poisson model, $\overline{R}$ = 145 years

$$P(X=x)=\frac{\mu^x\, e^{-\mu}}{x!}=\frac{(\Delta t/\overline{R})^x\, e^{-(\Delta t/\overline{R})}}{x!}$$

- The probability of one event in 30 years is:

$$P(X=1)=\frac{\left(\frac{30\ yrs}{145\ yrs}\right)^1 e^{-\left(\frac{30\ yrs}{145\ yrs}\right)}}{1!}=\frac{30}{145}e^{-30/145}=17\%$$

- The probability of no event in 30 years is:

$$P(X=0)=\frac{\left(\frac{30\,yrs}{145\ yrs}\right)^0 e^{-\left(\frac{30\,yrs}{145\ yrs}\right)}}{0!}=e^{-30/145}=81\%$$

- The probability of at least one event in 30 years is 1 - 0.81 = 19%

VII Exercise: probability of a large earthquake in southern California

8 If real earthquakes occurred but were not documented at Pallet creek, then the mean recurrence interval and standard deviation would be in error

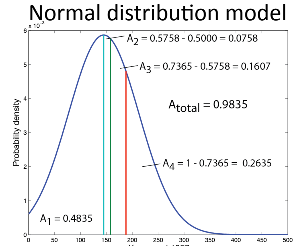
9 Calculations with synthetic data can test how robust calculated probabilities are

10 Probabilities depend on data and model

11 USGS estimated in 2007 a 46% probability of an earthquake ≥ M_w = 7.5 in southern California before 2037

http://www.scec.org/ucerf2/

Normal distribution model



Poisson distribution model