

## RESPONSE OF STRUCTURES (16)

### I Main Topics

A Seismic records

B Acceleration, velocity and displacement spectra

B Resonance and natural frequencies

C Response of structures

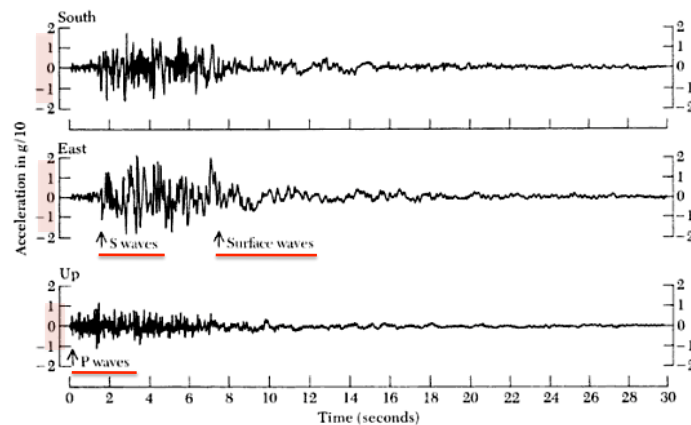
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## II Seismic time series records

A Accelerogram (plot of acceleration vs. time)



Three components of ground acceleration recorded from a site in Hollywood ~20 km from the 1971 San Fernando earthquake fault rupture. From Bolt, 1988.

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## B Time series records of acceleration, velocity, and displacement

- 1 Acceleration (a)
  - a  $a = dv/dt$
  - b Peak force at peak acceleration
  - c Peak acceleration at right:  $\sim 1100 \text{ cm/sec}^2$
  - d Time of peak acceleration:  $\sim 7.5 \text{ sec}$
- 2 Velocity (speed) (v)
  - a  $v = du/dt$
  - b Peak kinetic energy at peak velocity
  - c Peak velocity at right:  $\sim 115 \text{ cm/sec}$
  - d Time of peak velocity:  $\sim 3 \text{ sec}$
- 3 Displacement (u)
  - a Peak displacement at right:  $\sim 40 \text{ cm}$
  - b Time of peak displacement:  $\sim 5 \text{ sec}$

Recordings of N.  $14^\circ$  E. component of horizontal ground motion at Pacoima damsite for 1971 San Fernando earthquake (from Page and others, 1975).

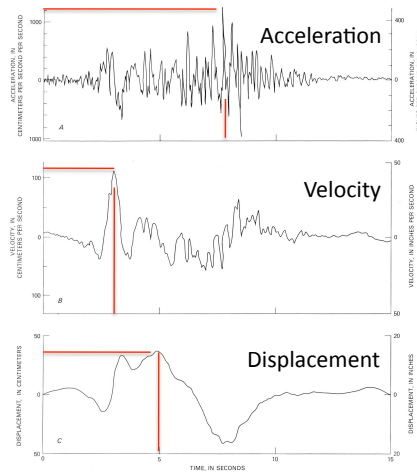


Fig. 31 from USGS Prof. Paper 1360

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## III Seismic spectra

- A Spectra represent parameters (e.g., displacement, velocity, and acceleration) as a function of wave frequency (or period), not time
- B Reveal the frequency (or period) and amplitude of the most energetic/forceful waves

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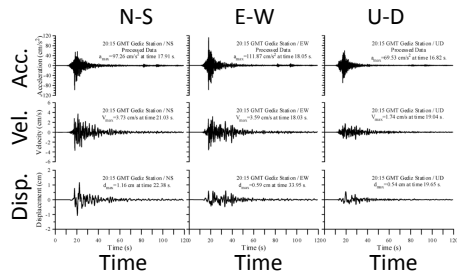
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### III Seismic Spectra

C Example: Simav earthquake, Turkey, 2011, M = 5.7-5.9

Time series records

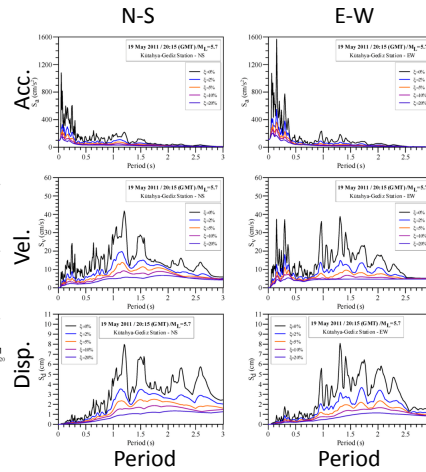


Doğangün et al., Buildings 2013, 3, 173-190; doi:10.3390/buildings3010173  
<http://www.mdpi.com/2075-5309/3/1/173/htm>

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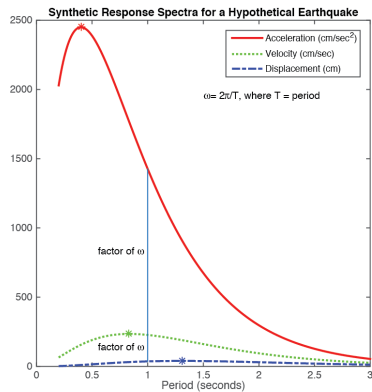
Seismic spectra for Simav event



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### III Seismic Spectra

D Seismic spectra envelopes  
 Example: Simav earthquake, Turkey, 2011, M = 5.7-5.9

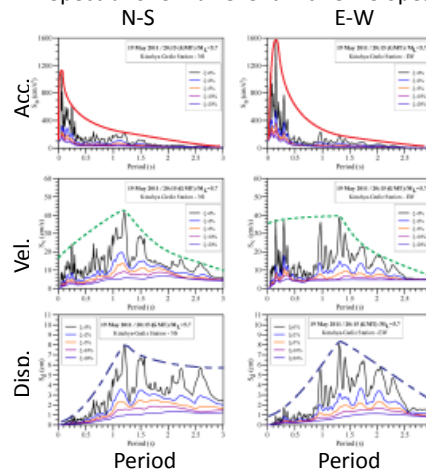


Envelopes give maximum amplitudes

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Spectra for Simav event with envelopes



Doğangün et al., Buildings 2013, 3, 173-190; doi:10.3390/buildings3010173  
<http://www.mdpi.com/2075-5309/3/1/173/htm>

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### III Seismic spectra

#### E Relationships among spectral envelopes

1  $y = A \sin[(2\pi/\lambda)(x + vt)]$

$v = \lambda/T = f\lambda; \lambda = v/f$

$y = A \sin[(2\pi f/v)(x + vt)]$

2 Now consider a particular point

$y_0 = y(x = 0)$

$= A \sin[(2\pi f/v)(vt)]$

$= A \sin(2\pi ft)$

3 Let  $\omega = 2\pi f = 2\pi/T = \text{angular frequency}$

$y_0 = A \sin(\omega t)$

$y_0' = \frac{d[A \sin(\omega t)]}{dt}$   
 $= A\omega \cos(\omega t)$

$y_0'' = \frac{d(y_0')}{dt} = \frac{d[A\omega \cos(\omega t)]}{dt}$   
 $= -A\omega^2 \sin(\omega t) = -\omega^2 y_0$

$\omega = \left( \left| \frac{y_0''}{y_0} \right| \right)^{1/2}$

### III Seismic spectra

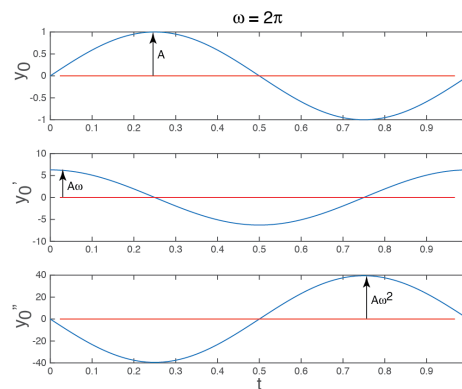
#### F Amplitude relationships among displacement, velocity, and accelerations for a single angular frequency

Let  $\omega = 2\pi f = 2\pi/T = \text{angular frequency}$

$y_0 = A \sin(\omega t)$

$y_0' = A\omega \cos(\omega t)$

$y_0'' = -A\omega^2 \sin(\omega t) = -\omega^2 y_0$

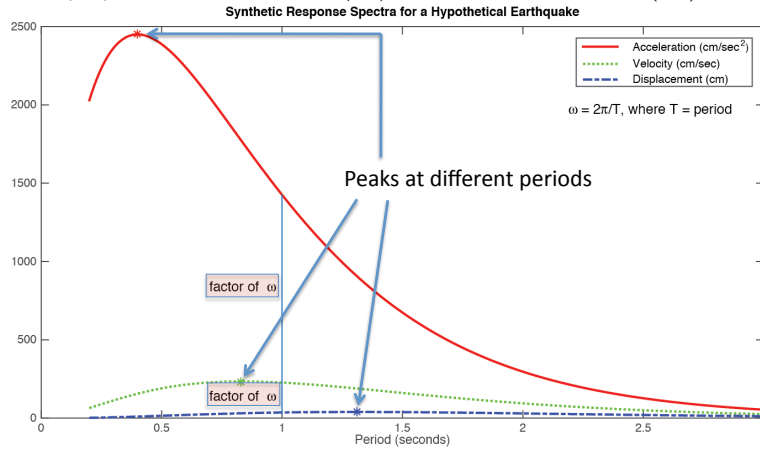


### III Seismic spectra

F Relationships among spectral envelopes for a frequency range

Let  $\omega = 2\pi f = 2\pi/T = \text{angular frequency}$

$$y_0 = A \sin(\omega t) \quad y_0' = A\omega \cos(\omega t) \quad y_0'' = -A\omega^2 \sin(\omega t) = -\omega^2 y_0$$



### III Seismic spectra

Frequency (f)	(Hz)	0.01*	0.1*	1*	10*	100
Angular frequency $\omega$		0.06	0.6	6	63	628
Maximum displacement (y)	(m)	$10^{-3}$	$10^{-3}$	$10^{-3}$	$10^{-3}$	$10^{-3}$
Maximum velocity $y'$ (= $\omega y$ )	(m/sec)	$6 \times 10^{-5}$	$6 \times 10^{-4}$	$6 \times 10^{-3}$	$6 \times 10^{-2}$	$6 \times 10^{-1}$
Max. acceleration $y''$ (= $ \omega^2 y $ )	(m/sec <sup>2</sup> )	$39 \times 10^{-5}$	$39 \times 10^{-4}$	$39 \times 10^{-3}$	$39 \times 10^{-2}$	$39 \times 10^{-1}$

G Max. acceleration (forces) commonly at 2-10 Hz (T=0.1-0.5 sec), max. velocities (kinetic energy) at 0.5-2 Hz (T=0.5-2 sec), and max. displacements at 0.006-0.5 Hz (T=2-160 sec)

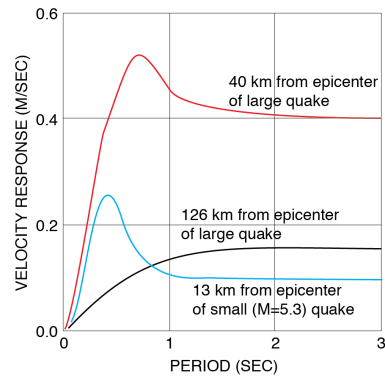
- 1 High frequency (small period) waves: high amplitudes of acceleration, small amplitudes of displacement
- 2 Low frequency (long period) waves: low amplitudes of acceleration, large amplitudes of displacement

\*Important frequency for design of large engineering structures

## III Seismic spectra

### H Effects of source and distance IDEALIZED UNDAMPED VELOCITY SPECTRUM CURVES

- 1 A small, nearby earthquake can affect short-period structures more than a larger, distant quake
- 2 A large, distant earthquake can affect long-period structures more than a smaller, nearby quake
- 3 Short-period (high frequency) waves attenuate with distance more readily than long-period (low frequency) waves



From Housner, 1970, in Wiegel, 1970

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## IV Resonance and natural frequencies

- A Resonance: vibration of large amplitude due to arrival of energy at a particular frequency
- B Natural frequency: The frequency at which a structure will resonate
- C Natural frequency of a pendulum
  - 1 Natural period:  

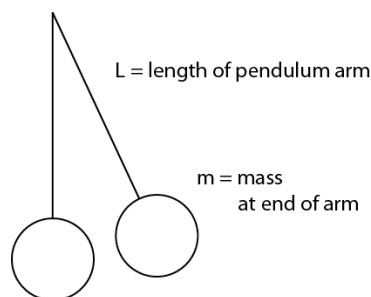
$$T = 2\pi(L/g)^{1/2}$$

Natural period increases with length
  - 2 Natural frequency:  

$$f = 1/T = (g/L)^{1/2}/(2\pi)$$
  - 3 Natural angular frequency:  

$$\omega = 2\pi f = (g/L)^{1/2}$$

Diagram of a simple pendulum



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## IV Resonance and natural frequencies

D Natural frequency of a mass on a spring (simple harmonic oscillator)

1 Natural period:

$$T = 2\pi(m/k)^{1/2}$$

2 Natural frequency:

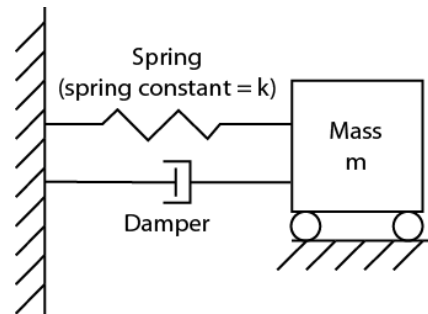
$$f = 1/T = (k/m)^{1/2}/(2\pi)$$

3 Natural angular frequency:

$$\omega = 2\pi f = (k/m)^{1/2}$$

\* No damping in these expressions

Diagram of a simple harmonic oscillator with damping



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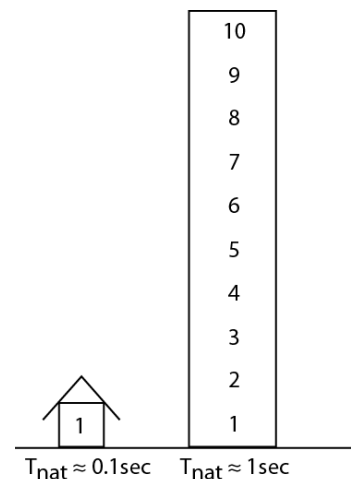
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## IV Resonance and natural frequencies

E Rule of thumb for buildings:  
natural period = # of stories/10

F Avoid structural designs with natural periods that match the natural period of the underlying materials (or the source)

G Previous experience helpful for step F



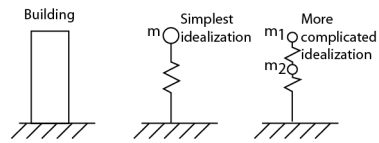
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## V Response of structures

- A Earthquakes commonly impart large shear forces at the base of a building
- B Bolt buildings to foundations and have sufficiently stiff ground floors
- C Asymmetric designs susceptible to twisting
- D Sophisticated models are used now to help design critical structures (beyond the scope of this course)



Collapsed General Hospital  
1985 Mexico City earthquake



[http://en.wikipedia.org/wiki/1985\\_Mexico\\_City\\_earthquake](http://en.wikipedia.org/wiki/1985_Mexico_City_earthquake)