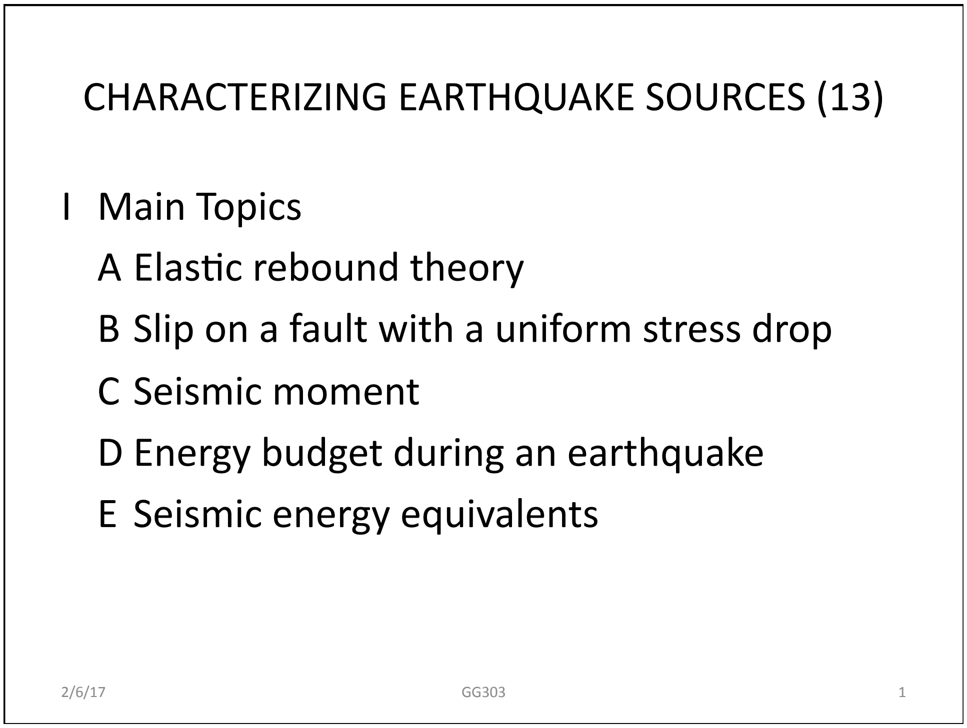
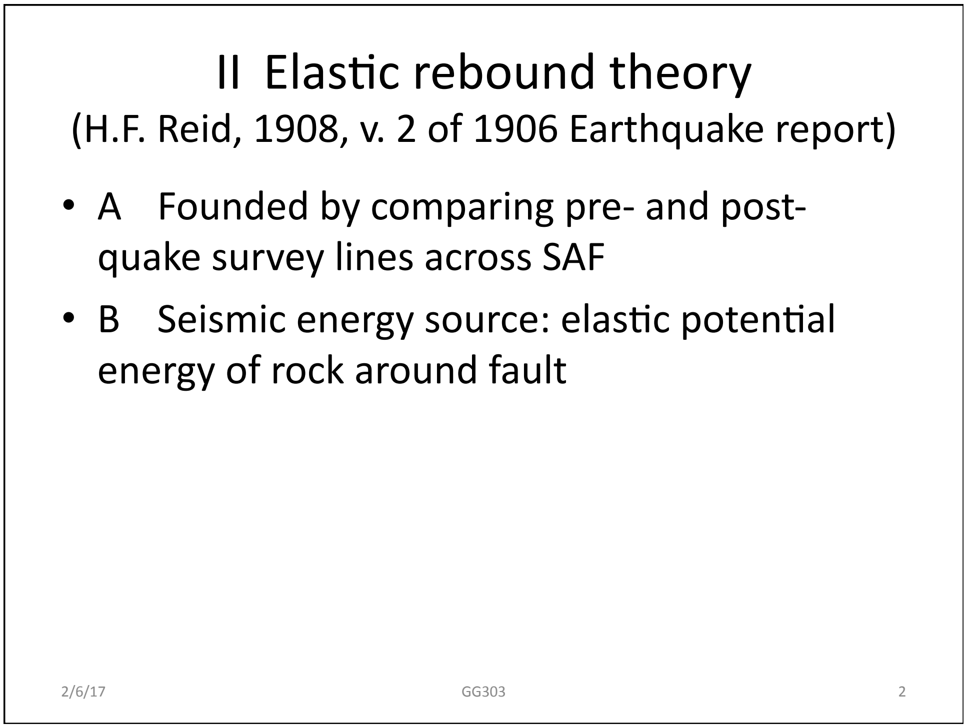


CHARACTERIZING EARTHQUAKE SOURCES (13)

I Main Topics

- A Elastic rebound theory
- B Slip on a fault with a uniform stress drop
- C Seismic moment
- D Energy budget during an earthquake
- E Seismic energy equivalents



II Elastic rebound theory

(H.F. Reid, 1908, v. 2 of 1906 Earthquake report)

- A Founded by comparing pre- and post-quake survey lines across SAF
- B Seismic energy source: elastic potential energy of rock around fault

II Elastic rebound theory (H.F. Reid, 1906 earthquake report)

Sequence of survey lines: actual (solid) and hypothetical (dashed)

1 Hypothetical survey line (1806)
- ~3m of far-field relative displacement likely between "1806" and 1851-1865

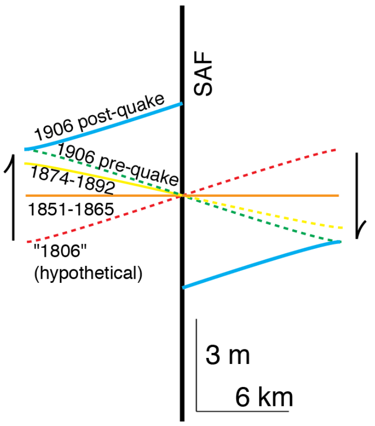
2 Real survey (1851-1865)
- ~1-2m far-field relative displacement from 1851-1865 to 1874-1892

3 Real survey (1874-1892)
- ~1-2m far-field relative displacement from 1874/1892 - 1906
- ~3m far-field relative displacement in ~50 years from 1851-1865 to 1906.

4 Hypothetical survey in 1906 before earthquake
- 6m slip in 1906 (2x the far-field relative displacement in previous ~50 years*

5 Real survey after 1906 quake (note apparent "overshoot" along fault relative to 1851-1865)

Diagrammatic survey lines across San Andreas fault (SAF)



* Basis for 100-year recurrence interval

III Slip (Δu) on a 2D fault with a uniform shear stress drop ($\Delta\tau = \tau_1 - \tau_2$)

A Rock is elastic, homogenous, isotropic, isothermal material

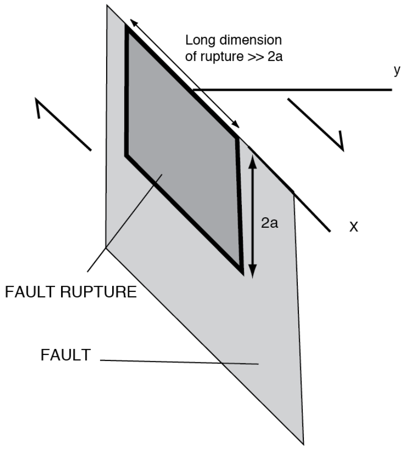
B Shear stress on fault in direction of slip prior to slip = τ_1; post-slip shear stress = τ_2

C Slip profile is related to the shape and size of the rupture

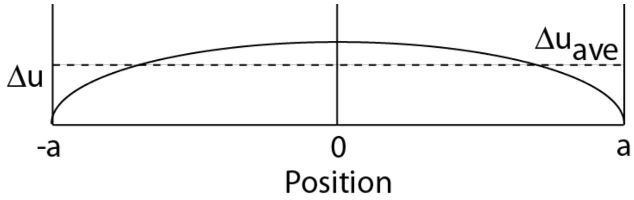
D Slip distribution is particularly sensitive to the <u>short</u> dimension

E For a "2-D" rupture (one dimension >> other dimension) suppose the short dimension = 2a

FAULT GEOMETRY AND FAULT RUPTURE GEOMETRY



III Slip (Δu) on a 2D fault with a uniform shear stress drop ($\Delta\tau = \tau_1 - \tau_2$)



μ = shear modulus
ν = Poisson's ratio

1 $\Delta u = 2(1-\nu)\,(\Delta\tau/\mu)\,(a^2 - x^2)^{1/2}$

2 $\Delta u_{max} = \Delta u(x = 0) = 2(1-\nu)\,(\Delta\tau/\mu)\,a \approx 3 \times 10^{-4}\,a$

3 $\Delta u_{min} = \Delta u(x = \pm a) = 0$

4 $$\Delta u_{ave} = \frac{\int_{-a}^{a} \Delta u\,dx}{2a} = \frac{\pi}{4}(\Delta umax)$$

IV Seismic moment M_o

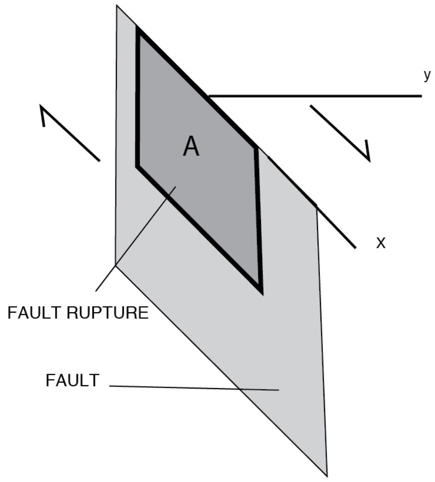
A $M_o = \mu\,\Delta u_{ave}\,A$
μ = shear modulus
A = rupture area

B M_o has dimensions of energy; used to measure earthquake size

C μ has been measured; geologists can estimate Δu_{ave} and A

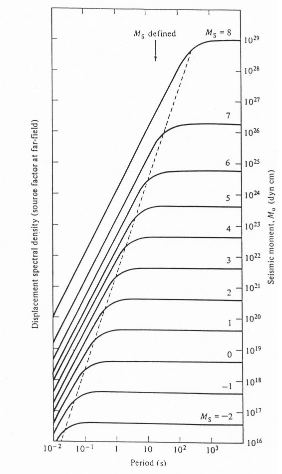
D Seismic moments can be predicted

E Moment can characterize a faulting earthquake of any size



IV Seismic moment M_o

F Scaling laws for earthquakes
  1 Elasticity theory predicts seismic wave amplitudes as a function of the wave period for fault ruptures of different size and slip
  2 These amplitude curves can be shifted and calibrated against real seismic records of previous earthquakes of assigned magnitudes
  3 Waves with very long periods yield the seismic moment
  4 $M_w \approx 2/3 \log M_o - 10.73$
  where M_o is in dyne-cm
  5 $M_w \approx 2/3 \log M_o - 6.07$
  where M_o is in Nm
  6 Amplitude hits a ceiling (saturates) for periods greater than a certain value for a given seismic moment

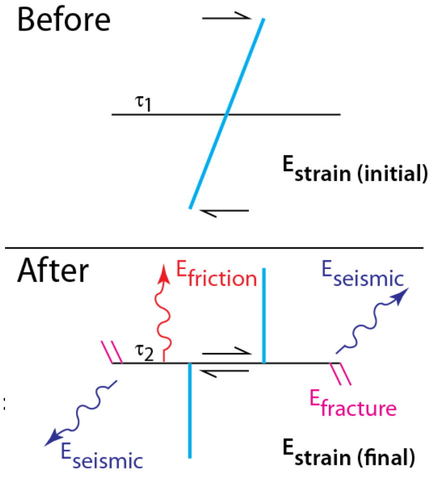


Seismic moment vs. wave period
for various earthquake magnitudes
(from Scholz, 2002, Fig. 4.8)

V Energy budget during an earthquake (From Scholz, 1990)

$$E_{seismic} + \Delta E_{strain} + \Delta E_{friction} + \Delta E_{fracture} + \Delta E_{chemical} + ? = 0$$

A Kinetic energy ($E_{seismic}$) in seismic waves
  1 $E_{seismic}$ varies with amplitude and wavelength
  2 Waves of a frequency of zero ("ultra-long period and wavelength") correspond to a static situation (“permanent” deformation)
  3 Seismic moment, which describes the “permanent” deformation after an earthquake, should be related to seismic energy release
  4 Empirical relationship of seismic energy ($E_{seismic}$) to magnitude (M_w):
  $E_{seismic}$ (joules)= $10(4.8 + 1.5\ M_w)$



V Energy budget during an earthquake (From Scholz, 1990)

B Strain energy (ΔE_{strain})

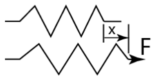
1 Energy in a linear spring

a Force in a spring

$F = kx$

k = spring constant

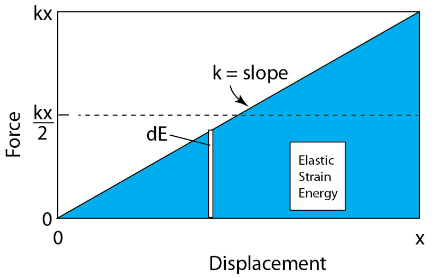
x = displacement



b Strain energy (ΔE_{strain}) equals area under a force-disp. curve

$\Delta E_{strain} > 0$ if spring stretches

$\Delta E_{strain} < 0$ if spring contracts



$$\Delta E_{strain} = \int_0^x F\,dx = \int_0^x kx\,dx = \left[\frac{1}{2}kx^2\right]_0^x = \frac{1}{2}kx^2$$

$$= \left(\frac{kx}{2}\right)(x)$$

Average force on spring during displacement

Displacement of spring end

V Energy budget during an earthquake (From Scholz, 2002)

1 $E_{seismic} + \Delta E_{strain} + \Delta E_{friction} + \Delta E_{fracture} = 0$

- The strain energy in the earth decreases after a quake, so $\Delta E_{strain} < 0$.
- Energy appears in the form of heat, so $\Delta E_{friction} > 0$.
- *If* the energy to create fractures is assumed to be negligible*, then

2 $E_{seismic} \approx -(\Delta E_{strain}) - (\Delta E_{friction})$

3 $E_{seismic} \approx \underbrace{(1/2)[\tau_1 + \tau_2]}_{\text{Average shear stress during slip}} [\Delta u_{ave}]\,[A] \; - \; (1/2)(2)(\underbrace{[\tau_2]}_{\text{Assumed shear stress during slip}} [\Delta u_{ave}]\,[A]$

4 $E_{seismic} \approx (1/2)[\tau_1 - \tau_2]\,[\Delta u_{ave}]\,[A]) \; = \; (1/2)\underbrace{[\Delta\tau]}_{\text{Shear stress drop during slip}}\,[\Delta u_{ave}]\,[A]$

5 So the seismic kinetic energy depends on the strength change on the fault $\Delta\tau$

* Substantial uncertainty

V Energy budget during an earthquake

C Formulas relating seismic energy release ($E_{seismic}$), moment (M_o), and moment magnitude (M_w)

1 $M_w \approx 2/3 \log M_o - 6.067$ M_o in Nm

2 $E_{seismic} \approx 10^{(4.8 + 1.5 M_s)}$ E_s in joules

3 $E_{seismic} \approx M_o/20{,}000$

4 $\log E_{seismic} \approx \log M_o - 4.3$

5 $\log M_o \approx 1.5 M_w + 9.1$

1 To dovetail with magnitude from surface waves

2 Calibration between magnitude and energy release done using nuclear tests

V Energy budget during an earthquake

D Energy Release vs. Magnitude

The empirical relationship between energy content in radiated seismic waves and magnitude is (Richter, 1958; Bolt, 1989):

E_s (joules) = $10^{(4.8 + 1.5 Ms)}$

M_s is the surface wave magnitude.

Consider two earthquakes that differ in magnitude by 1, where $M_{s1} = 1 + M_{s2}$. Then

E_{s1}/E_{s2} $= 10^{(4.8 + 1.5 [M_{s2} +1])}/10^{(4.8 + 1.5 M_{s2})}$

$= \{(10^{4.8})(10^{1.5 [M_{s2}]})(10^{1.5})\}/\{(10^{4.8})(10^{1.5 M_{s2}})\}$

$= 10^{1.5} \approx 31.6$

A unit increase in magnitude corresponds to (a) a factor of 10 increase in amplitude of shaking, and (b) a factor of 31.6 increase in energy release.

One magnitude 8 quake releases the energy of 1000 magnitude 6 quakes.

V Energy budget during an earthquake

E Relationship between seismic energy (M_w) and seismic moment (M_o)
$M_w = 2/3 \log M_o - 6.067$, where M_o is measured in Nm p. 249 of Bolt

This empirical relation dovetails magnitudes from surface waves and seismic moment (i.e. $M_s = M_w$). What is the relationship between M_o and $E_{seismic}$?

$E_{seismic} = 10^{(4.8 + 1.5\ Ms)}$ where E_s is in joules p. 179 of Scholz

$$
\begin{aligned}
E_{seismic} &= 10^{(4.8 + 1.5\ Ms)} \\
&= 10^{(4.8 + 1.5\ [2/3 \log Mo - 6.067])} \\
&= 10^{(4.8 +\ \log Mo - [1.5][6.067])} \\
&= 10^{(4.8 + \log Mo - 9.1)} \\
&= 10^{(\log Mo - 4.3)} \\
&= [10^{\log Mo}][10^{-4.3}] \\
E_{seismic} &= M_o/20{,}000]
\end{aligned}
$$

Alternatively, $E_{seismic} = [\Delta\tau/2][\Delta u_{ave}][A]$ p. 165 of Scholz
$M_o = [\mu][\Delta u_{ave}][A]$
$E_s/M_o = [\Delta\tau/2\mu]$ p. 179 of Scholz

Typically $\Delta\tau \approx 3$ MPa, and $2\mu = (2)(3 \times 10^4$ MPa), so $E_s/M_o = 1/20{,}000$

VI Seismic energy equivalents

Approximate Magnitude	kg TNT for Seismic Energy Yield	Joule equivalent	Example	Example
1.2	0.480	2.0×10^6	Stick of dynamite	
3.0	480	2.0×10^9	Oklahoma City bomb	
3.87	9.5×10^3	40×10^9	Chernobyl explosion	
6	15×10^3	63×10^{12}	Hiroshima bomb	
6.3	43×10^3	180×10^{12}	Christchurch, 2011	
7	480×10^3	2.8×10^{15}	Haiti, 2010	Java, 2009
8.35	50×10^6	210×10^{15}	Tsar Bomba	(H-bomb)
9	480×10^6	2.0×10^{18}	Japan, 2011	
9.2	950×10^6	4.0×10^{18}	Alaska, 1964	Sumatra, 2004
9.5	2.7×10^9	11×10^{21}	Chile, 1960	
13	100×10^{12}	420×10^{24}	Chicxulub impact	

http://en.wikipedia.org/wiki/Richter_magnitude_scale