FOLDED SURFACES & CLASSIFICATIONS

I Main Topics

A Curvature at a point along a curved surface

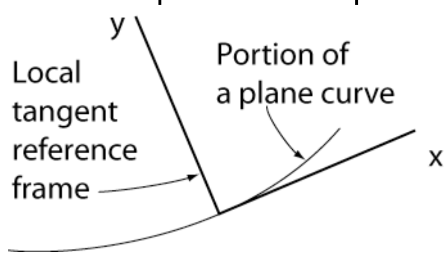
B Fold nomenclature and classification schemes

C Interference of folds

D Superposition of folds

II Curvature at a point along a curved surface

A Local equation of a plane curve <u>in a tangential reference frame</u>



At x= 0, y = 0.

At x= 0, y' = 0.

Express the plane curve as a power series of linearly independent terms:

1 $y = \left[\ldots + C_{-2}x^{-2} + C_{-1}x^{-1}\right] + \left[C_0 x^0\right] + \left[C_1 x^1 + C_2 x^2 + C_3 x^3 + \ldots\right]$.

As y is finite at x= 0, all the coefficients for terms with negative exponents must be zero. At x= 0, all the terms with positive exponents equal zero. Accordingly, since y = 0 at x = 0, C_0 = 0. So equation (1) simplifies:

2 $y = C_1 x^1 + C_2 x^2 + C_3 x^3 + \ldots$.

The constraint y' = 0 at x = 0 is satisfied at x = 0 only if C_1 = 0

3 $y' = C_1 x^0 + 2C_2 x^1 + 3C_3 x^2 + \ldots = 0$.

4 $y = C_2 x^2 + C_3 x^3 + \ldots$. Now examine the second derivative:

5 $y'' = 2C_2 + 6C_3 x^1 + \ldots$. Only the first term contributes as x → 0, hence

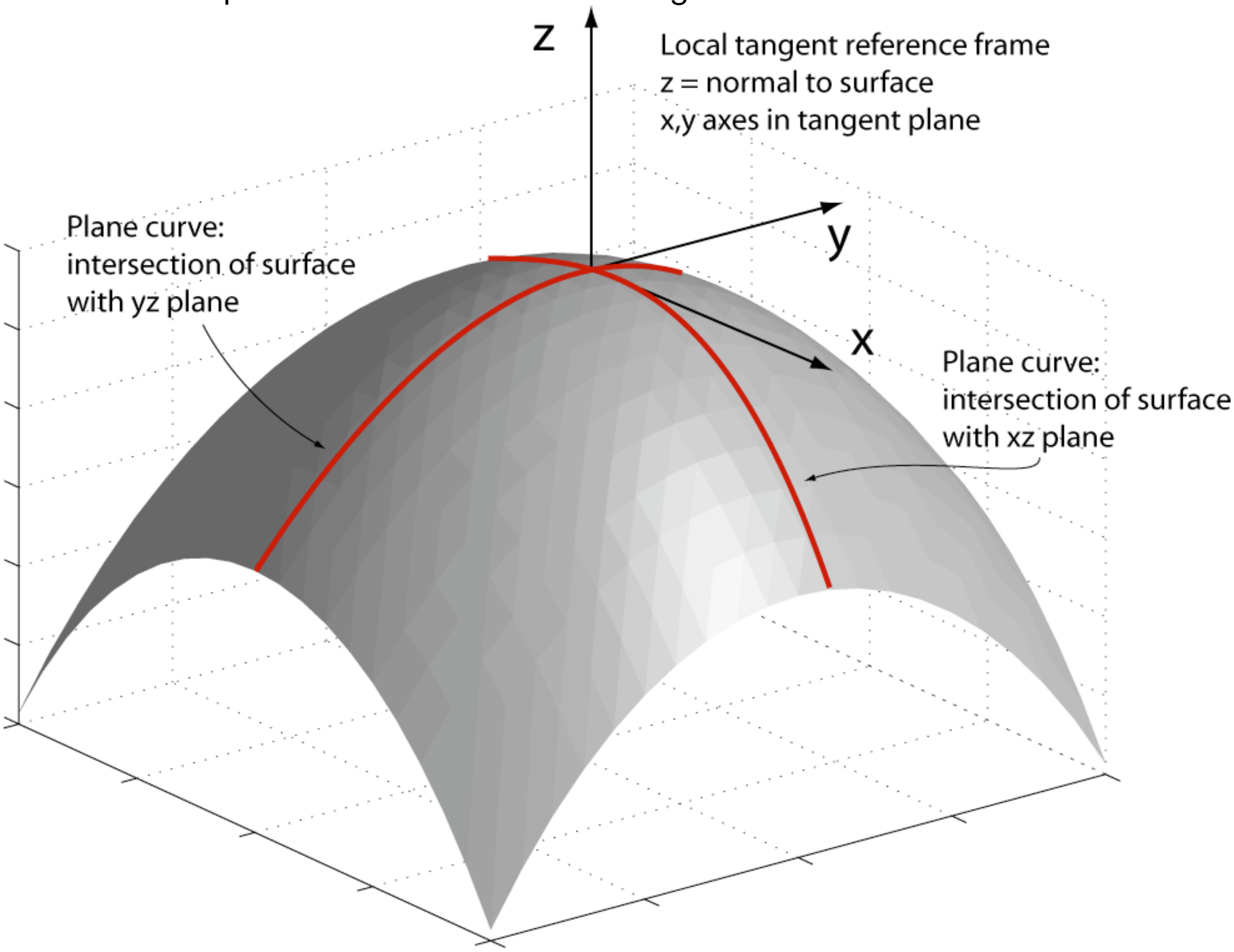
6 $\lim\limits_{x \to 0} y = C_2 x^2$.

<u>So near a point of tangency all plane curves are second-order (parabolic).</u>

At x = 0, x is the direction of increasing distance along the curve, so

7 $\lim\limits_{x \to 0} K = \left|y(s)''\right| = \left|y(x)''\right| = 2C_2$

B <u>Local equation of a surface in a tangential reference frame</u>



In this local reference frame, at (x= 0, y = 0), z = 0, ∂z/∂x = 0, ∂z/∂y = 0.

Plane curves locally all of second order pass through a point on a surface z = f(x,y) and contain the surface normal, so any continuous surface is locally 2nd order. The general form of such a surface in a tangential frame is

8 $z = Ax^2 + Bxy + Cy^2$,

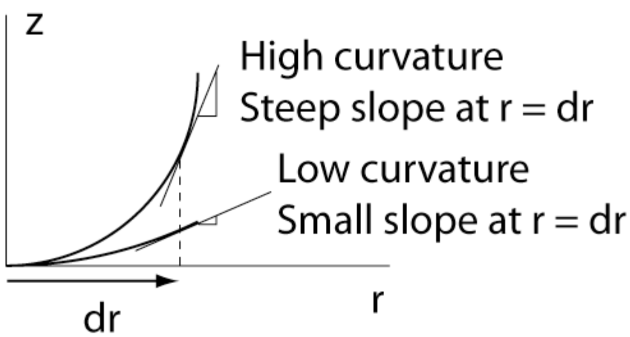
where at (x= 0, y = 0), z = 0, and the xy-plane is tangent to the surface. This is the equation of a paraboloid: <u>near a point all surfaces are second-order elliptical or hyperbolic paraboloids.</u>

Example: curve (*normal section*) in the arbitrary plane y = mx

9 $\lim_{x\to 0, y\to 0} z = Ax^2 + Bx(mx) + C(mx)^2 = (A + Bm + Cm^2)x^2$.

C Directions and magnitudes of principal curvatures for a surface

Consider the family of curves (i.e., *normal sections*) formed by a surface intersecting a series of planes through the surface normal at a point (see diagram above). The curve with the most positive tangent a short distance from the point of tangency (the local origin), as measured in a tangential reference frame, has a unit tangent that increases at the greatest rate (i.e., has the greatest curvature). The curve with the least positive tangent a short distance from the local origin has a unit tangent that increases at the smallest rate (i.e., has the least curvature). Near the point of tangency, the values of dx and dy determine the direction of various curves. We seek the direction, given by dx and dy, for which the curvature will be greatest (see diagram below, where dr is an incremental distance from the origin in the tangent plane and $dr^2 = dx^2 + dy^2$).



Cross section containing the normal to the surface (z). The intersection of the cross section with the surface yields a plane curve called a normal section. The r-direction is in the tangent plane, with z = 0 at r = 0. The first derivative at a small distance from the point of tangency will equal the second derivative multiplied by the distance dr.

The first partial differentials of a function z(x,y) represent the change in z (i.e., dz) for a given change in x (i.e., dx) or y (i.e., dy):

10a $$\frac{\partial^2 z}{\partial x^2}dx + \frac{\partial^2 z}{\partial x \partial y}dy = \frac{\partial(dz)}{\partial x}$$

10b $$\frac{\partial^2 z}{\partial y \partial x}dx + \frac{\partial^2 z}{\partial y^2}dy = \frac{\partial(dz)}{\partial y}$$

In the tangential frame $dz = z$, so (10a) and (10b) can be rewritten as

11a $$\frac{\partial^2 z}{\partial x^2}dx + \frac{\partial^2 z}{\partial x \partial y}dy = \frac{\partial z}{\partial x}$$

11b $$\frac{\partial^2 z}{\partial y \partial x}dx + \frac{\partial^2 z}{\partial y^2}dy = \frac{\partial z}{\partial y}$$

These can be written in matrix form:

12a $$\begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial x \partial y} \\ \frac{\partial^2 z}{\partial y \partial x} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{bmatrix}$$

Stress: traction equivalent

$$\begin{bmatrix} \sigma_{xx} & \sigma_{yx} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

The first derivatives of z (on the right side of equation 12a) at a small distance dr from the point of tangency equals second derivatives multiplied by dr, and the second derivative in a tangential reference frame is a normal curvature. Accordingly, equation (12a) can be rewritten in the form $Ax = \lambda x$:

13a $$\begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial x \partial y} \\ \frac{\partial^2 z}{\partial y \partial x} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} = k \begin{bmatrix} dx \\ dy \end{bmatrix}$$

Stress: traction equivalent

$$\begin{bmatrix} \sigma_{xx} & \sigma_{yx} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix} = T \begin{bmatrix} n_x \\ n_y \end{bmatrix}$$

<u>Solving equation (13a) yields the maximum and minimum curvatures and their directions (as measured in the tangent plane). The square matrix on the left side of equation (13a) is symmetric because $\partial^2 z/\partial x\partial y = \partial^2 z/\partial y\partial x$. Its eigenvalues (i.e., the principal curvatures) are given by the term *k* on the right side of equation (13a). Its eigenvectors, given by dx and dy, are the directions of the principal curvatures.</u> An analogy between principal curvatures and principal stresses is even more apparent if one writes $\partial^2 z/\partial x_i\partial y_j$ as k_{ij}:

14a $$\begin{bmatrix} k_{xx} & k_{yx} \\ k_{xy} & k_{yy} \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} = k \begin{bmatrix} dx \\ dy \end{bmatrix}$$

Stress: traction equivalent

$$\begin{bmatrix} \sigma_{xx} & \sigma_{yx} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix} = T \begin{bmatrix} n_x \\ n_y \end{bmatrix}$$

Since the square matrix in equations (13a) and (14a) is symmetric, <u>the principal curvatures are orthogonal</u> (see lecture notes on symmetric matrices). <u>The product of the principal curvatures is the Gaussian curvature ($K = k_1k_2$), and their mean is the mean curvature ($H = [k_1+k_2]/2$).</u>

D Euler's equation on normal curvature

By using the material in the previous section and in the notes on symmetric matrices, equation (8) can be re-written to eliminate the xy-term by using the reference frame of the principal curvatures and surface normal:

(15) $z = (1/2)\ (A^* x^{*2} + C^* y^{*2})$,

where x* and y* are the directions of the principal curvatures k_1 and k_2, respectively. If we let x* = r cosθ and y* = r sinθ, we obtain the equation of any normal section curve in the direction of θ

(16) $z = (1/2)\ (A^* \cos^2\theta + B^* \sin^2\theta) r^2$

The second derivative of z with respect to r gives the normal curvature

(17) $\partial^2 z/\partial r^2 = A^* \cos^2\theta + B^* \sin^2\theta$

The r-direction is tangent to the curve at the point we are evaluating the curvature, so the r- and s-directions coincide, so the second derivative of z with respect to r gives the normal curvature

(18) $k = k_1 \cos^2\theta + k_2 \sin^2\theta$

This is Euler's equation on normal curvature, developed in 1760. It shows that for any direction the normal curvature is bracketed by the maximum curvature and the minimum curvature.

II Fold nomenclature and classification schemes

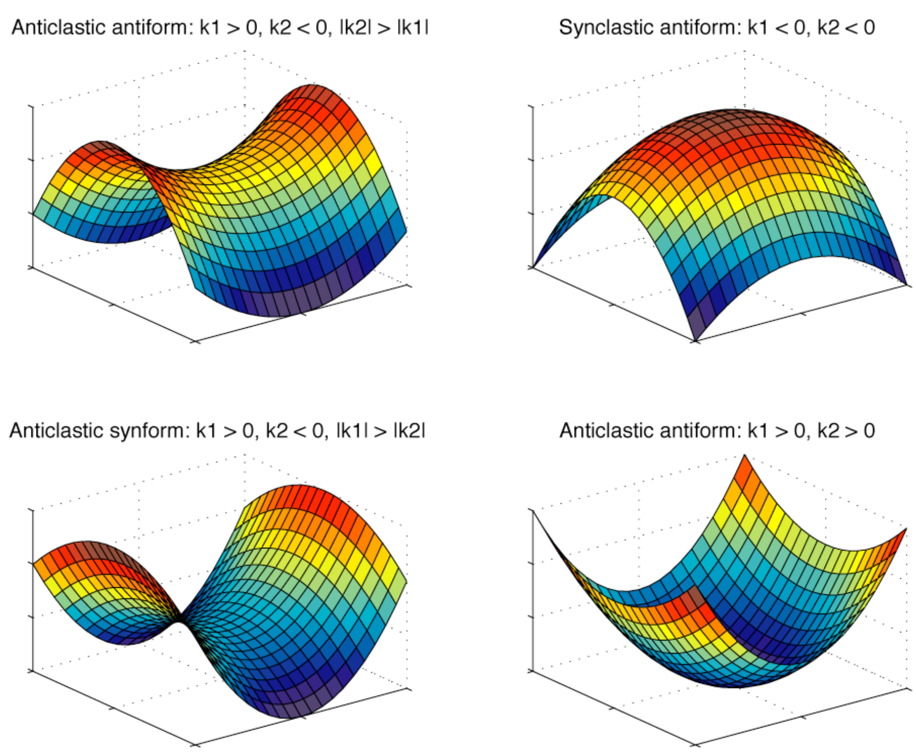
A Emerging fold terminology and classification

1 Classification of Lisle and Toimil, 2007*)

	$K < 0$ (Anticlastic) Principal curvatures have opposite signs	$K > 0$ (Synclastic) Principal curvatures have same signs
$H < 0$ ($\cap$) antiform	Anticlastic antiform $k_1 > 0$, $k_2 < 0$, $\lvert k_2 \rvert > \lvert k_1 \rvert$ "Saddle on a ridge"	Synclastic antiform $k_1 < 0$, $k_2 < 0$
$H > 0$ ($\cup$) synform	Anticlastic synform $k_1 > 0$, $k_2 < 0$, $\lvert k_1 \rvert > \lvert k_2 \rvert$ "Saddle in a valley"	Synclastic synform $k_1 > 0$, $k_2 > 0$

- * Lisle and Toimil (2007) consider convex curvatures as positive

Fold Classfication Scheme of Lisle and Toimil (2007)

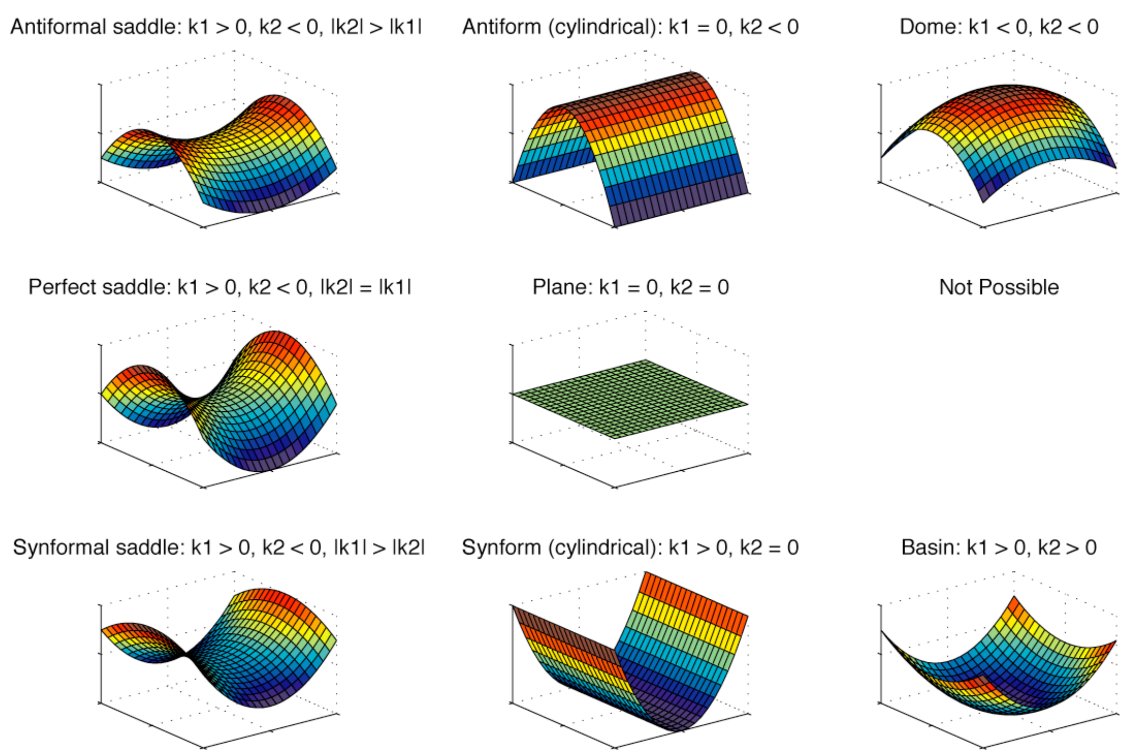


2 Classification of Mynatt et al., 2007

	K < 0 (saddle) Principal curvatures have opposite signs	K = 0	K > 0 (bowl or dome) Principal curvatures have same signs
H < 0 (∩) Antiform	Antiformal saddle $k_1 > 0,\ k_2 < 0,\ \lvert k_2\rvert > \lvert k_1\rvert$ "Saddle on a ridge"	Antiform $k_1 = 0,\ k_2 < 0$	Dome $k_1 < 0,\ k_2 < 0$
H = 0	Perfect saddle $k_1 > 0,\ k_2 < 0,\ \lvert k_2\rvert = \lvert k_1\rvert$	Plane $k_1 = 0,\ k_2 = 0$	Not possible
H > 0 (∪) Synform	Synformal saddle $k_1 > 0,\ k_2 < 0,\ \lvert k_1\rvert > \lvert k_2\rvert$ "Saddle in a valley"	Synform $k_1 > 0,\ k_2 = 0$	Basin $k_1 > 0,\ k_2 > 0$

- Mynatt et al., (2007) consider convex curvatures as positive

Fold Classfication Scheme of Mynat et al. (2007)



```
   function folds3d
% Prepares figures of 3D folds
x = -1:0.1:1;
y = x;
[X,Y] = meshgrid(x,y);


% Classification scheme of Lisle and Toimil (2007)
figure (1)

% Anticlastic antiform: k1 > 0, k2 < 0, |k2| > |k1|
k1 = 1; k2 = -2;
Z = 0.5*(k1*X.^2 + k2*Y.^2);
subplot(2,2,1)
surf(X,Y,Z)
title ('Anticlastic antiform: k1 > 0, k2 < 0, |k2| > |k1|')

% Synclastic antiform: k1 < 0, k2 < 0
k1 = -1; k2 = -2;
Z = 0.5*(k1*X.^2 + k2*Y.^2);
subplot(2,2,2)
surf(X,Y,Z)
title ('Synclastic antiform: k1 < 0, k2 < 0 ')

% Anticlastic synform: k1 > 0, k2 < 0, |k1| > |k2|
k1 = 2; k2 = -1;
Z = 0.5*(k1*X.^2 + k2*Y.^2);
subplot(2,2,3)
surf(X,Y,Z)
title ('Anticlastic synform: k1 > 0, k2 < 0, |k1| > |k2| ')

% Anticlastic antiform: k1 > 0, k2 > 0
k1 = 2; k2 = 1;
Z = 0.5*(k1*X.^2 + k2*Y.^2);
subplot(2,2,4)
surf(X,Y,Z)
title ('Anticlastic antiform: k1 > 0, k2 > 0 ')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Classification scheme of Mynatt et al. (2007)
figure (2)

% Antiformal saddle: k1 > 0, k2 < 0, |k2| > |k1|
k1 = 1; k2 = -2;
Z = 0.5*(k1*X.^2 + k2*Y.^2);
subplot(3,3,1)
surf(X,Y,Z)
title ('Antiformal saddle: k1 > 0, k2 < 0, |k2| > |k1| ')

% Antiform (cylindrical): k1 = 0, k2 < 0
k1 = 0; k2 = -2;
Z = 0.5*(k1*X.^2 + k2*Y.^2);
```

```
subplot(3,3,2)
surf(X,Y,Z)
title ('Antiform (cylindrical): k1 = 0, k2 < 0')

% Dome: k1 < 0, k2 < 0
k1 = -1; k2 = -2;
Z = 0.5*(k1*X.^2 + k2*Y.^2);
subplot(3,3,3)
surf(X,Y,Z)
title ('Dome: k1 < 0, k2 < 0')

% Perfect saddle: k1 > 0, k2 < 0, |k2| = |k1|
k1 = 1; k2 = -1;
Z = 0.5*(k1*X.^2 + k2*Y.^2);
subplot(3,3,4)
surf(X,Y,Z)
title ('Perfect saddle: k1 > 0, k2 < 0, |k2| = |k1|')

% Plane: k1 = 0, k2 = 0
k1 = 0; k2 = 0;
Z = 0.5*(k1*X.^2 + k2*Y.^2);
subplot(3,3,5)
surf(X,Y,Z)
title ('Plane: k1 = 0, k2 = 0')

% Not Possible
subplot(3,3,6)
title ('Not Possible')


% Synformal saddle: k1 > 0, k2 < 0, |k1| > |k2|
k1 = 2; k2 = -1;
Z = 0.5*(k1*X.^2 + k2*Y.^2);
subplot(3,3,7)
surf(X,Y,Z)
title ('Synformal saddle: k1 > 0, k2 < 0, |k1| > |k2|')

% Synform (cylindrical): k1 > 0, k2 = 0
k1 = 1; k2 = 0;
Z = 0.5*(k1*X.^2 + k2*Y.^2);
subplot(3,3,8)
surf(X,Y,Z)
title ('Synform (cylindrical): k1 > 0, k2 = 0')

% Basin: k1 > 0, k2 > 0
k1 = 2; k2 = 1;
Z = 0.5*(k1*X.^2 + k2*Y.^2);
subplot(3,3,9)
surf(X,Y,Z)
title ('Basin: k1 > 0, k2 > 0')
```

B "Traditional" Fold terminology and classification

1 ***Hinge point***: point of local maximum curvature.

2 ***Hinge line***: connects hinge points along a given layer.

3 ***Axial surface***: locus of hinge points in ***all*** the folded layers.

4 ***Limb***: surface of low curvature.

5 ***Cylindrical fold***: a surface swept out by moving a straight line parallel to itself

a Fold axis: line that can generate a cylindrical fold

b **Parallel fold**: top and bottom of layers are parallel and ***layer thickness is preserved*** (assumes bottom and top of layer were originally parallel).

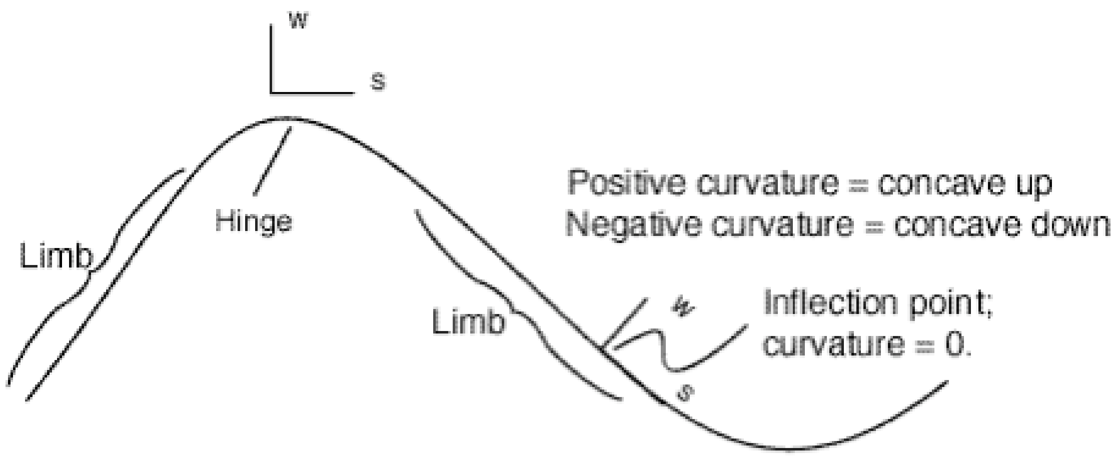
c Curved parallel fold: curvature is fairly uniform.

d Angular parallel fold: curvature is concentrated near the hinges and the limbs are relatively planar.

e Non-parallel fold: top and bottom of layers are not parallel; ***layer thickness is not preserved*** (assumes bottom and top of layer were originally parallel). Hinges typically thin and limbs thicken.

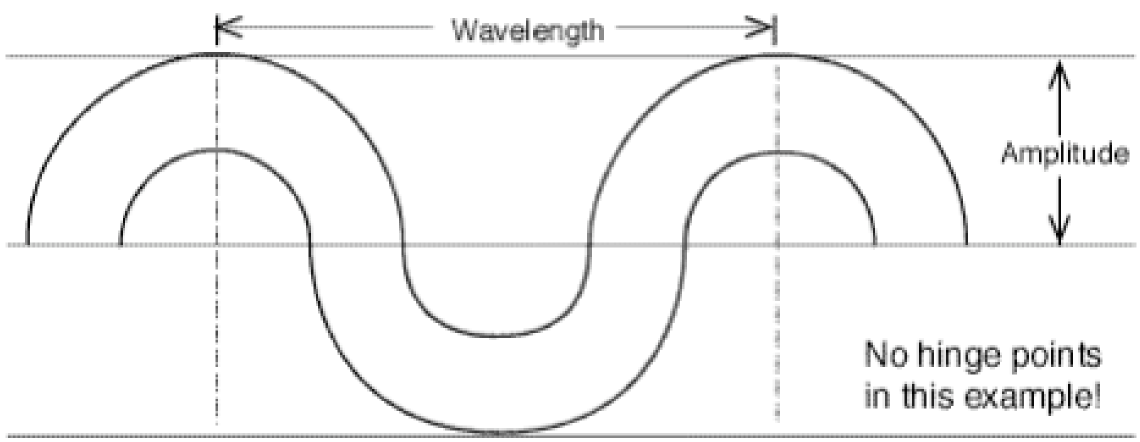
Limbs and Hinges along Folds

Fig. 27.4

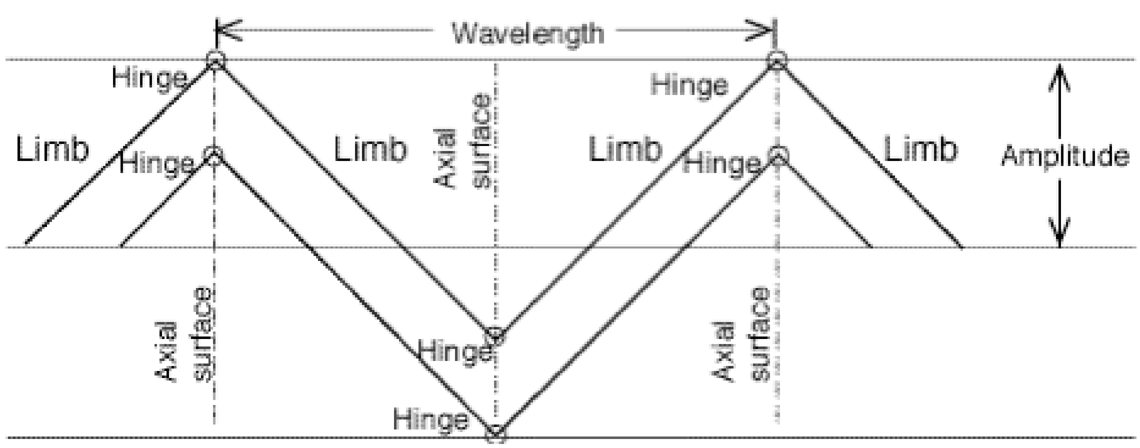


Radius of curvature is small(est) at the hinge, larg(est) on the limbs

Curved Parallel Fold



Angular Parallel Fold



E Non-cylindrical fold example: dome
B Anticlines, synclines, antiforms, synforms, and monoclines
B Kinks: folds with sharp, angular hinge regions
C "Tightness" of folds
D Classification by orientation of axial plane and plunge of fold axis
E Symmetrical folds vs. asymmetrical folds

III Ramsay's classification scheme; single-layer folds in profile
A Relates the curvature of the inner and outer surfaces of a fold.
B Dip isogons: lines that connect points of equal dip

Fold class	Curvature (C)	Comment
I	$C_{inner} > C_{outer}$	Dip isogons converge
1A		Orthogonal thickness on limbs exceeds thickness at hinge; uncommon
1B		Parallel folds
1C		Orthogonal thickness on limbs is less than thickness at hinge
2	$C_{inner} = C_{outer}$	Dip isogons are parallel Class 2 = **similar** folds)
3	$C_{inner} < C_{outer}$	Dip isogons are diverge

Class 1C (or 1B) folds commonly are stacked with class 3 folds.

IV Mechanical interaction of folds (See Fig. 9-57 of Suppe)
- A Layers far apart will not interact as they fold
- B Layers of similar properties that are close together will tend to fold as a single fold
- C Layers "near" each other will interact

V Superposition of folds
- A Can produce highly complicated geometries
- B Common in metamorphic rocks
- C "Demonstration" of z- and s- folds (parasitic)

<u>References</u>

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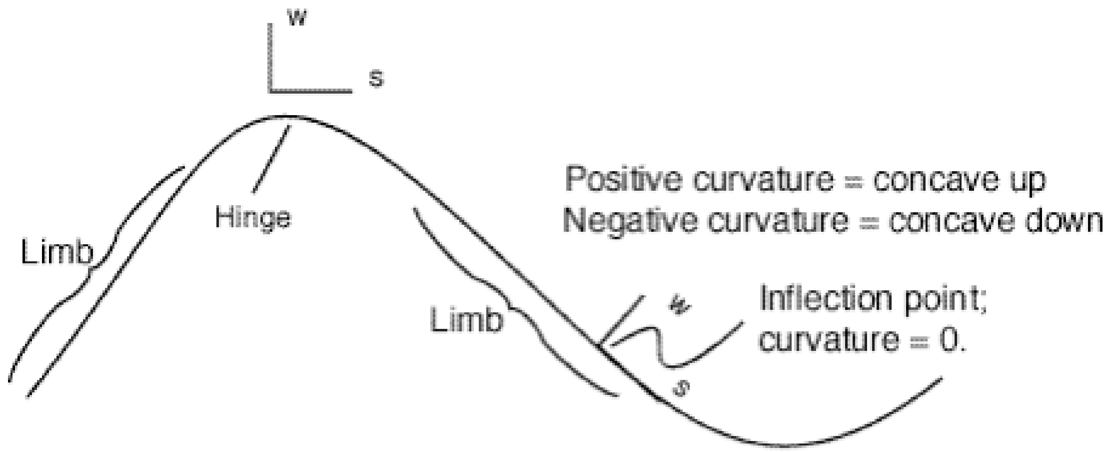
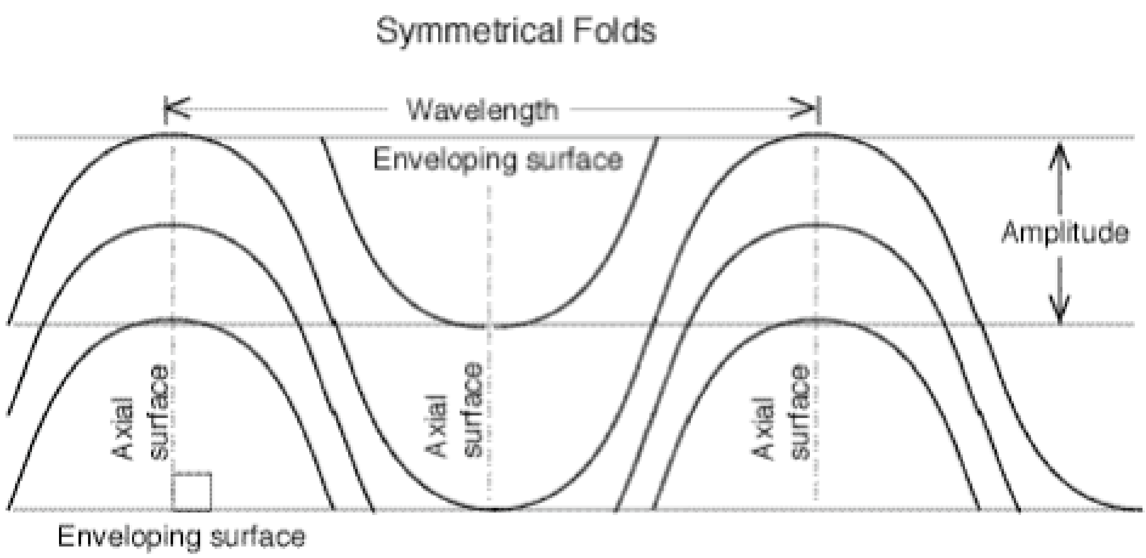
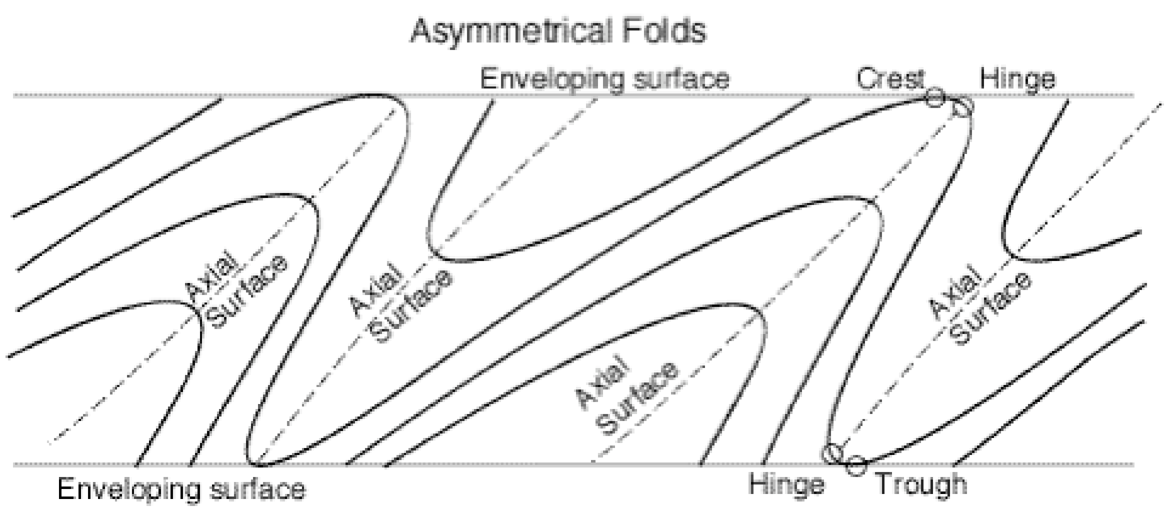
NOMENCLATURE FOR FOLDS

Fig. 28.1



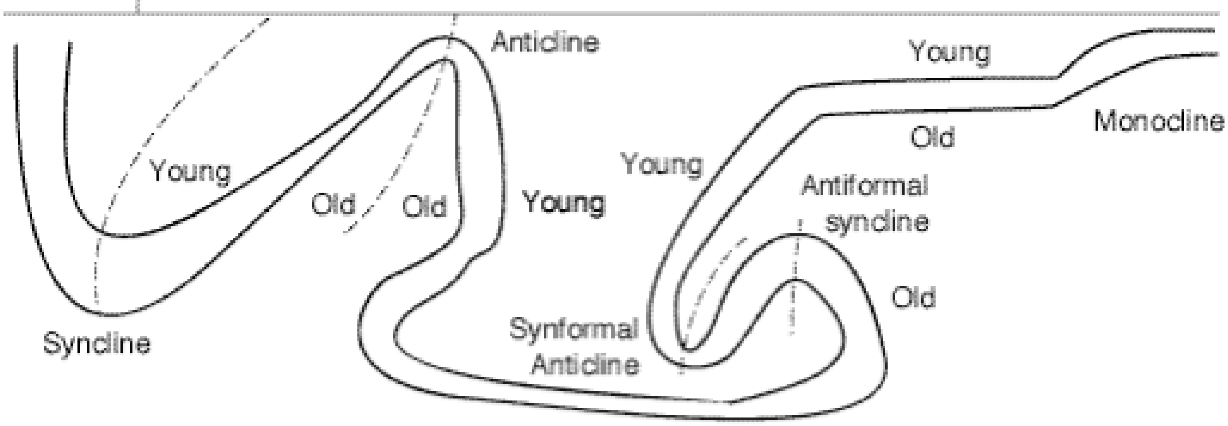
Radius of curvature is small(est) at the hinge, larg(est) on the limbs





NOMENCLATURE FOR FOLDS

Fig. 28.2



Syncline: fold where rocks become younger towards axial surface

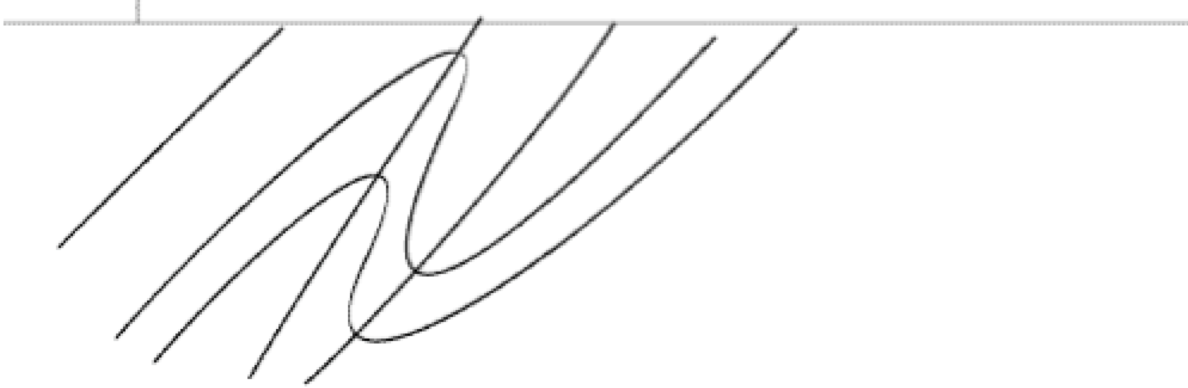
Anticline: fold where rocks become older towards axial surface

Synform: fold where limbs dip towards axial surface

Antiform: fold where limbs dip way from axial surface

Monocline: gentle anticline-syncline pair with horizontal outer limbs

Overturned folds



Overturned syncline: one limb of syncline is overturned

Overturned anticline: one limb of anticline: is overturned

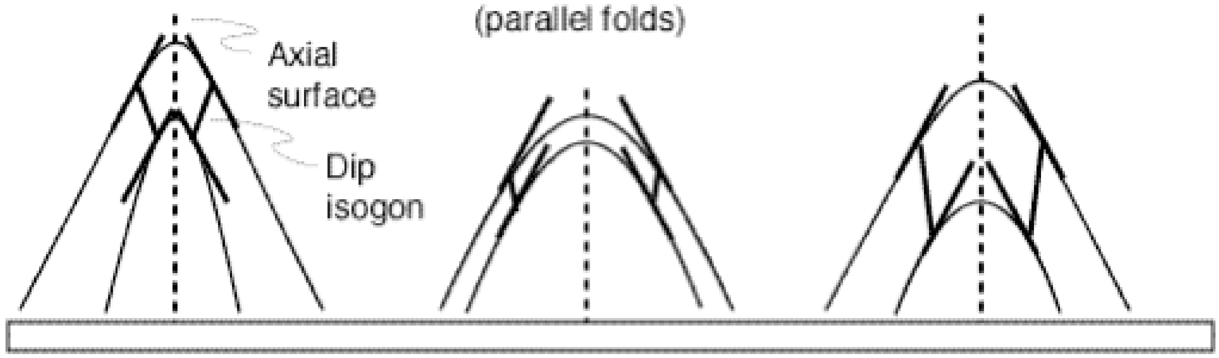
Ramsay's Fold Classification

Fig. 28.3

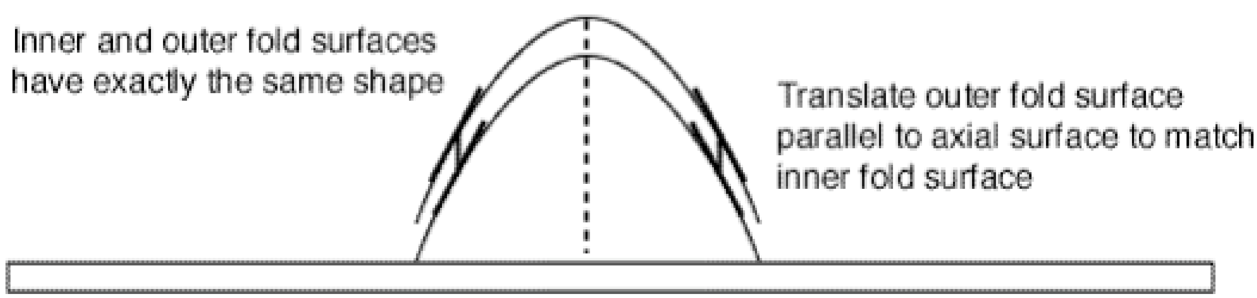
Dip Isogon: a line that connects points of equal dip on the top and bottom of a folded layer

Class 1: Dip isogons converge towards axial surface; $C_{inner} > C_{outer}$

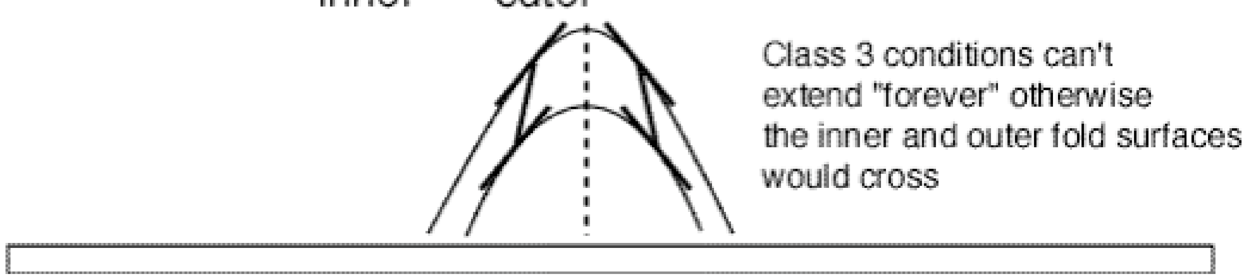
1A	1B	1C
Limbs thicker than hinges	Layer thickness is constant (parallel folds)	Limbs thinner than hinges



Class 2: Dip isogons parallel axial surface (similar folds); $C_{inner} = C_{outer}$

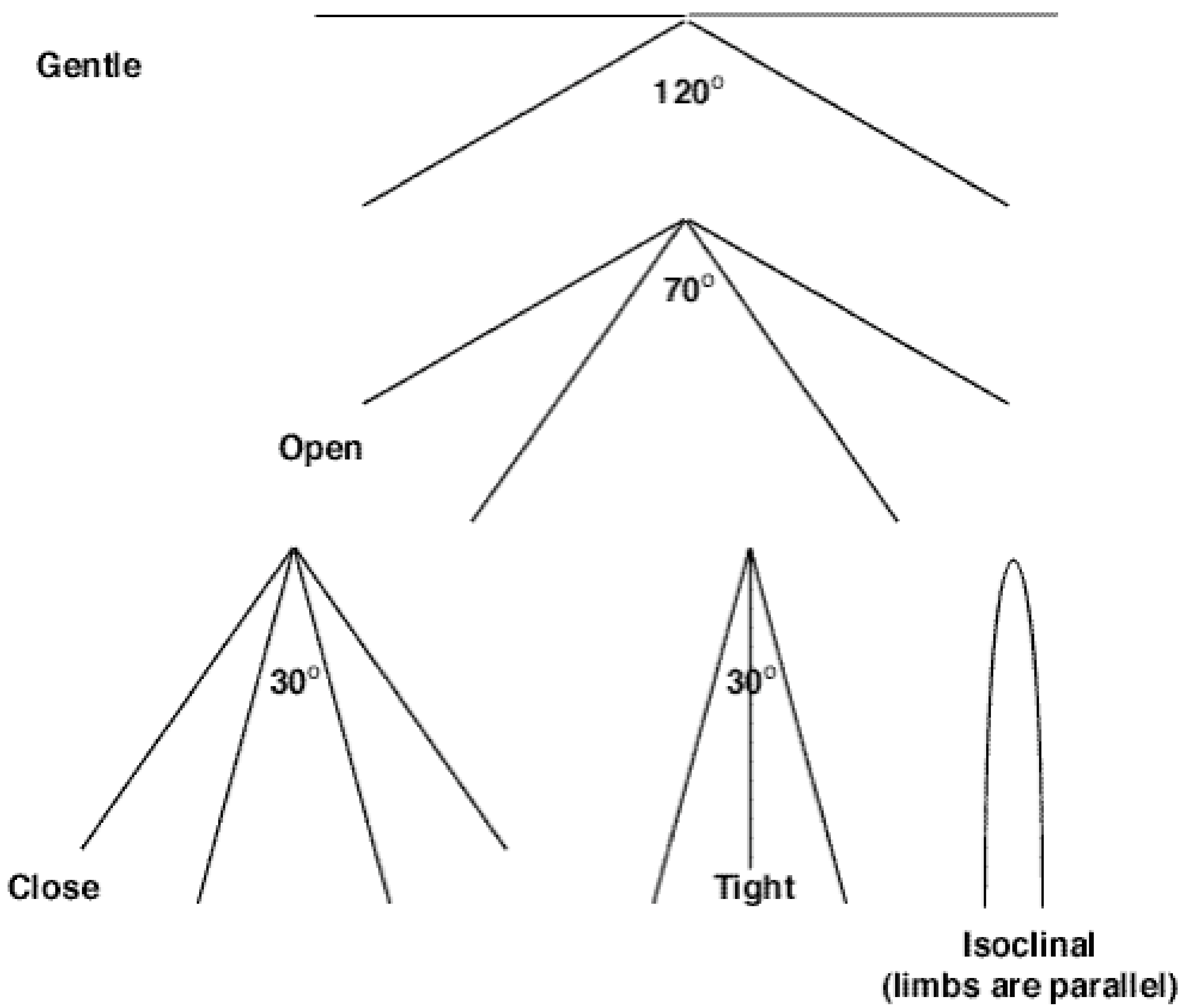


Class 3: Dip isogons diverge from axial surface; $C_{inner} < C_{outer}$



Terms for Describing the Tightness of Folds Fig. 28.4

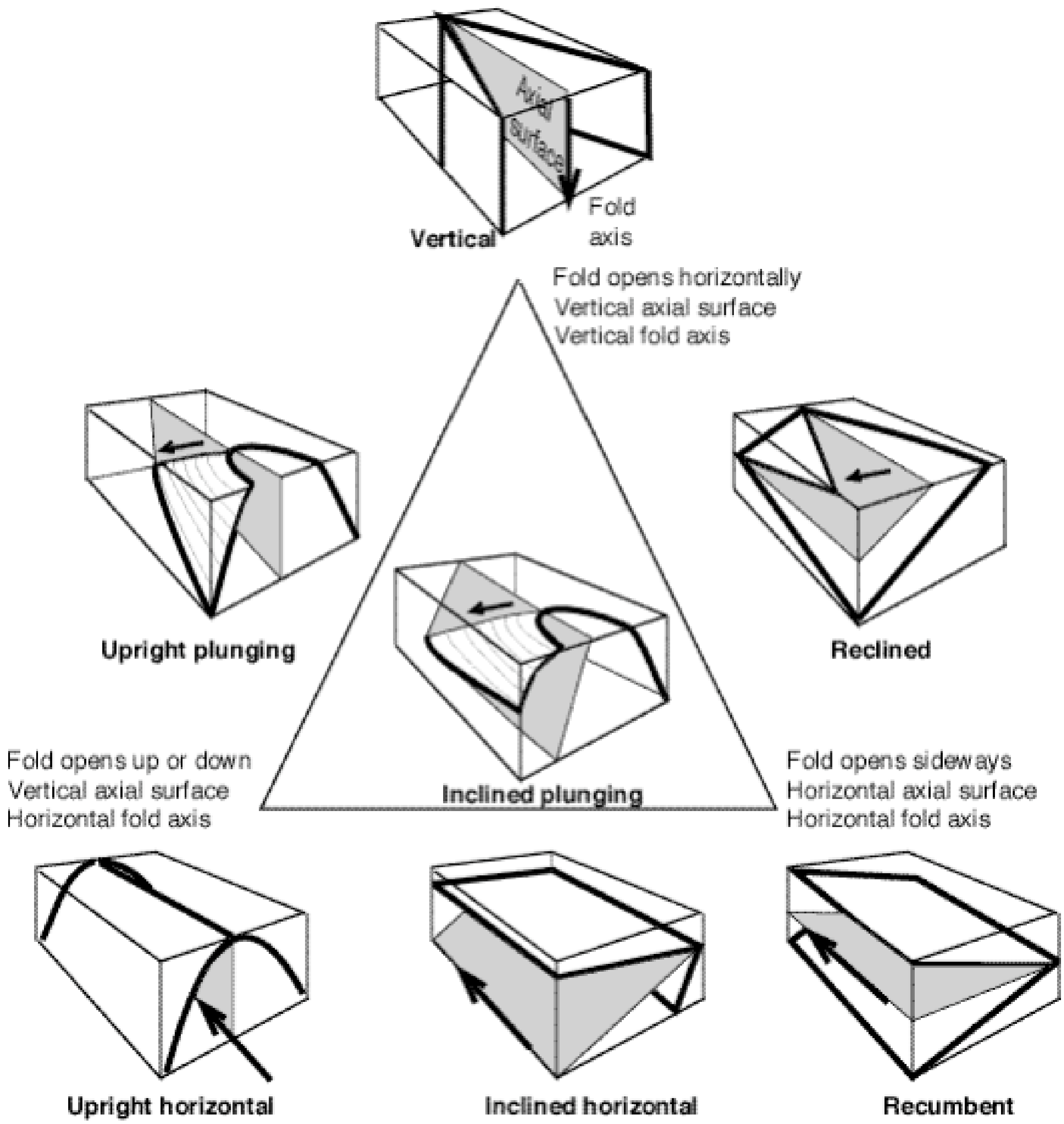
Interlimb angle	Description of fold
180° - 120°	Gentle
120° - 70°	Open
70° - 30°	Close
30° - 0°	Tight
"0°"	Isoclinal
Negative	Mushroom



Fold Classifications Fig. 28.5

(modified from Ragan, 1973, Figure 7.10)

Based on direction of fold concavity, axial suface orientation, and fold axis orientation



First modifier (e.g., "upright") describes orientation of axial surface
Second modifier (e.g., "horizontal") describes orientation of fold axis