FOLDS (I)

I Main Topics
- A What is a fold?
- B Fold geometry
- C Fold terminology and classification

II What is a fold?
- A Flexure (deformation-induced curvature) in rock (esp. layered)
- B <u>All</u> kinds of rocks can be folded, even granites

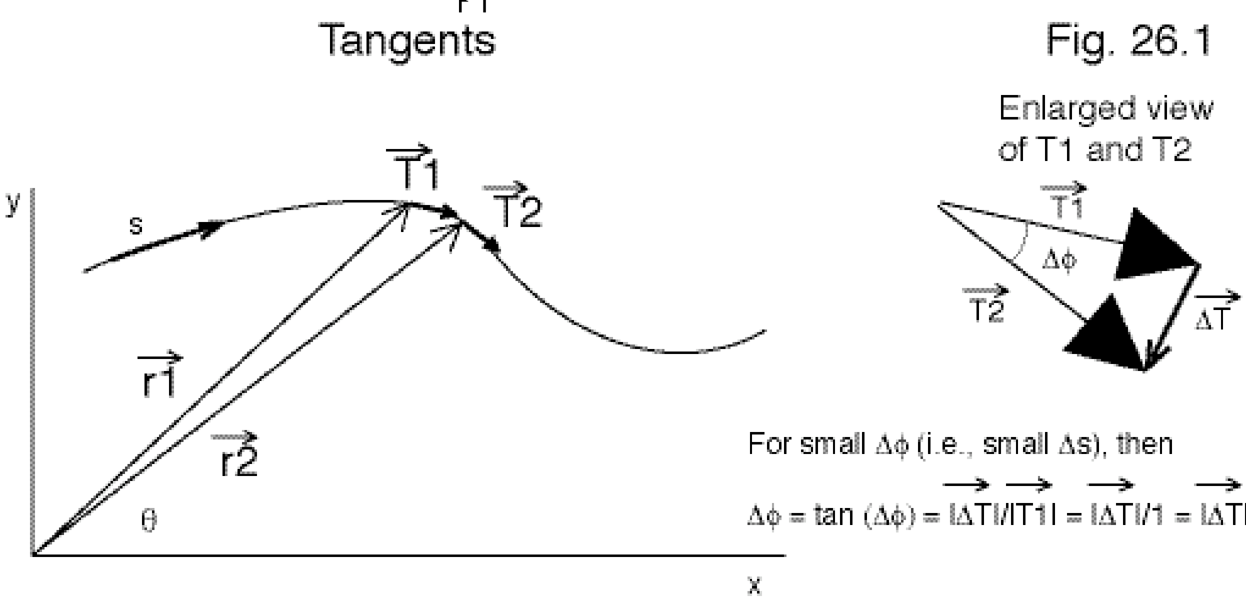
III Fold geometry

A Tangents

Consider a curve $\mathbf{r}(t)$, where t is <u>any</u> parameter, and $\mathbf{r}$ is a vector function that gives points on the curve

1 Tangent vector: $\mathbf{r}' = \frac{d\mathbf{r}}{dt}$

2 Unit tangent = $\mathbf{T} = \frac{\mathbf{r}'}{|\mathbf{r}'|}$



Let ϕ give the orientation of the unit vector T.

3 Example 1: parabola $\left(y = x^2\right)$

$$\vec{r}(x) = x\vec{i} + x^2\vec{j}$$

$$\mathbf{r}' = \frac{d\vec{r}}{dx} = \frac{d\left(x\vec{i} + x^2\vec{j}\right)}{dx} = \vec{i} + 2x\vec{j}$$

$$\vec{T} = \frac{\mathbf{r}'}{|\mathbf{r}'|} = \frac{1\vec{i} + 2x\vec{j}}{\sqrt{1^2 + (2x)^2}} = \frac{1\vec{i} + 2x\vec{j}}{\sqrt{1 + 4x^2}} \quad at \;\; x = 1, \; \vec{T} = \frac{\vec{i} + 2\vec{j}}{\sqrt{5}}$$

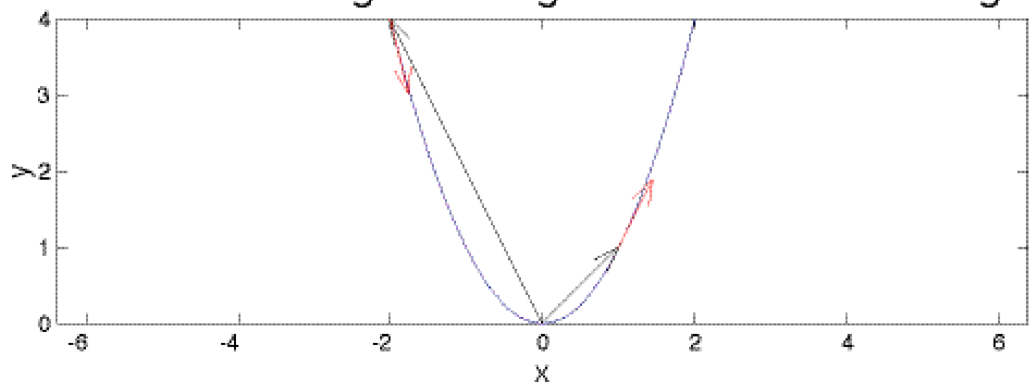
4 Example 2: unit circle $(x = \cos\theta, \;\; y = \sin\theta)$

$$\vec{r}(\theta) = \cos\theta\vec{i} + \sin\theta\vec{j}$$

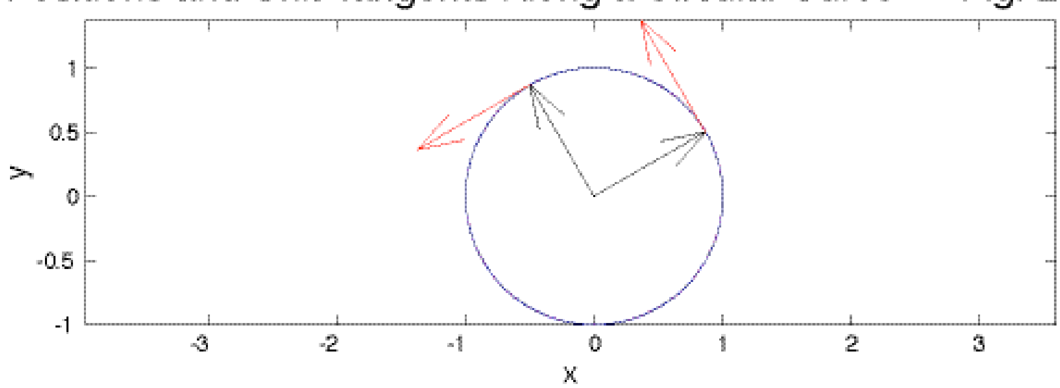
$$\mathbf{r}' = \frac{d\vec{r}}{d\theta} = \frac{d\left(\cos\theta\vec{i} + \sin\theta\vec{j}\right)}{d\theta} = -\sin\theta\vec{i} + \cos\theta\vec{j}$$

$$\vec{T} = \frac{\mathbf{r}'}{|\mathbf{r}'|} = \frac{-\sin\theta\vec{i} + \cos\theta\vec{j}}{\sqrt{(-\sin\theta)^2 + (\cos\theta)^2}} = -\sin\theta\vec{i} + \cos\theta\vec{j}$$

Positions and Unit Tangents Along a Parabolic Curve Fig. 26.2a



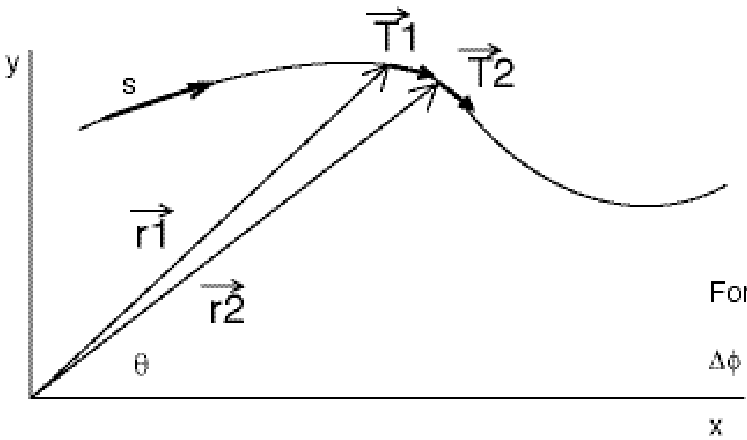
Positions and Unit Tangents Along a Circular Curve Fig. 26.2b



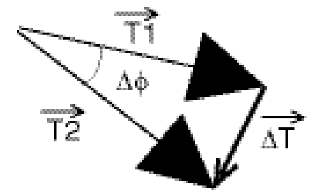
B Curvature = deviation from a straight line

Curvature

Fig. 26.3



Enlarged view of T1 and T2

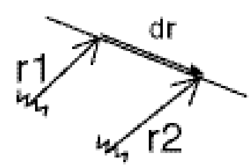


For small Δφ (i.e., small Δs), then

$\Delta\phi = \tan(\Delta\phi) = |\vec{\Delta T}|/|\vec{T1}| = |\vec{\Delta T}|/1 = |\vec{\Delta T}|$

Let φ give the orientation of the unit vector(s) T.

Then curvature $= \lim\limits_{\Delta s \to 0} \dfrac{\Delta\phi}{\Delta s}$ (i.e., change in orientation of unit tangent with distance along curve)

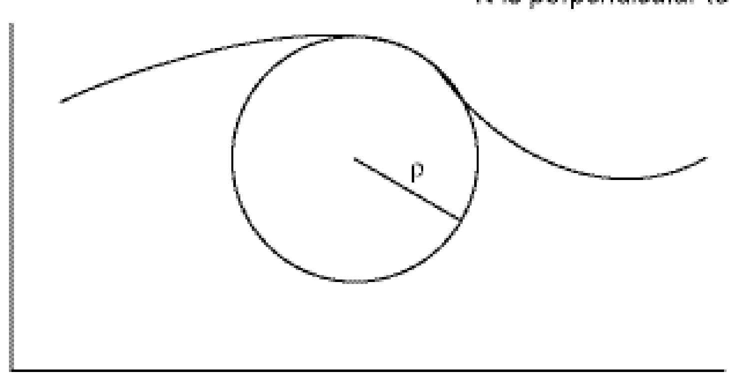


Curvature vector $= \vec{K} = \dfrac{d\vec{T}}{ds} = \dfrac{d\vec{T}/dt}{ds/dt}$, but $\dfrac{ds}{dt} = \left|\dfrac{d\vec{r}}{dt}\right|$

Here "t" is any parameter, for example θ. It can be much easier to express the equation of a curve in terms of some parameter other than arc length s, and this can simplify taking the necessary derivatives.

Curvature vector $= \vec{K} = \dfrac{\dfrac{d\vec{T}}{dt}}{\left|\dfrac{d\vec{r}}{dt}\right|}$ Curvature magnitude $= K = \left|\dfrac{\dfrac{d\vec{T}}{dt}}{\left|\dfrac{d\vec{r}}{dt}\right|}\right|$

$\vec{K}$ is perpendicular to $\vec{T}$.



ρ = radius of curvature

= 1/(curvature) = 1/K

1 Curvature along a curve is the first derivative (i.e., rate of change) of the unit tangent (i.e., slope) with respect to distance (s) along the curve;

2 Curvature vector $(\mathbf{K})$ = $\vec{\mathbf{K}}(\mathrm{t}) = \dfrac{\mathrm{d}\vec{\mathbf{T}}}{\mathrm{dt}} \Big/ \left|\dfrac{\mathrm{d}\vec{\mathbf{r}}}{\mathrm{dt}}\right|$

3 Curvature (K) = $K(\mathrm{t}) = |\mathbf{K}| = \left|\dfrac{\mathrm{d}\vec{\mathbf{T}}}{\mathrm{dt}} \Big/ \left|\dfrac{\mathrm{d}\vec{\mathbf{r}}}{\mathrm{dt}}\right|\right|$

4 K(s) = |T'(s)| = |r''(s)| (simpler version of (3), |dr| = ds)

5 Example 2: circle $(x = \rho\cos\theta,\ y = \rho\sin\theta)$

a $\vec{\mathrm{r}}(\theta) = \rho\cos\theta\vec{\mathrm{i}} + \rho\sin\theta\vec{\mathrm{j}}$

b $\mathbf{r}' = \dfrac{d\vec{r}}{d\theta} = \dfrac{d\left(\rho\cos\theta\vec{\mathrm{i}} + \rho\sin\theta\vec{\mathrm{j}}\right)}{d\theta} = -\rho\sin\theta\vec{\mathrm{i}} + \rho\cos\theta\vec{\mathrm{j}}$

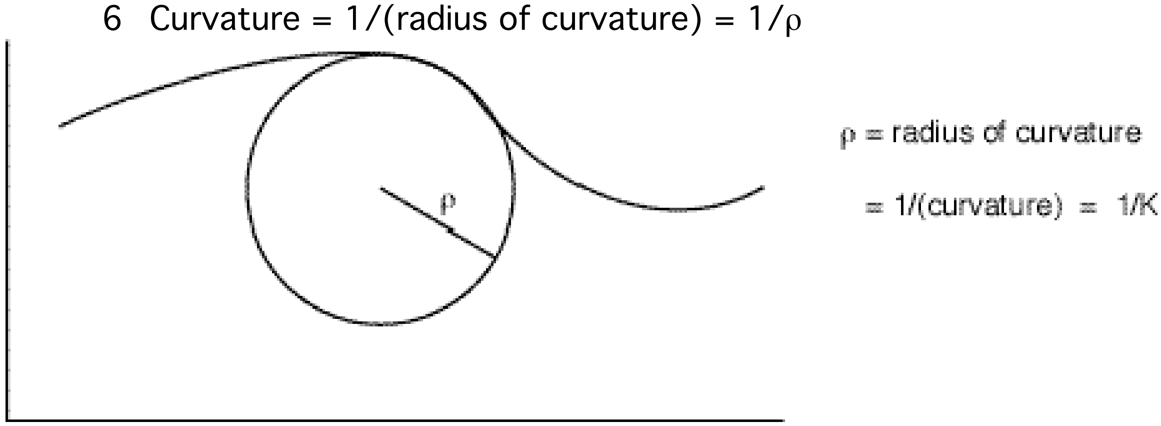
c $\vec{T} = \dfrac{\mathbf{r}'}{|\mathbf{r}'|} = \dfrac{-\rho\sin\theta\vec{i} + \rho\cos\theta\vec{j}}{\sqrt{(-\rho\sin\theta)^2 + (\rho\cos\theta)^2}} = \dfrac{-\rho\sin\theta\vec{i} + \rho\cos\theta\vec{j}}{\rho} = \sin\theta\vec{i} + \cos\theta\vec{j}$

d $\vec{\mathbf{K}} = \dfrac{\frac{\mathrm{d}\vec{\mathbf{T}}}{\mathrm{d}\theta}}{|\mathbf{r}'|} = \dfrac{\frac{\mathrm{d}\left(-\vec{\mathbf{i}}\sin\theta + \vec{\mathbf{j}}\cos\theta\right)}{\mathrm{d}\theta}}{\rho} = \dfrac{-\vec{\mathbf{i}}\cos\theta - \vec{\mathbf{j}}\sin\theta}{\rho} = -\dfrac{\vec{\mathbf{r}}}{\rho^2}$

e $\mathrm{K} = \left|\vec{\mathbf{K}}\right| = \dfrac{1}{\rho}\sqrt{(-\cos\theta)^2 + (-\sin\theta)^2} = \dfrac{1}{\rho}$

So K points opposite **r**, and the circle curvature = 1/ρ.

6 Curvature = 1/(radius of curvature) = 1/ρ



7 The curvature at a point on a curved <u>surface</u> depends on the direction of the path along the surface. The derivative of the curvature can be taken to yield the maximum and minimum curvatures. These turn out to be at right angles and are called principal curvatures. Gaussian curvature = $(C_{max})(C_{min})$. For a warped <u>but unstretched</u> sheet, C_G = const.

8 Curvature has an associated sign: "U" > 0; "Λ" < 0

9 Inflection point: Curvature = zero. Curvature changes from concave up to concave down.

IV Fold terminology and classification

A *Hinge point*: point of local maximum curvature.

B *Hinge line*: connects hinge points along a given layer.

C *Axial surface*: locus of hinge points in *all* the folded layers.

D *Limb*: surface of low curvature.

D *Cylindrical fold*: a surface swept out by moving a straight line parallel to itself

1 Fold axis: line that can generate a cylindrical fold

2 **Parallel fold**: top and bottom of layers are parallel and *layer thickness is preserved* (assumes bottom and top of layer were originally parallel).

a Curved parallel fold: curvature is fairly uniform.

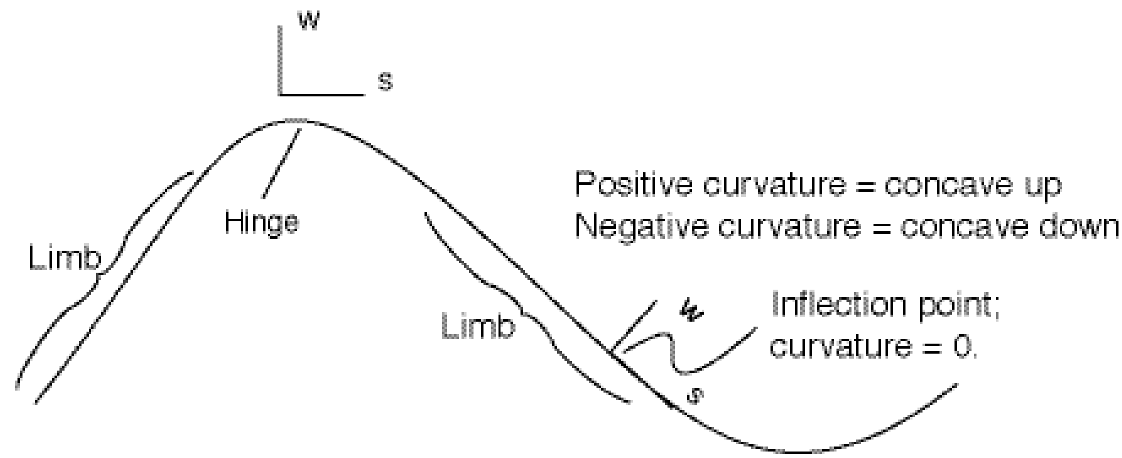
b Angular parallel fold: curvature is concentrated near the hinges and the limbs are relatively planar.

3 Non-parallel fold: top and bottom of layers are not parallel; *layer thickness is not preserved* (assumes bottom and top of layer were originally parallel). Hinges typically thin and limbs thicken.

E Non-cylindrical fold example: dome

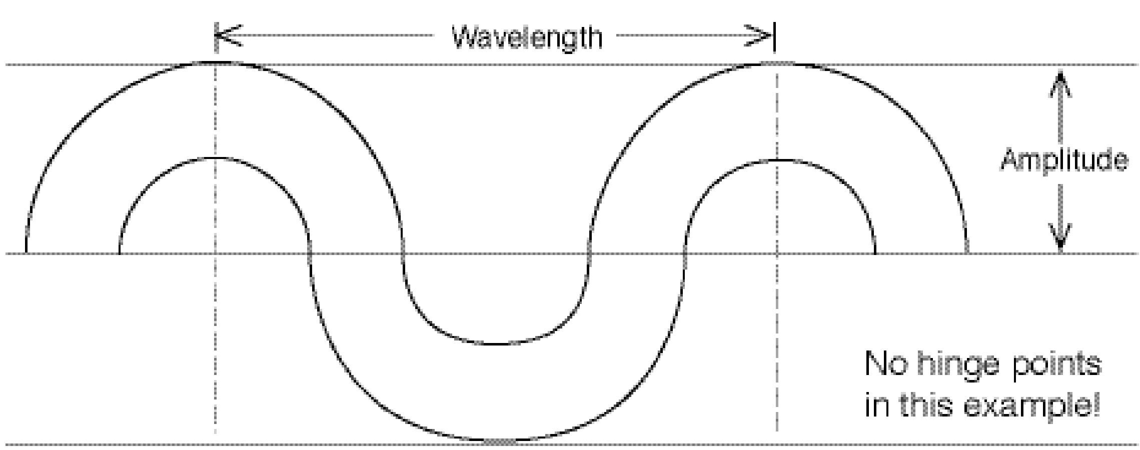
Limbs and Hinges along Folds

Fig. 26.4

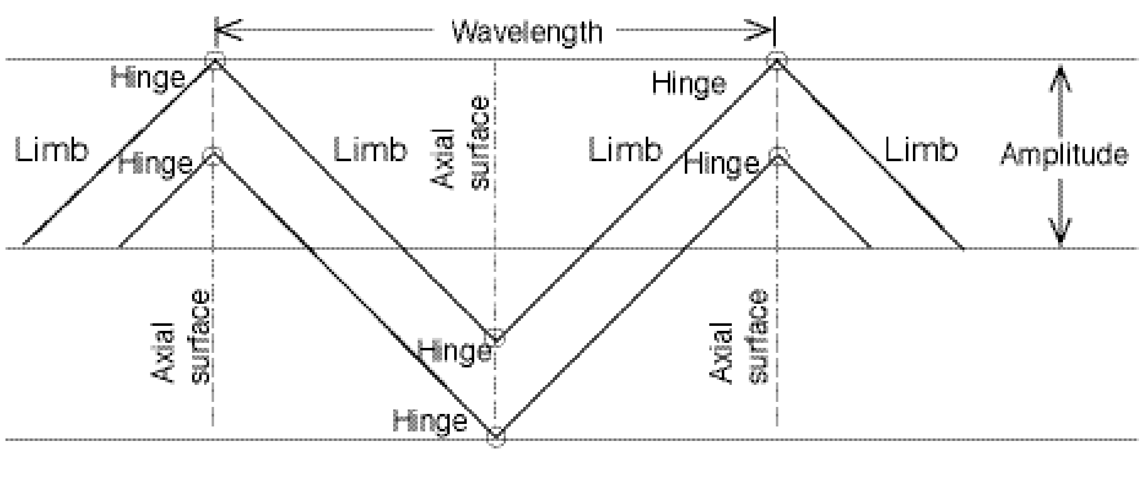


Radius of curvature is small(est) at the hinge, larg(est) on the limbs

Curved Parallel Fold



Angular Parallel Fold



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% Matlab script GG303_27_1.m

% Matlab script to produce plots for tangents to a parabola and a circle
% for Figure 27.2 of Lecture 27 of GG303

% Parabola problem (Example 1, curve r1, y = x^2)
x = -2:0.1:2;                        % This sets the range in x;
O1x = zeros(size(x));           % Defines origin for plotting position vectors;
O1y = zeros(size(x));           % Defines origin for plotting position vectors;
r1i = x;                % The "i" component of position r1 as a function of x;
r1j = x.^2;                     % The "j" component of position r1 as a function of x;
T1i = 1./sqrt(1+4*x.^2);        % The "i" component of unit tangent T1 as a function of x;
T1j = 2*x./sqrt(1+4*x.^2);% The "j" component of unit tangent T1 as a function of x;

% Circle problem (Example 2, curve r2, unit circle centered at origin)
thetad = 0:1:360;                    % This sets the range of theta in radians;
thetar = thetad*pi/180;              % This sets the range of theta in radians;
O2x = zeros(size(thetad));      % Defines origin for plotting position vectors;
O2y = zeros(size(thetad));      % Defines origin for plotting position vectors;
r2i = cos(thetar);              % The "i" component of position r2 as a function of theta;
r2j = sin(thetar);              % The "j" component of position r2 as a function of theta;
T2i = -sin(thetar);     % The "i" component of unit tangent T2 as a function of theta;
T2j = cos(thetar);      % The "j" component of unit tangent T2 as a function of theta;

% Now plot the figures on one page
figure(1)
clf

subplot(2,1,1)          % Prepare the first plot in a 2-row, 1column set of plots;
plot (r1i,r1j)          % Plot the curve
hold on;                        % Get ready for another plot
index1a = find(x ==  1);             % Find where x = 1;
index1b = find(x ==  -2);            % Find where x = -2;
index1 = [index1a, index1b];

% Draw black arrows from the origin to curve r with heads at the indexed points;
% The "stock" version of quiver produces arrow heads of different size:
% quiver ( O1x(index1),O1y(index1),r1i(index1),r1j(index1),0,'k'  );
% So I first plot black lines connecting the origin to the indexed points ...
line ([O1x(index1a),r1i(index1a)],[O1y(index1a),r1j(index1a)],'Color',[0  0 0] );
line ([O1x(index1b),r1i(index1b)],[O1y(index1b),r1j(index1b)],'Color',[0  0 0] );
% and then use quiver to draw arrows of UNIT length with heads where I want heads
scalefactor = sqrt( ( r1i(index1) - O1x(index1) ).^2 + ( r1j(index1) - O1y(index1) ).^2 );
newdx = ( r1i(index1) - O1x(index1) )./scalefactor;
newdy = ( r1j(index1) - O1y(index1) )./scalefactor;
quiver ( r1i(index1)-newdx,r1j(index1)-newdy,newdx,newdy,0,'k'  );
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```
% Draw red arrows for unit tangents to curve r1 with tails at the indexed points;
quiver ( r1i(index1),r1j(index1),T1i(index1),T1j(index1),0,'r'  );
axis('equal')                          % Set the x-scale and y-scale equal
xlabel('x','FontSize',14)
ylabel('y','FontSize',14)
title('Positions and Unit Tangents Along a Parabolic Curve      Fig. 27.2a','FontSize',18)

subplot(2,1,2)                         % Prepare the second plot in a 2-row, 1column set
of plots;
plot (r2i,r2j)                         % Plot the curve
hold on;                                     % Get ready for another plot
index2a = find(thetad == 30);          % Find where thetad = 30°;
index2b = find(thetad == 120);         % Find where thetad = 120°;
index2 = [index2a, index2b];
% Draw black arrows from the origin to curve r with heads at the indexed points;
quiver ( O2x(index2),O2y(index2),r2i(index2),r2j(index2),0,'k');
% Draw red arrows for unit tangents to curve r with tails at the indexed points;
quiver ( r2i(index2),r2j(index2),T2i(index2),T2j(index2),0,'r'  );
axis('equal')                          % Set the x-scale and y-scale equal
xlabel('x','FontSize',14)
ylabel('y','FontSize',14)
title('Positions and Unit Tangents Along a Circular Curve      Fig. 27.2b','FontSize',18)

print -dill Fig_27.1.ill           % Save figure as an Adobe Illustrator file
```