DISLOCATIONS

I Main Topics
- A Dislocations and other defects in solids
- B Significance of dislocations
- C Planar dislocations
- D Displacement and stress fields for a screw dislocation (mode III)

II Dislocations and other defects in solids
- A Dislocations
  - 1 Originally, extra (or missing) planes or partial planes of material (e.g., atoms)
  - 2 Surfaces across which displacements are discontinuous
  - 3 Evidence for dislocations from electron microscopy
- B Point defects
  - 1 Originally, extra (or missing) volumes (e.g., atoms)
  - 2 Displacements are discontinuous across point defects

III Significance of dislocations
- A They account for permanent plastic deformation in crystals
- B They account for the low observed strength of crystals relative to theoretical predictions
- B They provide useful quantitative description of relative motions across surfaces across a broad range of scale (crystals [10^{-6} m] to plate boundaries [10^{6} m]) – ~12 orders of magnitude!
- C They induce tremendous stress concentrations and account for large deformations under small “average” stresses

IV Planar dislocations

A Represented mathematically as infinitely long cut with a straight edge

B ***Relative*** displacement (of one side of the dislocation relative to the other) across a dislocation is called the Burger's vector *b*.

C Screw dislocation

1 Accommodate a tearing motion

2 Displacement is exclusively parallel to the dislocation edge

3 Analogy: a lock washer or a 360° spiral staircase

4 Macroscopic geologic use: to model faults

D Edge dislocation

1 Accommodate opening or sliding motions

2 Displacement is exclusively perpendicular to the dislocation edge

3 Displacement can be parallel or perpendicular to the dislocation plane

4 Analogy: an extra row of corn kernels on a cob of corn

5 Macroscopic geologic use: to model dikes or faults

V Displacement and stress fields for a screw dislocation (mode III)

A Displacement parallel to the dislocation edge increases uniformly along a spiral-like circuit from one side of the dislocation to the other (for a right-handed screw dislocation, point your right thumb along the dislocation edge; displacement parallel to the edge increases in the direction your fingers curl.

B Angular position: $\theta = \tan^{-1}(y/x)$

C Expressions for displacements and strains

1 Cartesian displacements: $u = u_x \quad v = u_y \quad w = u_z$

2 Normal strains: $\varepsilon_{xx} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}\right) \quad \varepsilon_{yy} = \frac{1}{2}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial y}\right) \quad \varepsilon_{zz} = \frac{1}{2}\left(\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z}\right)$

3 Shear strains: $\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \quad \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \quad \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$

4 Cylindrical displacements: $u_r \quad u_\theta \quad u_z = w$

5 Normal strains: $\varepsilon_{rr} = \frac{1}{2}\left(\frac{\partial u_r}{\partial r} + \frac{\partial u_r}{\partial r}\right) \quad \varepsilon_{\theta\theta} = \frac{u}{r} + \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta}\right) \quad \varepsilon_{zz} = \frac{1}{2}\left(\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z}\right)$

6 Shear strains: $\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) \varepsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) \varepsilon_{zr} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right)$

Polar coordinates | Cartesian coordinates

a $u_r = 0$ | $u = 0$

b $u_\theta = 0$ | $v = 0$

c $w = b\frac{\theta_z}{2\pi}$ | $w = \frac{b}{2\pi}\tan^{-1}\frac{y}{x}$

2 Strain

Polar coordinates | Cartesian coordinates

a $\varepsilon_{r\theta} = \varepsilon_{\theta r} = 0$ | $\varepsilon_{xy} = \varepsilon_{yx} = 0$

b $\varepsilon_{\theta z} = \varepsilon_{z\theta} = \frac{b}{2\pi r}$ | $\varepsilon_{yz} = \varepsilon_{zy} = \frac{b}{2\pi}\frac{x}{x^2 + y^2} = \frac{b}{2\pi}\frac{x}{r^2}$

c $u_{rz} = u_{zr} = 0$ | $\varepsilon_{xz} = \varepsilon_{zx} = \frac{-b}{2\pi}\frac{y}{x^2 + y^2} = \frac{-b}{2\pi}\frac{y}{r^2}$

d $\varepsilon_{rr} = 0$ | $\varepsilon_{xx} = 0$

e $\varepsilon_{\theta\theta} = 0$ | $\varepsilon_{yy} = 0$

f $\varepsilon_{zz} = 0$ | $\varepsilon_{zz} = 0$

3 Stress (G = shear modulus)

a $\sigma_{r\theta} = \sigma_{\theta r} = 0$ | $\sigma_{xy} = \sigma_{yx} = 0$

b $\sigma_{\theta z} = \sigma_{z\theta} = \frac{Gb}{2\pi r}$ | $\sigma_{yz} = \sigma_{zy} = \frac{Gb}{2\pi}\frac{x}{x^2 + y^2} = \frac{Gb}{2\pi}\frac{x}{r^2}$

c $\sigma_{r\theta} = \sigma_{\theta r} = 0$ | $\sigma_{xz} = \sigma_{zx} = \frac{-Gb}{2\pi}\frac{y}{x^2 + y^2} = \frac{-Gb}{2\pi}\frac{y}{r^2}$

d $\sigma_{rr} = 0$ | $\sigma_{xx} = 0$

e $\sigma_{\theta\theta} = 0$ | $\sigma_{yy} = 0$

f $\sigma_{zz} = 0$ | $\sigma_{zz} = 0$

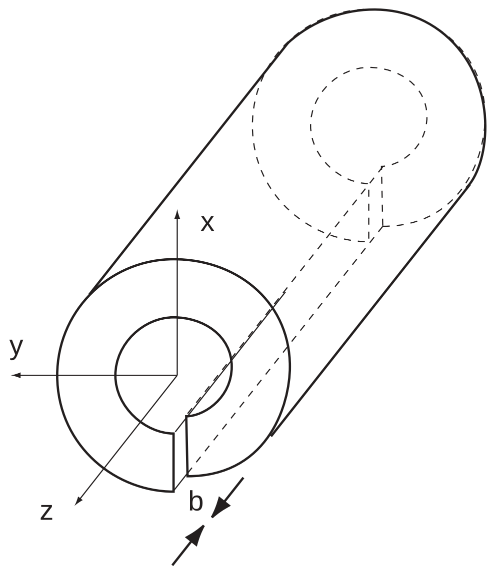
4 Key points

a Only the shear stresses acting on or in the z direction are non-zero

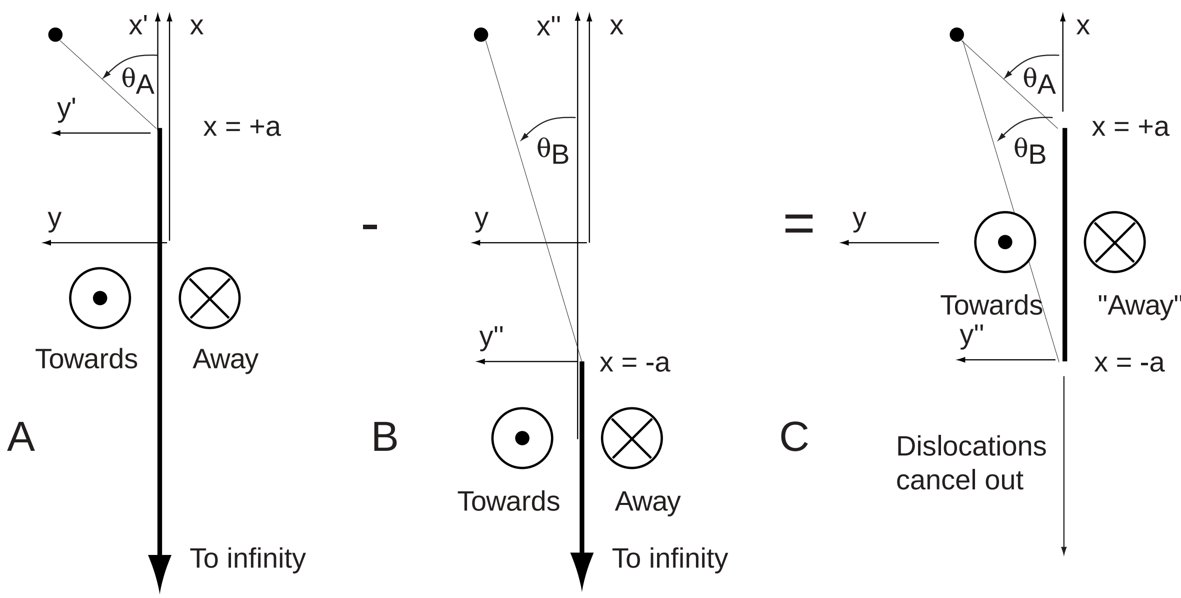
b The stresses are singular (i.e., go to infinity) near the dislocation end: a powerful stress concentration exists there.

c This theoretical singular stress concentration exists no matter how small the relative displacement ***b*** is.

SCREW DISLOCATIONS



SUPERPOSTION OF TWO (INFINITE) SCREW DISLOCATIONS (A,B)
TO FORM A FINITE *DISPLACEMENT DISCONTINTUITY (C)*
(View along the -z direction)



$w_A = b\theta_A/(2\pi)$

$= (b/2\pi)\ \tan^{-1}(y'/x')$

$= (b/2\pi)\ \tan^{-1}(y/[x-a])$

$w_B = b\theta_B/(2\pi)$

$= (b/2\pi)\ \tan^{-1}(y''/x'')$

$= (b/2\pi)\ \tan^{-1}(y/[x+a])$

$-\pi \le \theta \le \pi$

$$w_C = b(\theta_A - \theta_B)/(2\pi)$$

$$= (b/2\pi)\ [\tan^{-1}(y/(x-a)) - \tan^{-1}(y/(x+a))]$$

$w_C\ (\theta_A = -\pi, \theta_B = 0) = -B/2$
$w_C\ (\theta_A = 0, \theta_B = 0) = 0$
$w_C\ (\theta_A = \pi, \theta_B = 0) = B/2$
$w_C\ (\theta_A = \pi, \theta_B = \pi) = 0$