RHEOLOGY & LINEAR ELASTICITY

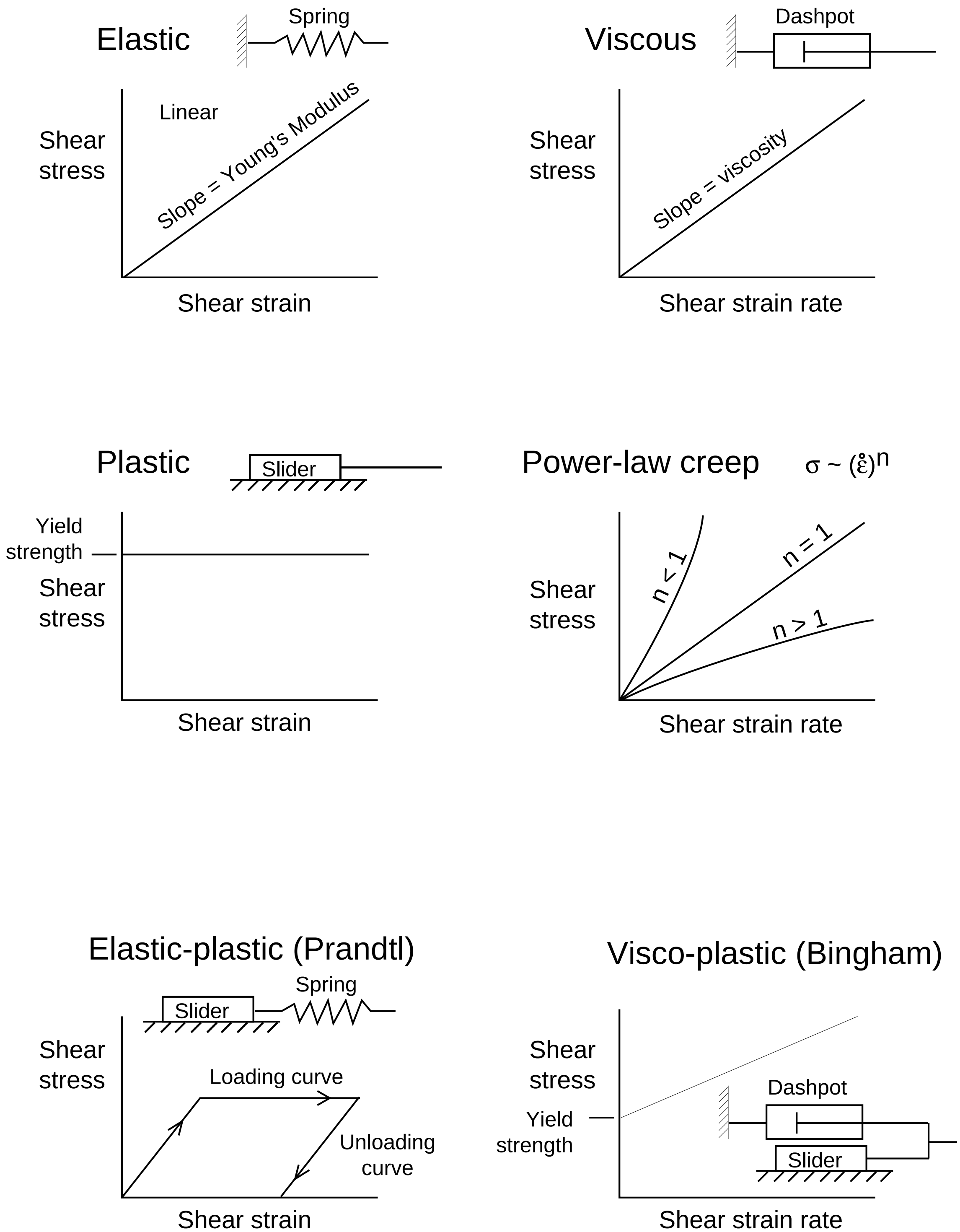
I Main Topics
  A Rheology: Macroscopic deformation behavior
  B Importance of fluids and fractures in deformation
  C Linear elasticity for homogeneous isotropic materials

II Rheology: Macroscopic deformation behavior
  A Elasticity
    1 Deformation is reversible when load is removed
    2 Stress (σ) is related to strain (ε)
    3 Deformation ***is not*** time dependent if load is constant
    4 Examples: Seismic (acoustic) waves, rubber ball
  D Viscosity
    1 Deformation is irreversible when load is removed
    2 Stress (σ) is related to strain ***rate_***($\dot{\varepsilon}$)
    3 Deformation ***is*** time dependent if load is constant
    4 Examples: Lava flows, corn syrup
  C Plasticity
    1 No deformation until yield strength is ***locally*** exceeded; then irreversible deformation occurs under a constant load
    2 Stress is related to strain
    3 Deformation can increase with time under a constant load
    4 Examples: plastics, soils
  D Other rheologies
    1 Elastoplastic and viscoplastic (e.g., paint) rheologies
    2 Power-law creep $\{\dot{\varepsilon} = (\sigma_1 - \sigma_3)^n \exp(-Q/RT)\}$ (e.g., rock salt)
  E Linear vs. nonlinear behavior
  F Rheology = $\mathbf{f}(\sigma_{ij}, \textit{fluid pressure}, \dot{\varepsilon}, \textit{chemistry}, \textit{temperature})$
  G Rheologic equation of real rocks = ?

Common Rheologic Models

(Displacements & forces below anaolgous to strain & stress)

Fig. 20.1



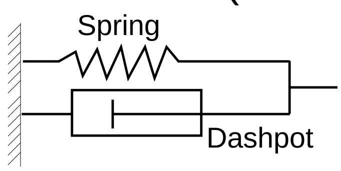
Common Visco-elastic Rheologic Models

Fig. 20.2

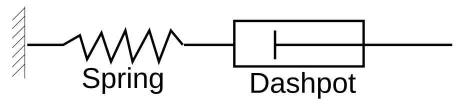
(Displacements & forces below anaolgous to strain & stress)

From Fung, 1977

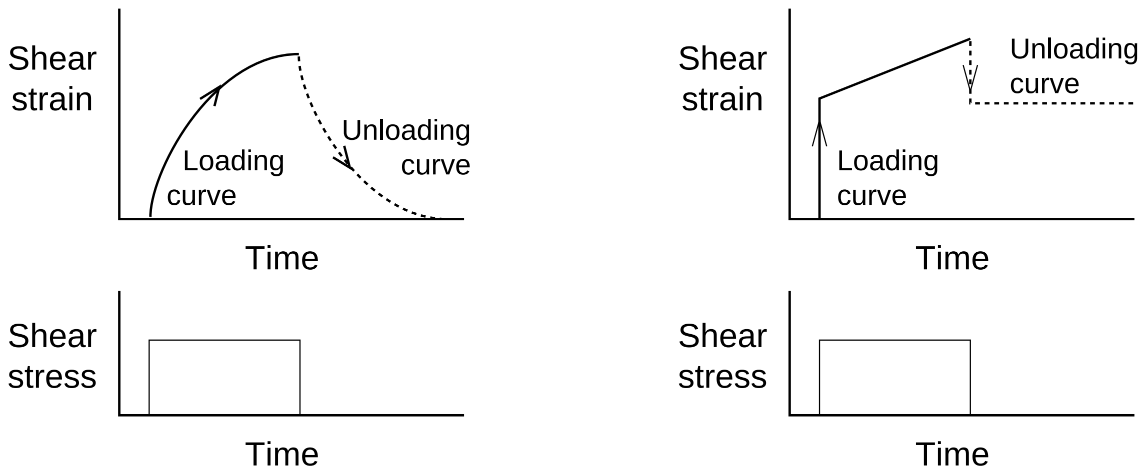
Visco-elastic (Kelvin)



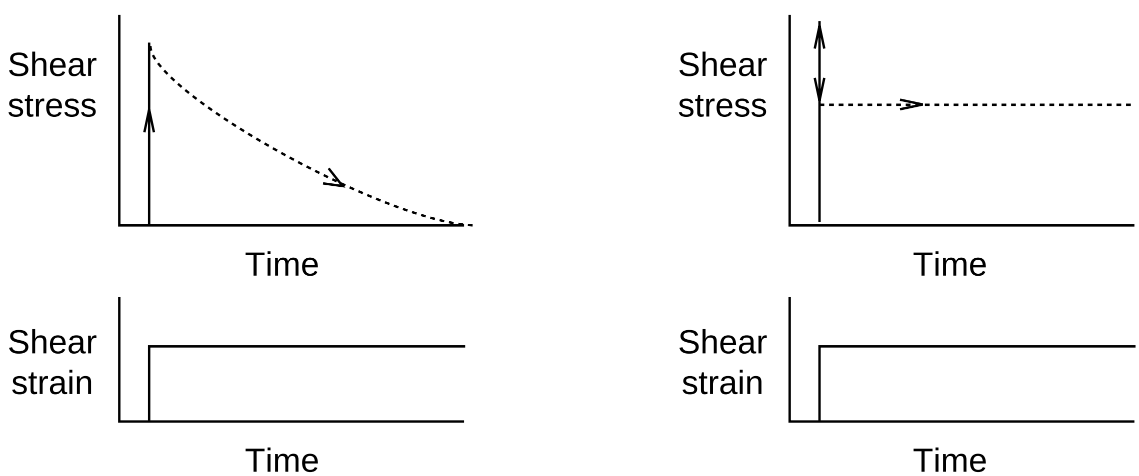
Visco-elastic (Maxwell)



Creep functions in response to step loads in stress



Relaxation functions under constant strain



III Importance of fluids and fractures in deformation

A Observed effects of water on rocks

1 Lowers long term strength

2 Dissolves/precipitates minerals

3 Increases reaction rates by orders of magnitude

B Evidence for fluid-assisted mass/volume change in deformed rocks

1 Martinsburg shale: "pressure solution" considered responsible for 50% volume loss based on strain recorded by fossils.

2 Profound implications for "balanced cross sections" which are constructed assuming conservation of volume of deformed rock

3 Effect of cracks on "pressure solution": cracks greatly enhance the area of a rock mass that can be exposed to fluids

C Effect of cracks on fluid flow

1 Limited influence where fractures are not interconnected

2 Can increase flow rates by several orders of magnitude where fractures are connected

D Veins provide evidence for episodic fluid flow and fracturing

IV Linear elastic stress-strain relationships

A Force and displacement of a spring (from Hooke, 1676)

1 F = kx: F= force, k = spring constant, x = displacement

2 $\text{Elastic potential energy} = \int_0^x F dx = \int_0^x kx \; dx = k\int_0^x x \; dx = \frac{1}{2}kx^2$

Equation 1 can be recast in terms of stress and strain:

3 $\sigma_{11} A = k \int_0^L \varepsilon_{11} dx$

where ε_{11} = du/dx, A = x-section area, and L = spring length

If ε_{11} is a constant along the length of the spring, then

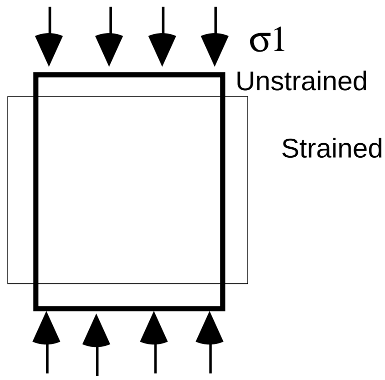
4 $\sigma_{11} A = k\varepsilon_{11} \int_0^L dx = k\varepsilon_{11} L$

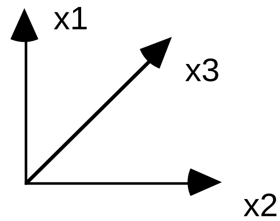
5 $\sigma_{11} = \frac{kL}{A}\varepsilon_{11}$; **stress:strain relationship is linear.** OK for small strains.

B Linear elasticity

1 Constitutive laws for relating stress and (infinitesimal) strain

2 Uniaxial stress: $\sigma_{11} = \sigma_1 \neq 0$; $\sigma_{22} = \sigma_{33} = 0$





a $\varepsilon_{11} = \sigma_{11}/E$ Hooke's Law (or $\varepsilon_1 = \sigma_1/E$)

E = Young's modulus; dimensions of stress

b $\varepsilon_{22} = \varepsilon_{33} = -\nu\,\varepsilon_{11}$ (or $\varepsilon_2 = \varepsilon_3 = -\nu\,\varepsilon_1$)

i ν = Poisson's ratio; dimensionless

ii Strain in one direction tends to induce strain in another

3 Linear elasticity in 3 dimensions

From superposition of Hooke's Law for homogeneous isotropic materials

a $\varepsilon_{11} = \sigma_{11}/E - (\sigma_{22} + \sigma_{33})(\nu/E)$

b $\varepsilon_{22} = \sigma_{22}/E - (\sigma_{11} + \sigma_{33})(\nu/E)$

c $\varepsilon_{33} = \sigma_{33}/E - (\sigma_{11} + \sigma_{22})(\nu/E)$

d Directions of principal stresses and principal strains coincide

e Extension in one direction can occur without tension

f Compression in one direction can occur without shortening

D Isotropic (hydrostatic) stress: $\sigma_1 = \sigma_2 = \sigma_3$; no shear stress

E Uniaxial strain: $\varepsilon_{11} = \varepsilon_1 \neq 0$; $\varepsilon_{22} = \varepsilon_{33} = 0$

F <u>Plane stress</u>: $\sigma_{33} = \sigma_3 = 0$; $\sigma_{11} \neq 0$; $\sigma_{22} \neq 0$

Thin plate approximation; stress perpendicular to plane are zero

G <u>Plane strain</u>: $\varepsilon_{33} = \varepsilon_3 = 0$; $\varepsilon_{11} \neq 0$; $\varepsilon_{22} \neq 0$

1 Displacement in the 3-direction is constant (zero)

2 Plate is confined between rigid walls $\perp$ to x_3

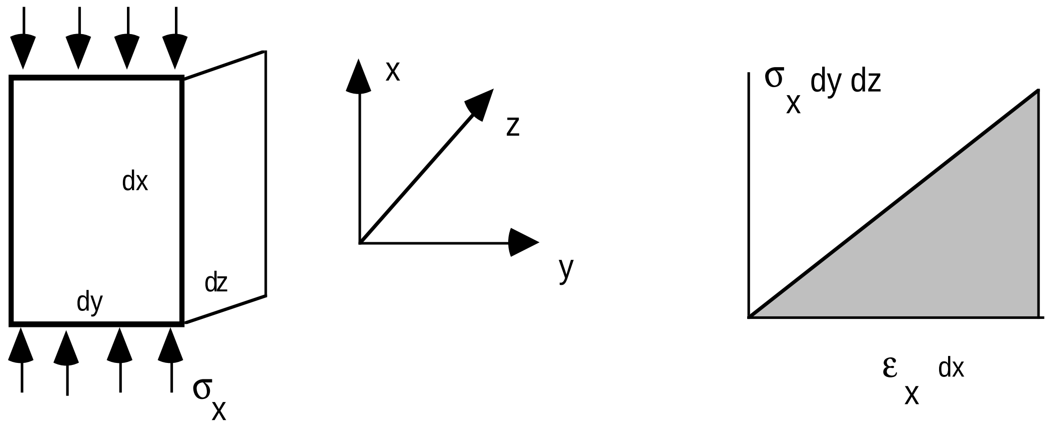
3 Thick plate approximation

H <u>Pure shear stress</u>: $\sigma_1 = -\sigma_2$; $\sigma_3 = 0$

IV Strain energy

A W = (F)(u) = E

B Uniaxial stress



1 dW = (F)(du) = 1/2 (σ_x dy dz) ($\varepsilon_x$dx) = 1/2 ($\sigma_x\,\varepsilon_x$) (dx dy dz)

2 Strain energy density = W_0 = dW/(dx dy dz)= 1/2 ($\sigma_x\,\varepsilon_x$)

C Strain energy associated with three-dimensional loading (in terms of principal stresses and principal strains)

1 dW = 1/2 ($\sigma_1\,\varepsilon_1 + \sigma_2\,\varepsilon_2 + \sigma_2\,\varepsilon_2$) (dx dy dz)

2 W_0 = dW/(dx dy dz) = 1/2 ($\sigma_1\,\varepsilon_1 + \sigma_2\,\varepsilon_2 + \sigma_2\,\varepsilon_2$)

V Relationships among different elastic moduli

1 G = μ = shear modulus = E/(2[1+ν]); $\varepsilon_{xy} = \sigma_{xy}/2G$

2 λ = Lame' constant = Eν/([1 + ν][1 - 2ν])

3 K = bulk modulus = E/(3[1 - 2ν])

4 β = compressibility = 1/K

$\Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$ = -p/K; p = pressure

5 P-wave speed: $V_p = \sqrt{\left(K + \frac{4}{3}\mu\right)/\rho}$

6 S-wave speed: $Vs = \sqrt{\mu / \rho}$

Additional References

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