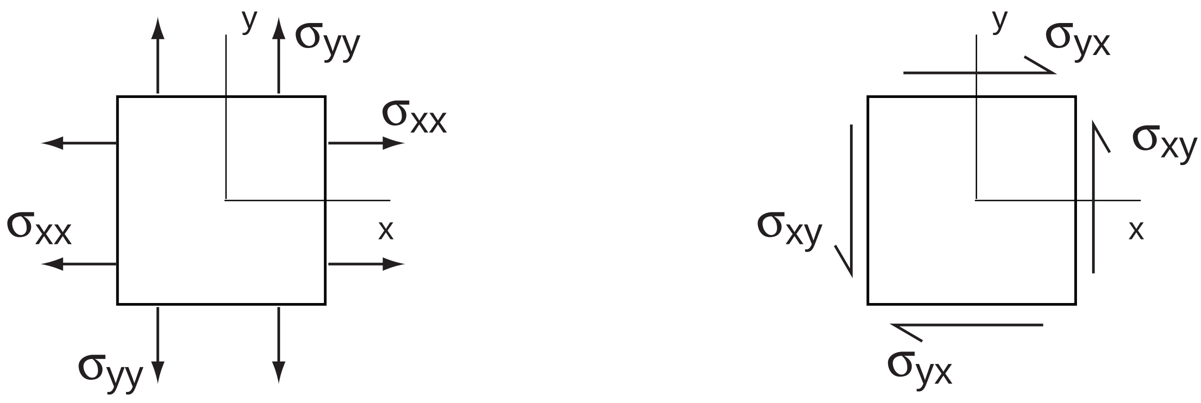
STRESS AT A POINT

I Main Topics
  A Stress vector (traction) on a plane
  B Stress at a point
  C Principal stresses
  D Initial 2-D stress transformation equations

II Stress vector (traction) **on a particular plane**: $\vec{\tau}$
  A $\vec{\tau} = \lim_{A \to 0} \vec{F}/A$. Dimensions of force per unit area
  B Traction vectors can be added vectorially.
  C A traction vector can be resolved into normal and shear components.
    1 A normal traction $\vec{\tau}_n$ acts perpendicular to a plane
    2 A shear traction $\vec{\tau}_s$ acts parallel to a plane
  D The magnitudes of tractions depend on the orientation of the plane

III Stress at a point
  A Stresses refer to **balanced** internal "forces (per unit area)". They differ from force vectors, which, if unbalanced, cause accelerations
  B **"On -in convention": The stress component σ_{ij} acts on the plane normal to the i-direction and acts in the j-direction**
    1 Normal stresses: i=j
    2 Shear stresses: i≠j



Normal stresses Shear stresses

C $\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$ or $\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$

D In 3-D, nine components are needed to define the state of stress at a point. These nine components are the traction components that act on three perpendicular planes (planes with normals in the x-, y-, and z-directions)

E The state of stress can (and usually does) vary from point to point.

F For rotational equilibrium, $\sigma_{xy} = \sigma_{yx}$, $\sigma_{xz} = \sigma_{zx}$, $\sigma_{yz} = \sigma_{zy}$

IV **Principal Stresses** (t<u>hese have magnitudes and orientations</u>)

A Principal stresses act on planes which feel no shear stress

B The principal stresses are normal stresses.

C Principal stresses act on perpendicular planes

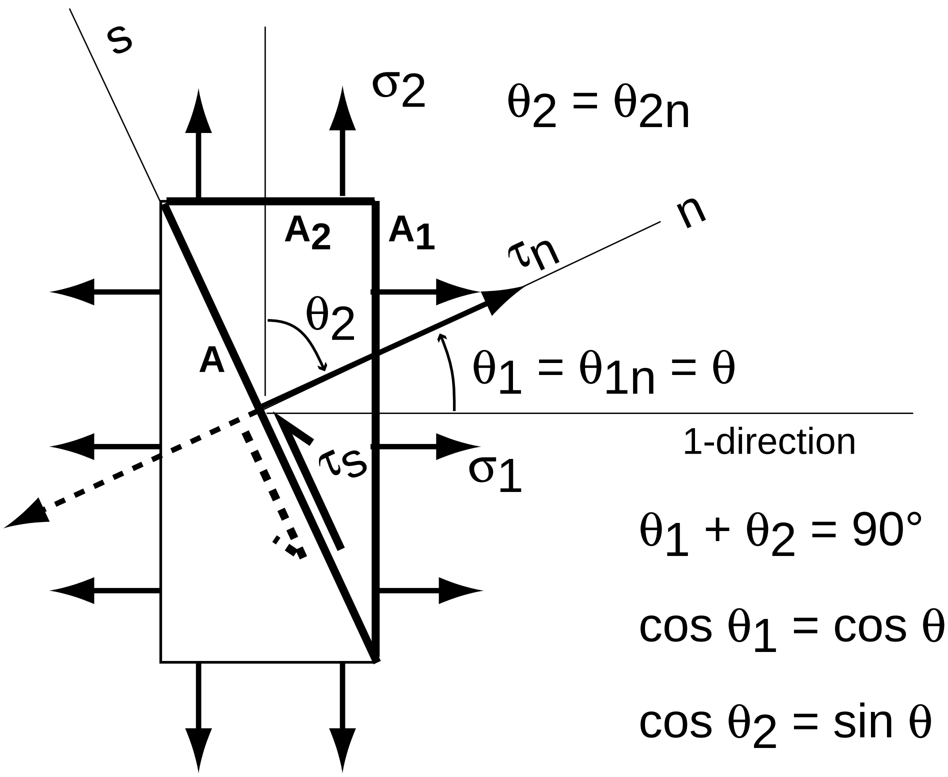
D The maximum, intermediate, and minimum principal stresses are usually designated σ_1, σ_2, and σ_3, respectively. Note that the principal stresses have a single subscript.

E <u>The principal stresses represent the stress state most simply.</u>

F $\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$ 3-D or $\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$ 2-D

V Initial 2-D stress transformation equations

A Consider three planes A, A1, and A2 that form the sides of a triangular prism; these have normals in the n-, 1-, and 2-directions, respectively. Plane A1 is acted on by known normal stress σ_1. Plane A2 is acted on by known normal stress σ_2. The n-direction is at angle θ_1 (=θ) with respect to the 1-direction, and at angle θ_2 with respect to the 2-direction. The s-direction is at a counter-clockwise 90° angle relative to the n-direction (like y and x).



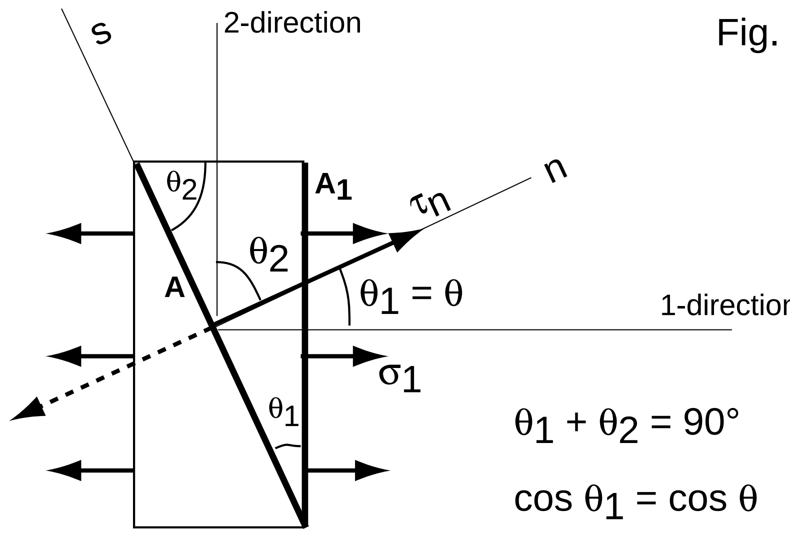
To find the normal traction, τ_n that acts on plane A, we determine how much σ1 and σ2 each contribute individually to τ_n and then sum the contributions (Figures 16.1, 16.2). Similarly, to find the shear traction, τ_s that acts on plane A, we determine how much σ1 and σ2 each contribute to τ and then sum those contributions (Figures 16.3, 16.4). The results are:

1 $\tau_n = \sigma_1 \cos^2\theta + \sigma_2 sin^2\theta$

2 $\tau_s = (\sigma_2 - \sigma_1) \sin\theta \cos\theta$

Contribution of σ_1 to τ_n

Fig. 1



$\theta_1 + \theta_2 = 90°$

$\cos\theta_1 = \cos\theta$

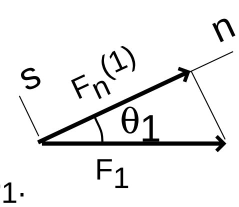
$\cos\theta_2 = \sin\theta$

What does σ_1 on face A_1 of area A_1 contribute to τ_n on face A of area A?

Start with the definition of stress: $\tau_n^{(1)} = F_n^{(1)}/A$.

The unknown quantities F_n and A must be found from the known quantities σ_1 and θ.

To do this we first find the force F_1 associated with σ_1:



Force = (stress)(area)

$F_1 = \sigma_1 A_1$

The component of force F_1 that acts along the n-direction is $F_1 \cos\theta_1$.

$F_n^{(1)} = F_1 \cos\theta_1$

As can be seen from the diagram atop the page $A_1 = A \cos\theta_1$, so

$A = A_1/\cos\theta_1$

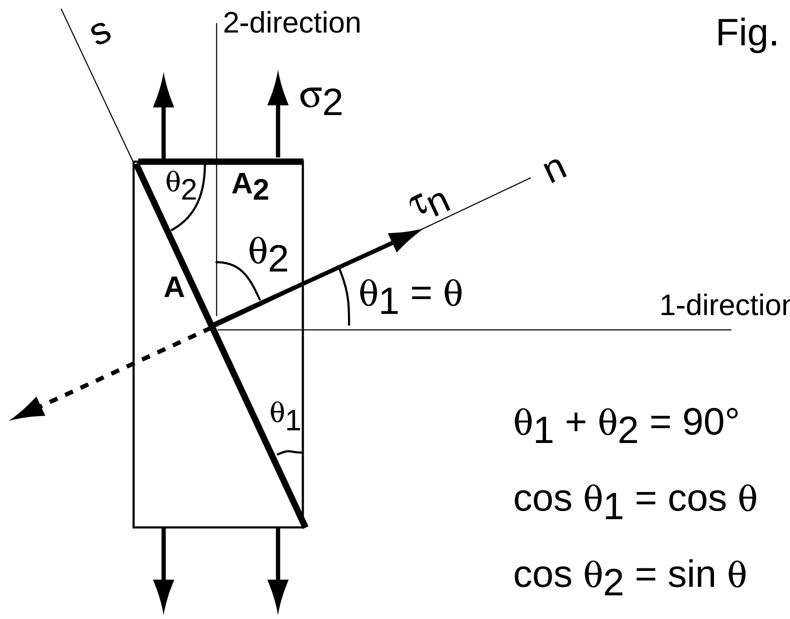
So the contribution to σ_n by σ_1 is:

$\tau_n^{(1)} = F_n^{(1)} / A = F_1 \cos\theta_1 / (A_1/\cos\theta_1) = (F_1 / A_1) \cos\theta_1 \cos\theta_1 = \sigma_1 \cos\theta_1 \cos$

$\tau_n^{(1)} = \sigma_1 \cos\theta \cos\theta$ ← First contribution to τ_n

Fig. 16.2

Contribution of σ_2 to τ_n



$\theta_1 + \theta_2 = 90°$

$\cos \theta_1 = \cos \theta$

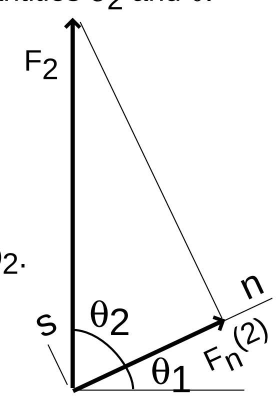
$\cos \theta_2 = \sin \theta$

What does σ_2 on face A_2 of area A_2 contribute to τ_n on face A of area A?

Start with the definition of stress: $\tau_n^{(2)} = F_s^{(2)}/ A.$

The unknown quantities F_s and A must be found from the known quantities σ_2 and θ.

To do this we first find the force F_2 associated with σ_2:



Force = (stress)(area)

$F_2 = \sigma_2 A_2$

The component of force F_2 that acts along the n-direction is $F_2 \cos \theta_2$.

$F_n^{(2)} = F_2 \cos \theta_2$

As can be seen from the diagram atop the page $A_2 = A \cos \theta_2$, so

$A = A_2/\cos \theta_2$

So the contribution to σ_n by σ_2 is:

$\tau_n^{(2)} = F_n^{(2)} / A = F_2 \cos \theta_2 / (A_2/\cos \theta_2) = (F_2 / A_2) \cos \theta_2 \cos \theta_2 = \sigma_2 \cos \theta_2 \cos \theta_2$

$\tau_n^{(2)} = \sigma_2 \sin \theta \sin \theta$ ← Second contribution to σ_n

$\tau_n = \tau_n^{(1)} + \tau_n^{(2)} = \sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta$ ← Total value of σ_n

Contribution of σ_1 to τ_s

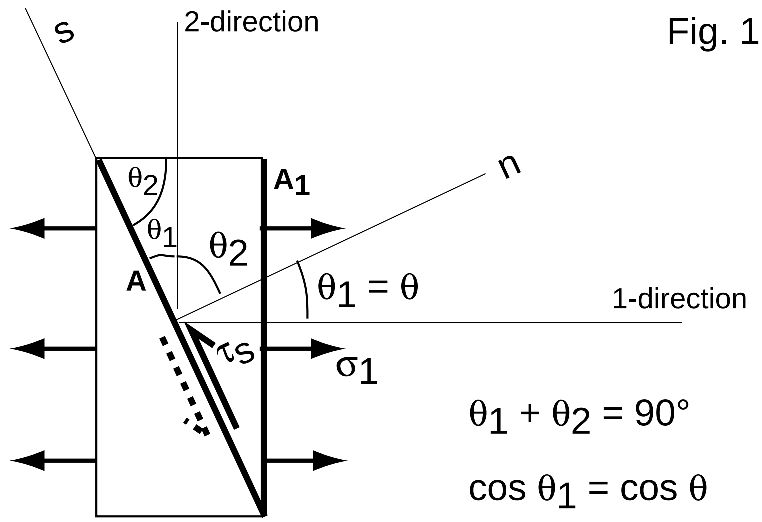


Fig. 16.3

$\theta_1 + \theta_2 = 90°$

$\cos\theta_1 = \cos\theta$

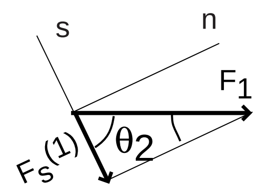
$\cos\theta_2 = \sin\theta$

What does σ_1 on face A_1 of area A_1 contribute to τ_s on face A of area A?

Start with the definition of stress: $\tau_s^{(1)} = F_s^{(1)} / A$

The unknown quantities F_s and A must be found from the known quantities σ_2 and θ.

To do this we first find the force F_1 associated with σ_1:



Force = (stress)(area)

$F_1 = \sigma_1 A_1$

The component of force F_1 that acts along the s-direction is $- F_1 \cos\theta_2$.

$F_s^{(1)} = - F_1 \cos\theta_2$

As can be seen from the diagram atop the page $A_1 = A \cos\theta_1$, so

$A = A_1 / \cos\theta_1$

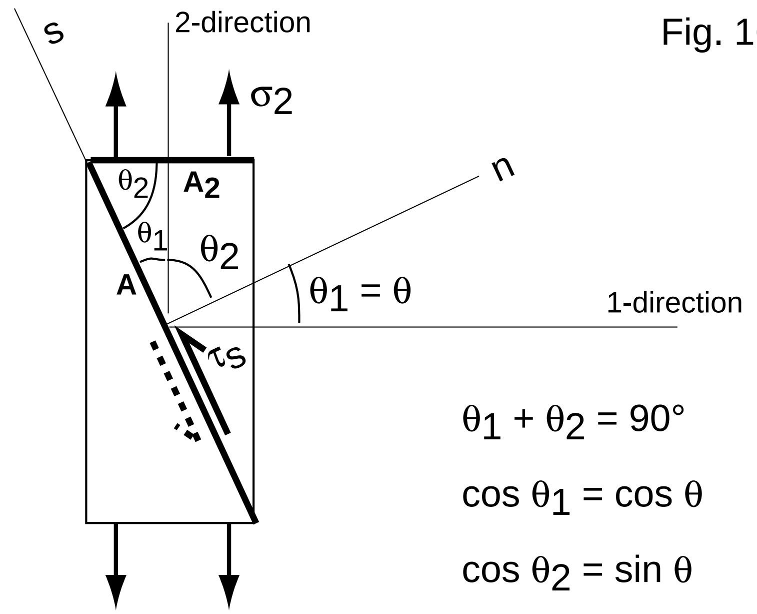
So the contribution to τ by σ_1 is:

$$\tau_s^{(1)} = F_s^{(1)} / A = - F_1 \cos\theta_2 / (A_1/\cos\theta_1) = (- F_2 / A_2) \cos\theta_2 \cos\theta_1 = - \sigma_1 \cos\theta_2 \cos\theta_1$$

$\tau_s^{(1)} = - \sigma_1 \sin\theta \cos\theta$ ← First contribution to τ_s

Fig. 16.4

Contribution of σ_2 to τ_s



$\theta_1 + \theta_2 = 90°$

$\cos\theta_1 = \cos\theta$

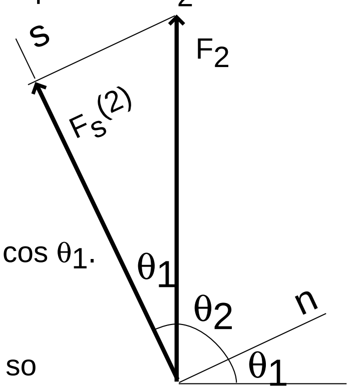
$\cos\theta_2 = \sin\theta$

What does σ_2 on face A_2 of area A_2 contribute to τ_s on face A of area A?

Start with the definition of stress: $\boxed{\tau_s^{(2)} = F_s^{(2)}/ A}$

The unknown quantities F_s and A must be found from the known quantities σ_2 and θ.

To do this we first find the force F_2 associated with σ_2:



Force = (stress)(area)

$\boxed{F_2 = \sigma_2 A_2}$

The component of force F_2 that acts along the s-direction is $F_2 \cos\theta_1$.

$\boxed{F_s^{(2)} = F_2 \cos\theta_1}$

As can be seen from the diagram atop the page $A_2 = A \cos\theta_2$, so

$\boxed{A = A_2/\cos\theta_2}$

So the contribution to τ by σ_2 is:

$\tau_s^{(2)} = F_s^{(2)} / A = F_2 \cos\theta_1 / (A_2/\cos\theta_2) = (F_2 / A_2) \cos\theta_1 \cos\theta_2 = \sigma_2 \cos\theta_2 \cos\theta_1$

$\boxed{\tau_s^{(2)} = \sigma_2 \sin\theta \cos\theta}$ ← Second contribution to τ_s

$\boxed{\tau_s = \tau_s^{(1)} + \tau_s^{(2)} = (\sigma_2 - \sigma_1)(\sin\theta \cos\theta)}$ ← Total value of τ_s