ROTATIONS (II)

I Main Topics
- A Rotations using a stereonet
- B Comments on rotations using stereonets and matrices
- C Uses of rotation in geology (and engineering) II

II Rotations using a stereonet
- A Best uses
  - 1 Rotation axis is vertical
  - 2 Rotation axis is horizontal (e.g., to restore tilted beds)
- B **Construction technique**
  - 1 Find orientation of rotation axis
  - 2 Find angle of rotation and rotate **pole** to plane (or a linear feature) along a **small circle** perpendicular to the rotation axis.
    - a For a horizontal rotation axis the small circle is vertical
    - b For a vertical rotation axis the small circle is horizontal
  - 3 **WARNING**: DON'T rotate a plane by rotating its dip direction vector; this doesn't work (see handout)

III Comments on rotations using stereonets and matrices

	Advantages	Disadvantages
Stereonets	Good for visualization Can bring into field	Relatively slow Need good stereonets, paper
Matrices	Speed and flexibility Good for multiple rotations	Computer really required to cut down on errors

- A **General comments** about stereonets vs. rotation matrices
  - **1 With stereonets, an object is usually considered to be rotated and the coordinate axes are held fixed.**
  - **2 With rotation matrices, an object is usually considered to be held fixed and the coordinate axes are rotated.**
- B To return tilted bedding to horizontal choose a rotation axis that coincides with the direction of strike. The angle of rotation is the **negative** of the dip of the bedding (right-hand rule!) if the bedding is to be rotated back to horizontal and **positive** if the axes are to be rotated to the plane of the tilted bedding. Only one rotation is typically used; seldom are beds then rotated about a vertical axis.

IV Orientations of features from drill cores **(see handout)**

A Orientations measured relative to core are called apparent

B To determine the in-situ (true) orientations of features from the apparent orientations, two rotations are needed (see handout):

1 Rotate the core (or the coordinate axes) about a vertical axis. The rotation angle involves the trend of the core.

2 Rotate the core (or the coordinate axes) about a horizontal axis perpendicular to the trend of the core . The rotation angle is the involves the plunge of the core .

3 The rotations are given by

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) & 0 \\ -\sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} \cos(-[90-\phi]) & 0 & -\sin(-[90-\phi]) \\ 0 & 1 & 0 \\ \sin(-[90-\phi]) & 0 & \cos(-[90-\phi]) \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} \right)$$

or

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos(-\theta)\cos(-[90-\phi]) & \sin(-\theta) & -\cos(-\theta)\sin(-[90-\phi]) \\ -\sin(-\theta)\cos(-[90-\phi]) & \cos(-\theta) & \sin(-\theta)\sin(-[90-\phi]) \\ \sin(-[90-\phi]) & 0 & \cos(-[90-\phi]) \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix}$$

or

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos\theta\sin\phi & -\sin\theta & \cos\theta\cos\phi \\ \sin\theta\sin\phi & \cos\theta & \sin\theta\cos\phi \\ -\cos\phi & 0 & \sin\phi \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix}$$

4 In the approach described in the handout the primed axes are rotated to coincide with the unprimed axes. An alternative to rotate the unprimed axes to coincide with the primed axes. This can be accomplished by the following two steps:

a Rotate the unprimed axes about the vertical x_3 axis by angle θ. The x_2 axis becomes the x_2^a axis.

b Rotate the unprimed axes about the x_2^a axis by angle $(90°-\phi)$.

c The rotations are given by

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} \cos(90°-\phi) & 0 & -\sin(90°-\phi) \\ 0 & 1 & 0 \\ \sin(90°-\phi) & 0 & \cos(90°-\phi) \end{bmatrix} \left(\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right)$$

or

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} \sin\phi\cos\theta & \sin\phi\sin\theta & -\cos\phi \\ -\sin\theta & \cos\theta & 0 \\ \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

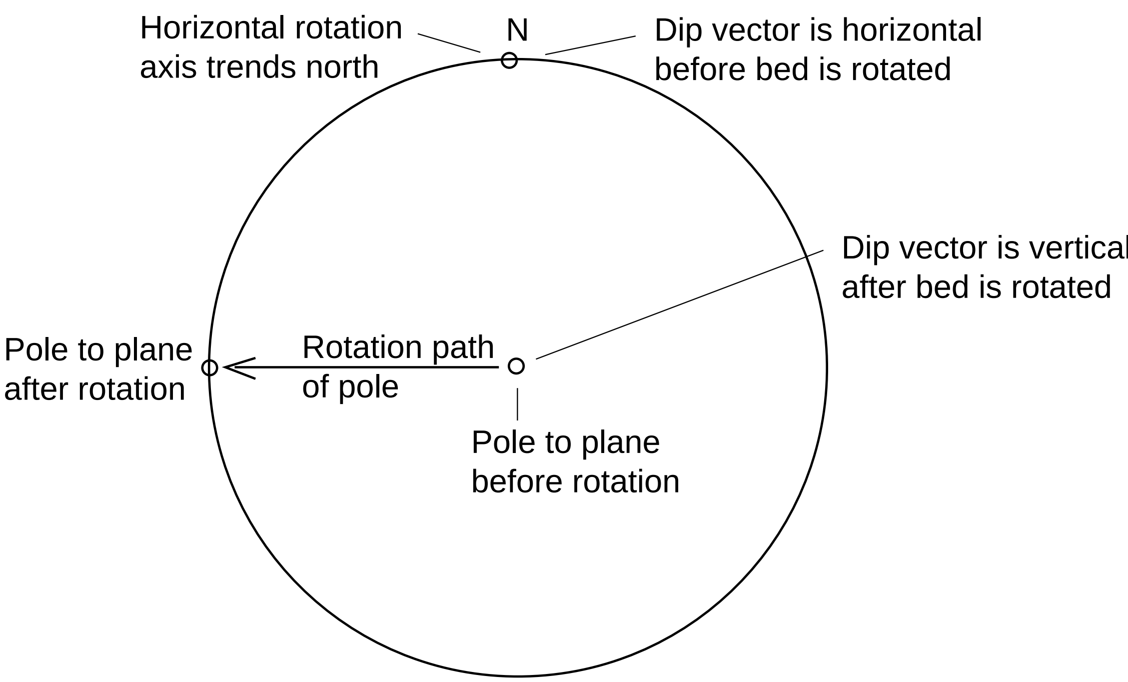
d From (2c), if [x] = [a][x'], then, from (3c), [x'] = [a]^T[x]

Fig. 11.1

Rotations of planes using poles; Problem with attempted rotation of planes using dip vectors

Consider a horizontal plane. We will consider it to strike to the north and dip to the east at 0°; these directions are consistent with a right-hand rule. The pole to the plane trends west (270°) and plunges 90°. The dip vector trends east (90°) and plunges 0°.

Suppose we wish to rotate the plane by +90° about a horizontal axis that trends north. We can visualize that after the rotation the plane will still strike to the north but will dip 90°. How do the pole to the plane and the dip vector rotate?



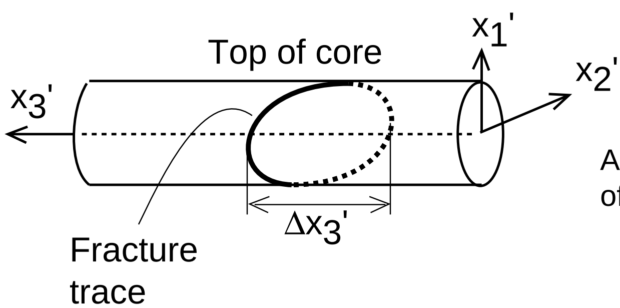
Equal area projection

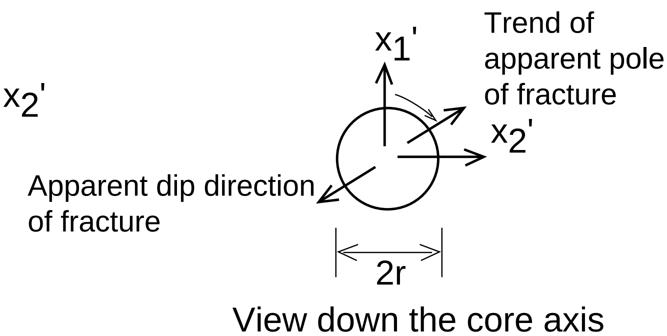
The pole will rotate about the rotation axis and yield a result consistent with the final orienation of the the rotated plane. **The original dip vector will not rotate about the rotation axis**, so there is no rotation path to link the pre-rotation dip vector for the plane to the post-rotation dip vector for the plane.

Bottom line: do rotations with poles, not dip vectors

Conversion of apparent orientation scheme of Goodman to in-situ orientations (I)

Let the coordinate system for the apparent orientations be the x_1', x_2', x_3' system, where x_3' points down the core axis and x_1' points towards a line scribed on the top surface of the core. In this reference frame the apparent dip direction of a fracture = α and the apparent dip angle of the fracture = β.

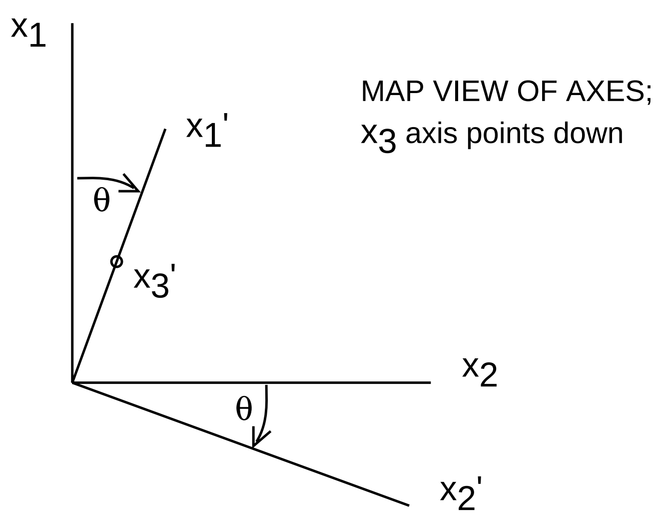




In practice, this means the "apparent orientations" are measured with the core held vertically and its top line facing north.

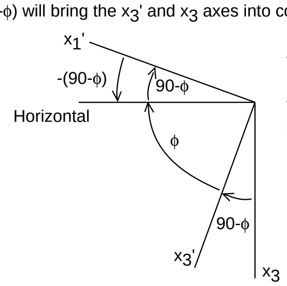
If the radius of the core is r, and the limits of the fracture trace in the x_3' direction is Δx_3, then $\underline{\beta = \tan^{-1}(\Delta x_3'/2r)}$

Suppose the in-situ geographic coordinate system is chosen to be x_1 = north, x_2 = east, and x_3 = down. The vertical plane containing the borehole contains the x_1' and x_3' axes, and the x_2' axis is horizontal. The trend and plunge of the x_3' axis coincide with the trend (θ) and plunge (ϕ) of the borehole.



Conversion of apparent orientation scheme of Goodman
to in-situ orientations (II)

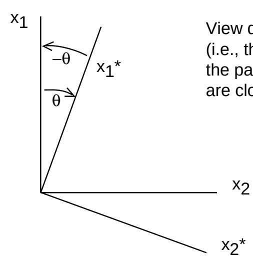
Rotation 1
Rotating the primed axes about the horizontal x_2' axis by the angle **-**(90°-ϕ) will bring the x_3' and x_3 axes into coincidence.



View down along the x_2' axis (i.e., the x_2' axis points into the page). Positive angles are clockwise.

The x_3' axis becomes the x_3^* axis.

Rotation 2
Rotating the starred axes about the vertical x_3^* axis by the angle (-θ) will bring all the axes into coincidence.



View down along the x_3^* axis (i.e., the x_3^* axis points into the page). Positive angles are clockwise.

Let the coordinate system for the apparent orientations be the x_1',x_2',x_3' system, where x_3' points down the core axis and x_1' points towards a line scribed on the top surface of the core. In this reference frame the apparent dip direction of a fracture = α and the apparent dip angle of the fracture = β.

If the radius of the core is r, and the limits of the fracture trace in the x_3' direction is Δx_3, then $\beta = \tan^{-1}$ ($2r / \Delta x_3'$)

Suppose the in-situ geographic coordinate system is chosen to be x_1 = north, x_2 = east, and x_3 = down. The vertical plane containing the borehole contains the x_1' and x_3' axes, and the x_2' axis is horizontal. The trend and plunge of the x_3' axis coincide with the trend (θ) and plunge (ϕ) of the borehole.