ROTATIONS (I)

I Main Topics
  A Uses of rotation in geology (and engineering) I
  B Concepts behind rotation

II Uses of rotation in geology (and engineering) I
  A Plate tectonics
  B Evaluating stresses and strains
  C Return tilted bedding to horizontal to examine:
    1 pre-tilting orientation of sedimentary structures (e.g. ripples)
    2 pre-tilting orientation of beds below angular unconformities
    3 pre-tilting orientation of paleomagnetic orientations
  D To determine the orientations of features from drill cores
  E Need to consider whether object is rotated and coordinate axes are fixed or whether the object is fixed and coordinate axes are rotated. This affects the sign(s) and sequence of the angle(s) of rotation.

III Concepts behind rotation
  A Any orthogonal coordinate system with axes x_1, x_2, x_3 can be rotated to coincide with another orthogonal coordinate system x_1', x_2', x_3' by using the direction cosines relating the axes of the two systems.

Dir. cosines	x_1 axis	x_2 axis	x_3 axis
x_1' axis	$a_{11'} = a_{1'1}$	$a_{12'} = a_{2'1}$	$a_{13'} = a_{3'1}$
x_2' axis	$a_{21'} = a_{1'2}$	$a_{22'} = a_{2'2}$	$a_{23'} = a_{3'2}$
x_3' axis	$a_{31'} = a_{1'3}$	$a_{32'} = a_{2'3}$	$a_{33'} = a_{3'3}$

Note: $a_{13'}$ is not equal to $a_{1'3}$ because $\theta_{13'}$ is not equal to $\theta_{1'3}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_{11'} & a_{12'} & a_{13'} \\ a_{21'} & a_{22'} & a_{23'} \\ a_{31'} & a_{32'} & a_{33'} \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} a_{1'1} & a_{1'2} & a_{1'3} \\ a_{2'1} & a_{2'2} & a_{2'3} \\ a_{3'1} & a_{3'2} & a_{3'3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Right-hand rule: If thumb is along rotation axis, then fingers curl in positive θ direction

B <u>Any</u> orthogonal coordinate system with axes x_1,x_2,x_3 can always be rotated to coincide with another orthogonal coordinate system by the following **three** steps (Cayley):

1 <u>Rotate the xyz system about the x_1 axis by angle θ_1</u>,
so x_1,x_2,x_3 -> x_1',x_2',x_3' (Note that $x_1=x_1'$)

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} a_{1'1} & a_{1'2} & a_{1'3} \\ a_{2'1} & a_{2'2} & a_{2'3} \\ a_{3'1} & a_{3'2} & a_{3'3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

2 Rotate the x_1',x_2',x_3' system about the x_2' axis by angle θ_2,
so x_1',x_2',x_3' -> x_1'',x_2'',x_3'' (Note that $x_2'=x_2''$)

$$\begin{bmatrix} x_1'' \\ x_2'' \\ x_3'' \end{bmatrix} = \begin{bmatrix} a_{1''1'} & a_{1''2'} & a_{1''3'} \\ a_{2''1'} & a_{2''2'} & a_{2''3'} \\ a_{3''1'} & a_{3''2'} & a_{3''3'} \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix}$$

3 Rotate the x_1'',x_2'',x_3'' system about the x_3'' axis by angle θ_3,
so x_1'',x_2'',x_3'' -> x_1''',x_2''',x_3''' (Note that $x_3''=x_3'''$)

$$\begin{bmatrix} x_1''' \\ x_2''' \\ x_3''' \end{bmatrix} = \begin{bmatrix} a_{1'''1''} & a_{1'''2''} & a_{1'''3''} \\ a_{2'''1''} & a_{2'''2''} & a_{2'''3''} \\ a_{3'''1''} & a_{3'''2''} & a_{3'''3''} \end{bmatrix} \begin{bmatrix} x_1'' \\ x_2'' \\ x_3'' \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1'' \\ x_2'' \\ x_3'' \end{bmatrix}$$

Note that the rotation matrices can be obtained by permutation:
x_1 -> x_2, x_2 -> x_3, and x_3 -> x_1.

The direction cosines relating the x''' and x systems are:

Dir. cosines	x_1 axis	x_2 axis	x_3 axis
x_1''' axis	$a_{1'''1} = a_{11}'''$	$a_{1'''2} = a_{21}'''$	$a_{1'''3} = a_{31}'''$
x_2''' axis	$a_{2'''1} = a_{12}'''$	$a_{2'''2} = a_{22}'''$	$a_{2'''3} = a_{32}'''$
x_3''' axis	$a_{3'''1} = a_{13}'''$	$a_{3'''2} = a_{23}'''$	$a_{3'''3} = a_{33}'''$

C <u>Any</u> orthogonal coordinate system with axes x_1, x_2, x_3 can always be rotated to coincide with another orthogonal coordinate system by one rotation about a specially chosen axis (Euler's theorem). This single rotation can be found by successively applying the three rotations listed above:

$$\begin{bmatrix} x_1''' \\ x_2''' \\ x_3''' \end{bmatrix} = \begin{bmatrix} a_{1'''1''} & a_{1'''2''} & a_{1'''3''} \\ a_{2'''1''} & a_{2'''2''} & a_{2'''3''} \\ a_{3'''1''} & a_{3'''2''} & a_{3'''3''} \end{bmatrix} \left(\begin{bmatrix} a_{1''1'} & a_{1''2'} & a_{1''3'} \\ a_{2''1'} & a_{2''2'} & a_{2''3'} \\ a_{3''1'} & a_{3''2'} & a_{3''3'} \end{bmatrix} \left(\begin{bmatrix} a_{1'1} & a_{1'2} & a_{1'3} \\ a_{2'1} & a_{2'2} & a_{2'3} \\ a_{3'1} & a_{3'2} & a_{3'3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) \right)$$

$$\begin{bmatrix} x_1''' \\ x_2''' \\ x_3''' \end{bmatrix} = \begin{bmatrix} a_{1'''1} & a_{1'''2} & a_{1'''3} \\ a_{2'''1} & a_{2'''2} & a_{2'''3} \\ a_{3'''1} & a_{3'''2} & a_{3'''3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Dir. cosines	x_1 axis	x_2 axis	x_3 axis
x_1''' axis	$+\cos\theta_3 \cos\theta_2$	$+\sin\theta_3 \cos\theta_2$	$-\sin\theta_2$
x_2''' axis	$-\sin\theta_3 \cos\theta_1$ $+\cos\theta_3 \sin\theta_2 \sin\theta_1$	$+\cos\theta_3 \cos\theta_1$ $+\sin\theta_3 \sin\theta_2 \sin\theta_1$	$+\cos\theta_2 \sin\theta_1$
x_3''' axis	$+\sin\theta_3 \sin\theta_1$ $+\cos\theta_3 \sin\theta_2 \cos\theta_1$	$-\cos\theta_3 \sin\theta_1$ $+\sin\theta_3 \sin\theta_2 \cos\theta_1$	$+\cos\theta_2 \cos\theta_1$

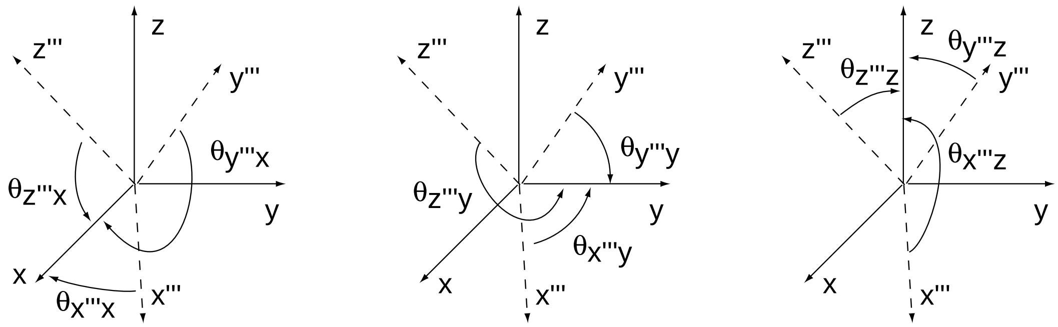
Let the axis of rotation be called axis **N** and the direction cosines relating the N axis to **<u>either</u>** the x_1, x_2, x_3 system **<u>or</u>** the x_1''', x_2''', x_3''' system be η_1, η_2, and η_3, respectively. If the angle of rotation about **N** is θ, then the nine direction cosines in the rotation matrix above can be expressed as:

Dir. cosines	x_1 axis	x_2 axis	x_3 axis
x_1''' axis	$\eta_1\eta_1(1-\cos\theta)$ $+ \cos\theta$	$\eta_1\eta_2(1-\cos\theta)$ $+ \eta_3 \sin\theta$	$\eta_1\eta_3(1-\cos\theta)$ $- \eta_2 \sin\theta$
x_2''' axis	$\eta_2\eta_1(1-\cos\theta)$ $- \eta_3 \sin\theta$	$\eta_2\eta_2(1-\cos\theta)$ $+ \cos\theta$	$\eta_2\eta_3(1-\cos\theta)$ $+ \eta_1 \sin\theta$
x_3''' axis	$\eta_3\eta_1(1-\cos\theta)$ $+ \eta_2 \sin\theta$	$\eta_3\eta_2(1-\cos\theta)$ $- \eta_1 \sin\theta$	$\eta_3\eta_3(1-\cos\theta)$ $+ \cos\theta$

Methods of Rotation

Fig. 10.1

1 All the angles between the x,y,z axes and x''',y''',z''' axes are known

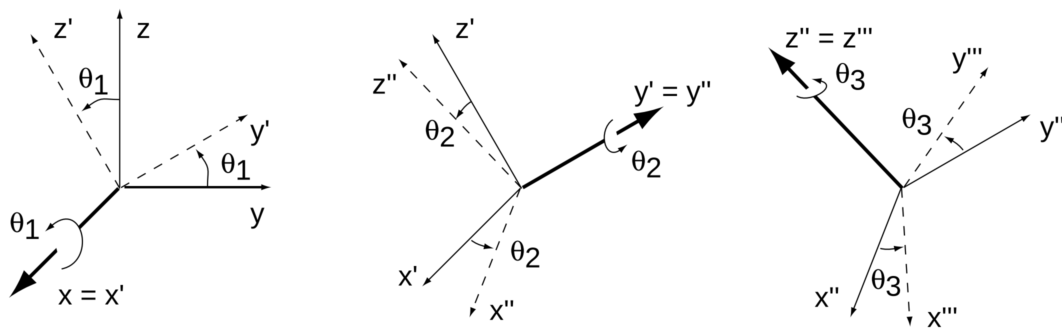


$a_{x'''x} = \cos \theta_{x'''x}$ $a_{x'''y} = \cos \theta_{x'''y}$ $a_{x'''z} = \cos \theta_{x'''z}$

$a_{y'''x} = \cos \theta_{y'''x}$ $a_{y'''y} = \cos \theta_{y'''y}$ $a_{y'''z} = \cos \theta_{y'''z}$

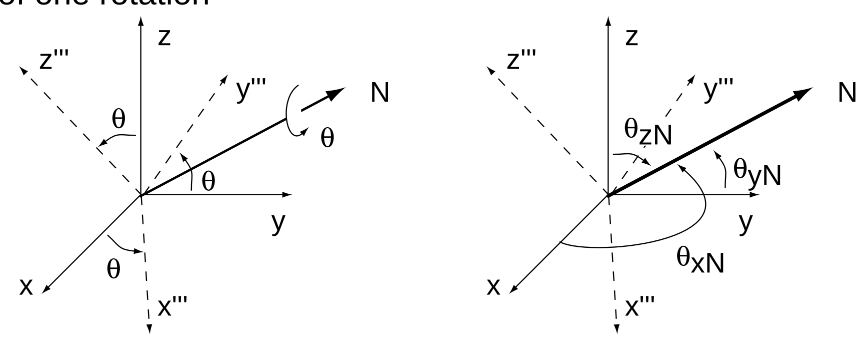
$a_{z'''x} = \cos \theta_{z'''x}$ $a_{z'''y} = \cos \theta_{z'''y}$ $a_{z'''z} = \cos \theta_{z'''z}$

2 Method of three rotations



A: Rotate about x-axis B: Rotate about y'-axis C: Rotate about z''-axis

3 Method of one rotation

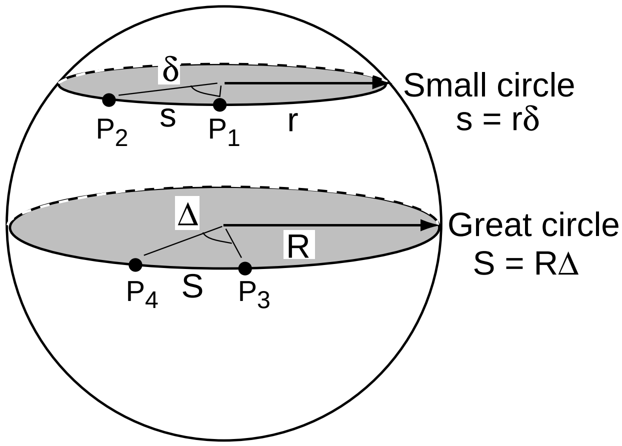


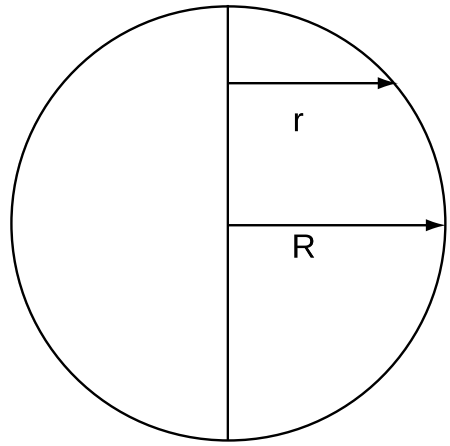
This shows the angle of rotation about the rotation axis

This shows the orientation of the rotation axis relative to the three coordinate axes

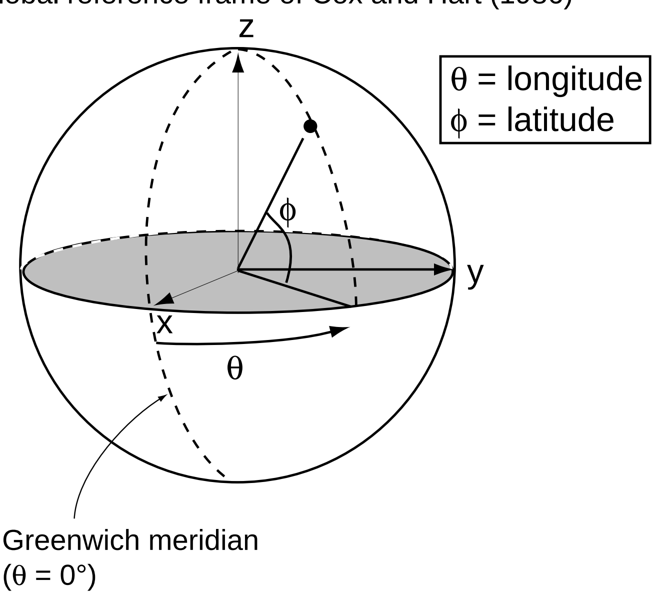
$\eta_1 = \cos \theta_{xN}$ $\eta_2 = \cos \theta_{yN}$ $\eta_3 = \cos \theta_{zN}$

Geometry on a Sphere for Plate Tectonics Fig. 10.2





Global reference frame of Cox and Hart (1986)



<u>Direction cosines</u>

$\alpha = \cos \omega_x = (\cos \phi)(\cos \theta)$

$\beta = \cos \omega_y = (\cos \phi)(\sin \theta)$

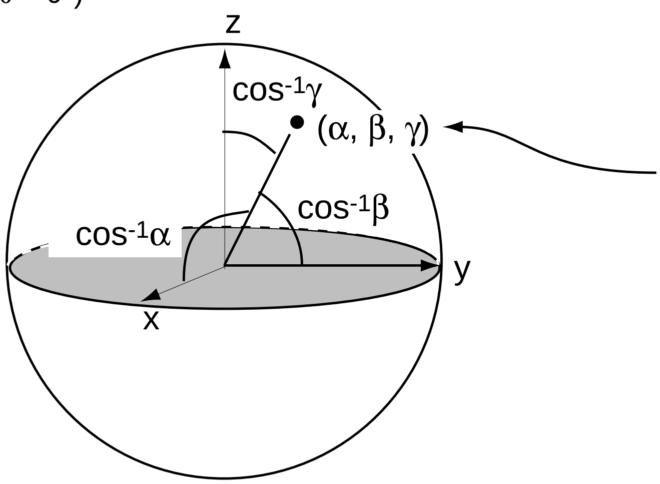
$\gamma = \cos w_z = \sin \phi$

East longitude: positive
West longitude: negative
North latitude: positive
South latitude: negative

To get x,y, and z coordinates, multiply α, β, and γ by R, respectively .

$\phi = \sin^{-1}\gamma$

$\theta = \operatorname{atan2}(\beta,\alpha)$



The x,y,z coordinates of a point on a <u>unit sphere</u> are α, β, and γ, respectively .