SPHERICAL PROJECTIONS (II)

Schedule Updates and Reminders: Bring tracing paper &needles for Lab 5

I Main Topics
- A Angles between lines and planes
- B Determination of fold axes
- C Equal- angle and equal- area projections
- D Equal-angle projections of circles
- E Computer programs for spherical projections

II Angles between lines and planes (see Fig. 8.3)
- A Angle between lines is measured in the plane (great circle) containing the lines. On a spherical projection, this angle is measured along the cyclographic trace of the unique great circle representing the plane containing the two lines. The angle is obtained by counting across the small circles the cyclographic traces crosses.
- B The angle between two planes is the angle between the poles to the planes. This angle is measured along the cyclographic trace of the unique great circle representing the plane containing the poles to the two planes. This also could be done using the dot product or cross product of the poles.

III Fold axes of cylindrical folds (see Fig. 8.3)
- A The fold axis is along the line of intersection of beds (β diagram).
- B The fold axis is perpendicular to the plane containing the poles to beds (π diagram); this approach works better than a β diagram if many poles to beds are being considered. For two poles, the π diagram procedure is analogous to finding the cross product between the bedding plane poles.

IV Types of spherical projections
- A Equal angle projection (Wulff net) See handout
- B Equal area projection (Schmidt net) See handout

C Comparison of Equal Angle and Equal Area projections (see Fig. 9.1)

(From Hobbs, Means, and Williams, 1976, An Outline of Structural Geology)

Property	Equal angle projection	Equal area projection
Net type	Wulff net	Schmidt net
Projection preserves ...	Angles	Areas
Projection does not preserve ...	Areas	Angles
A line project as a ...	Point	Point
Great circle projection	Circle	Fourth-order quadric
Small circle projection	Circle	Fourth-order quadric
Distance from center of primitive circle to cyclographic trace measured in direction of dip	$R\tan\left(\frac{\pi}{4}-\frac{dip}{2}\right)$	$R\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{dip}{2}\right)$
Distance from center of primitive circle to pole of plane measured in the direction opposite to that of the dip	$R\tan\left(\frac{dip}{2}\right)$	$R\sqrt{2}\sin\left(\frac{dip}{2}\right)$
Distance from center of primitive circle to point that represents a plunging line	$R\tan\left(\frac{\pi}{4}-\frac{plunge}{2}\right)$	$R\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{plunge}{2}\right)$
Best use	Measuring angular relations	Contouring orientation data

For equal-angle projections alone, the angle between two planes equals the angle between tangent lines where the cyclographic traces of two planes intersect (hence the name of the projection)

V Equal-angle projections of circles (see Figs. 9.2-9.4)

VI Free computer programs for spherical projections

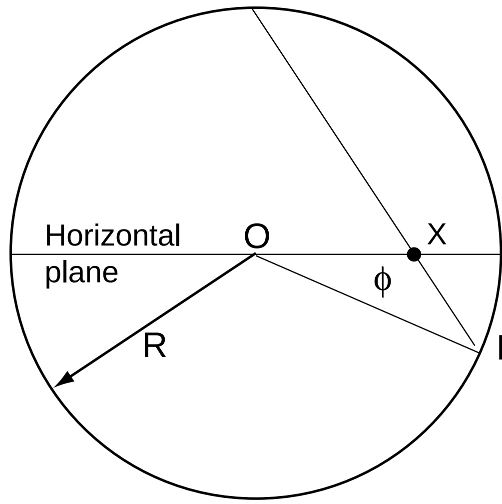
1 S.J. Martel's Matlab code for spherical projections

2 "Stereonet" by R. Allmendinger at Cornell University (for the Mac)

Fig. 9.1

Spherical Projections

Equal-angle projection (Sterographic Projection)

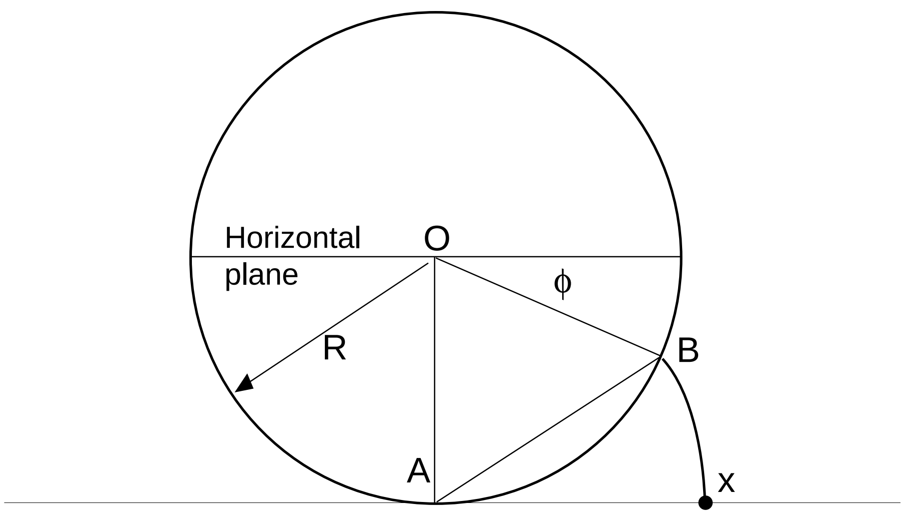


Horizontal plane through center of sphere is projection plane

B is the "original" point
X is the projection

The **shapes** of plane shapes on the surface of the sphere **are preserved** in this projection, but the relative **areas are altered**. Good for measuring the angles between the cyclographic traces of planes.

Equal-area projection

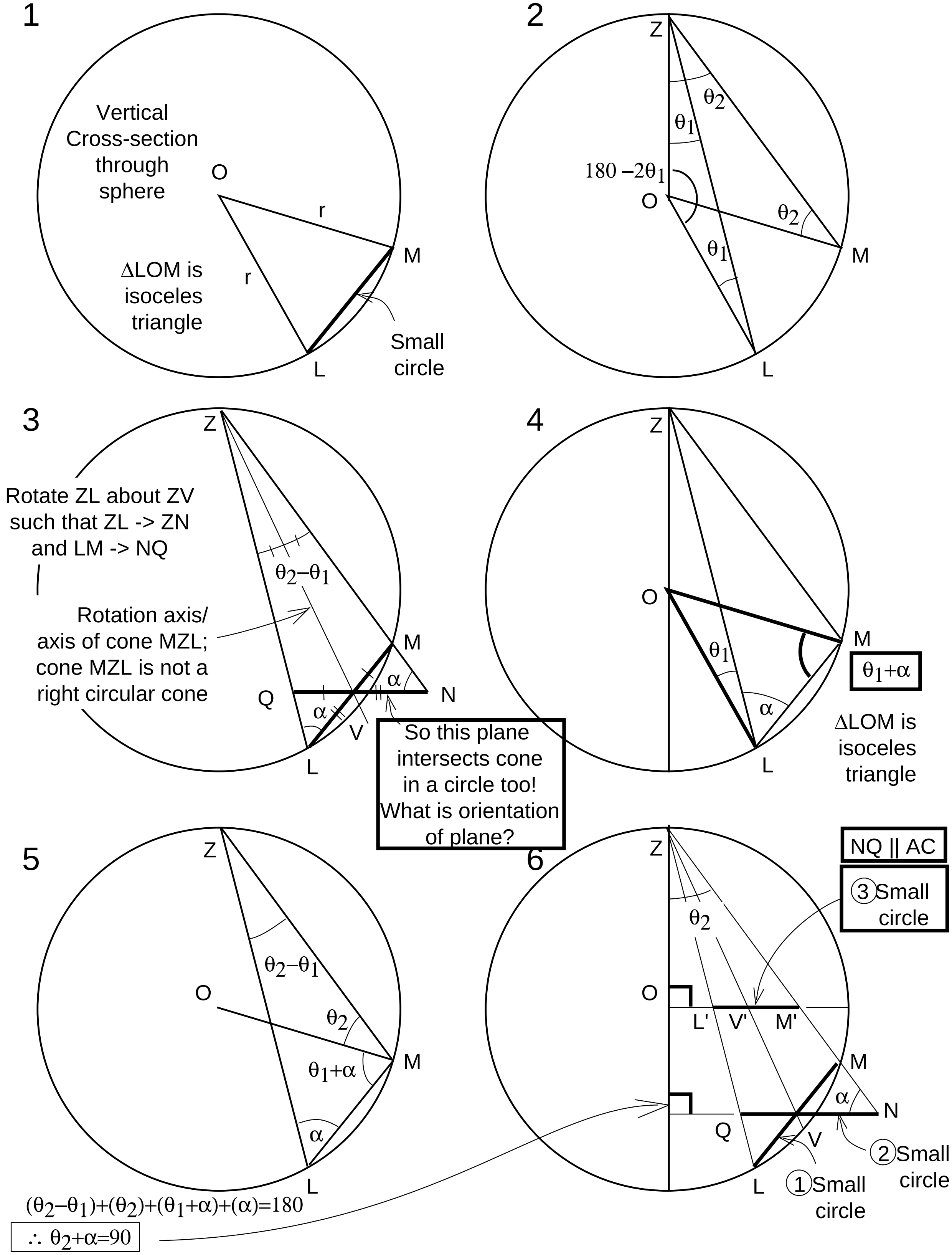


B is the "original" point
X is the projection

The relative **areas** of plane shapes on the surface of the sphere **are preserved** in this projection, but the **shapes are altered**. Good for representing the density of poles.

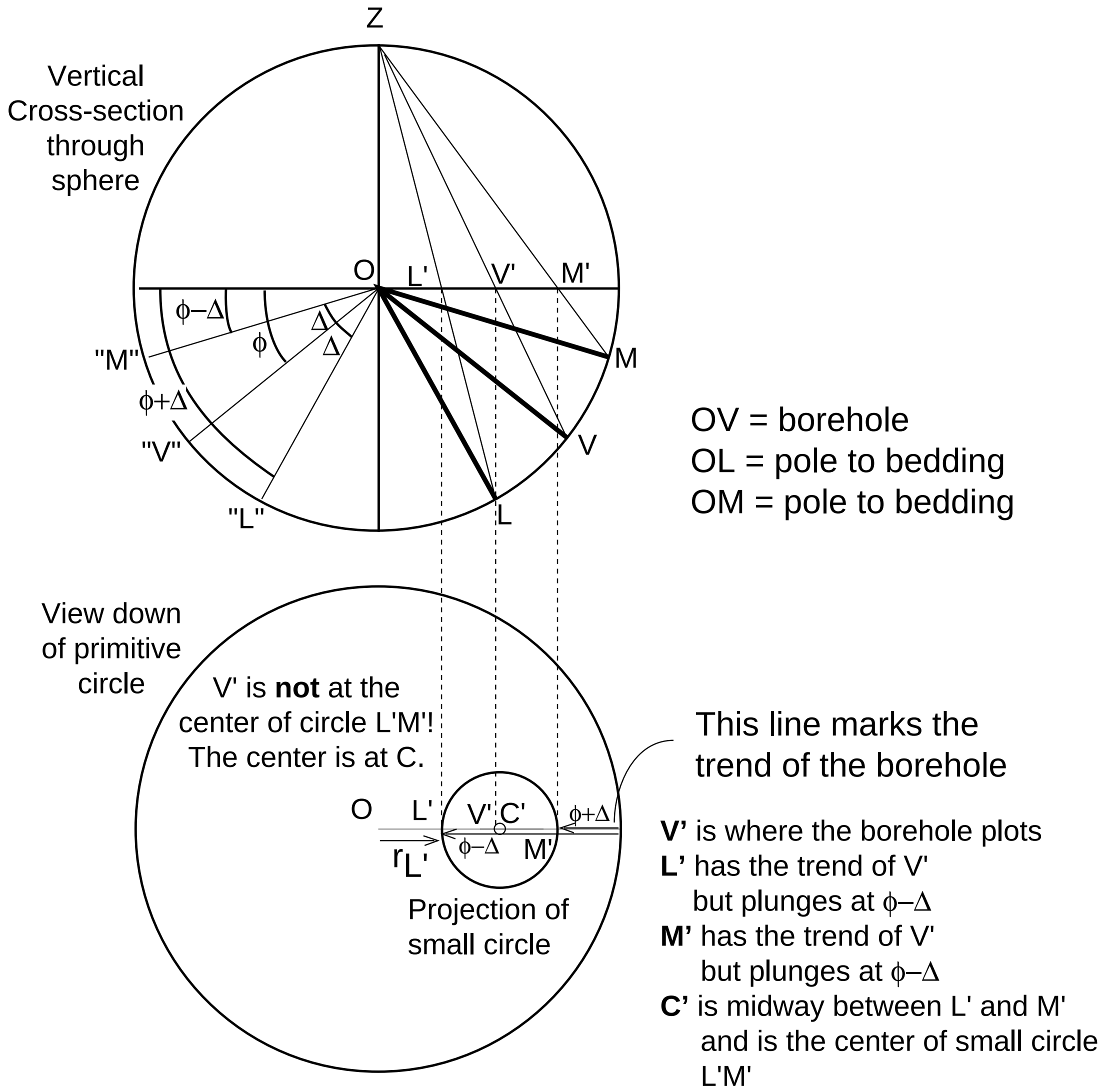
Equal-Angle Projection of a Small Circle

Fig. 9.2



Equal-Angle Projection of a Small Circle (II)

Fig. 9.3L



$$\frac{r_{L'}}{R} = \tan\left[\frac{90° - (\phi+\Delta)}{2}\right] \qquad r_{L'} = R\tan\left[\frac{90° - (\phi+\Delta)}{2}\right] \qquad \phi+\Delta = 90° - 2(\tan^{-1}[r_{L'}/R])$$

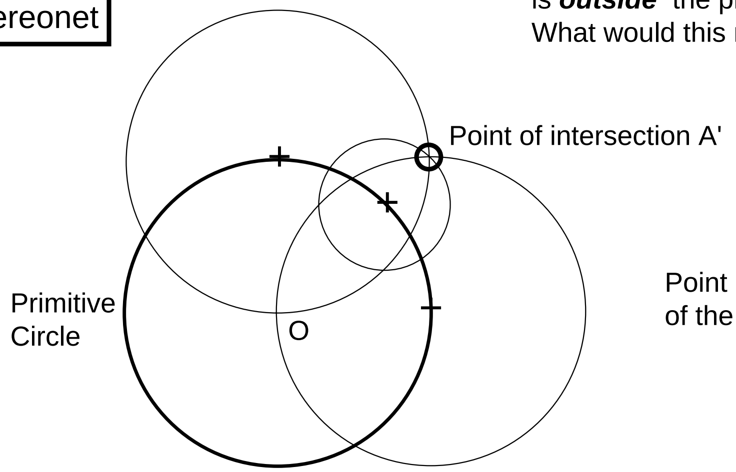
$$\frac{r_{M'}}{R} = \tan\left[\frac{90° - (\phi-\Delta)}{2}\right] \qquad r_{M'} = R\tan\left[\frac{90° - (\phi-\Delta)}{2}\right] \qquad \phi-\Delta = 90° - 2(\tan^{-1}[r_{M'}/R])$$

$$r_C = \frac{r_{L'}+r_{M'}}{2} \qquad \text{radius of proj. small circle} = \frac{r_{L'} - r_{M'}}{2}$$

Points Outside a Primitive Circle in Equal-angle Projections Fig 9.4L

View down onto stereonet

The point of intersection of three circles is ***outside*** the primitive circle. What would this mean?

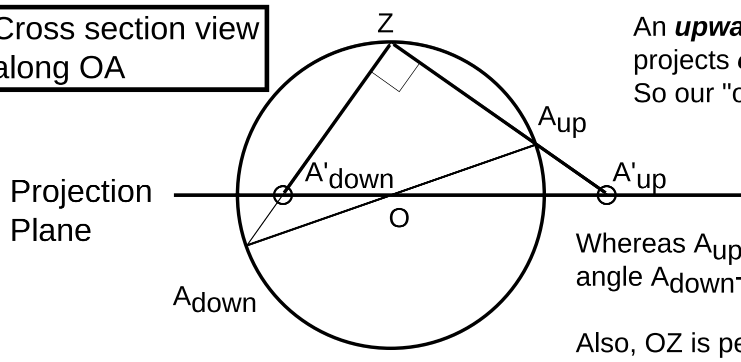


Point O is the center of the primitive circle

Let's return to how the projection is done to answer the question

Cross section view along OA

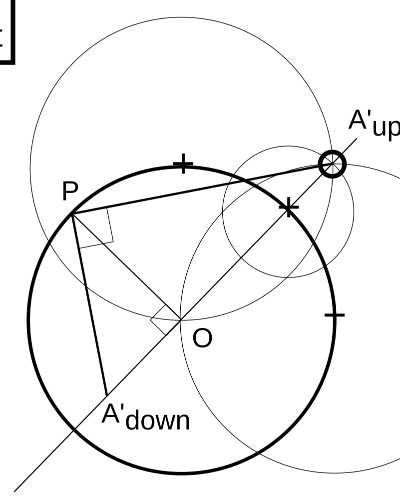
An ***upward*** pointing line projects ***outside*** the primitive circle! So our "outside" point A' is really A'_{up}.



Whereas A_{up}-A_{down} is a diameter, then angle A_{down}-Z-A_{up} must be a right angle.

Also, OZ is perpendicular to A'_{up}.

View down onto stereonet



To plot the downward-pointing pole corresponding to A'_{up}, we turn the equal-angle projection method "on its side":

1 Draw a line from A'_{up} through O

2 Draw line OP perpendicular to line OA'_{up}

3 Draw line OA'_{down} perpendicular to OA'up. Points A'down, O , and A'up lie on one line