EQUATIONS OF LINES & PLANES

I Main Topics

A Direction cosines

B Lines

C Planes

II Direction cosines

A The cosines of the angles between a line and the coordinate axes

B The coordinates of the endpoint of a vector of unit length

C The ordered projection lengths of a line of unit length onto the x,y, and z axes

III Lines

A Defined by 2 points

Two-point form: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

where (x_1,y_1) and (x_2,y_2) are two known points on the line

B Defined by 1 point (e.g., x_0,y_0,z_0) and a direction

Slope-intercept form (2-D): $y = mx + b$

General form (2-D): $Ax + By + C = 0$

$y = mx + b$

$mx - y + b = 0$

$(1/n)\ (mx - y + b) = 0$

$Ax + By + C = 0$

Parametric form: $x = x_0 + t\alpha$, $y = y_0 + t\beta$, $z = z_0 + t\gamma$,

where $\alpha = \cos\omega_x$, $\beta = \cos\omega_y$, and (for 3-D) $\gamma = \cos\omega_z$;

α, β, and γ are direction cosines. Note that in 2-D, $\cos\omega_x = \sin\omega_y$

C Defined by the intersection of two planes

IV Planes

1 **Defined by three points**

2 **Defined by two intersecting lines**

3 Defined by two parallel lines

4 Defined by a line and a point not on the line

5 **Defined by a distance and direction (or pole) from a point**

A General form: $Ax + By + Cz + D = 0$

B Normal form: $\alpha x + \beta y + \gamma z = d$, where

$$\alpha = \frac{A}{\pm\sqrt{A^2+B^2+C^2}}, \quad \beta = \frac{B}{\pm\sqrt{A^2+B^2+C^2}}, \quad \gamma = \frac{C}{\pm\sqrt{A^2+B^2+C^2}},$$

$$d = \frac{-D}{\pm\sqrt{A^2+B^2+C^2}}$$

The sign of the denominator is opposite to the sign of D, **so d >0**.

$\alpha = \cos\omega_x$, $\beta = \cos\omega_y$, and $\gamma = \cos\omega_z$.

C Vector expression of normal form: $\mathbf{n} \bullet \mathbf{V} = d$, where

V is a vector from a given point O to the plane,

n (**bold**) is the unit normal to the plane given by direction cosines α, β, and γ; **n** also goes through point O.

d (unbolded) is the distance from the point to the plane along the normal vector **n**, and

• refers to the dot product:

$\langle x_1, y_1, z_1\rangle \bullet \langle x_2, y_2, z_2\rangle = x_1 x_2 + x_2 y_2 + z_1 z_2$

The equation of "C" can be understood as follows: "The distance from the reference point to a plane (as measured along a direction perpendicular to the plane) is d." If the normal points from the reference point to the plane, then d>0. Otherwise, d<0.

Direction Cosines from Geologic Angle Measurements (Spherical coordinates)

Positive z-axis up
y = north; x = east

xy plane is horizontal plane

RIGHT-HANDED COORDINATES

Remember: Trends are azimuths and are measured in a horizontal plane. Plunges are inclinations and are measured in a vertical plane.

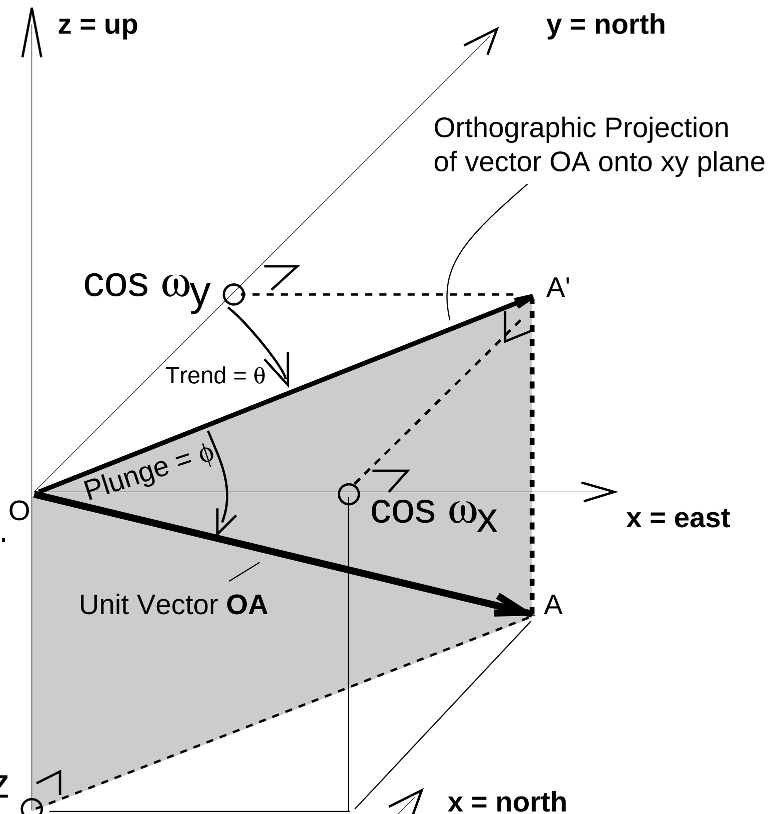
The direction cosines α and β are determined from OA', the length of OA' being |OA'| =|OA| cos φ = cos φ.

$\alpha = \cos\omega_x = (\cos\phi)\,(\sin\theta)$

$\beta = \cos\omega_y = (\cos\phi)\,(\cos\theta)$

$\gamma = \cos\omega_z = -\,(\sin\phi)$

$\alpha^2 + \beta^2 + \gamma^2 = 1$



Positive z-axis down
x = north; y = east

xy plane is horizontal plane

RIGHT-HANDED COORDINATES

Remember: Trends are azimuths and are measured in a horizontal plane. Plunges are inclinations and are measured in a vertical plane.

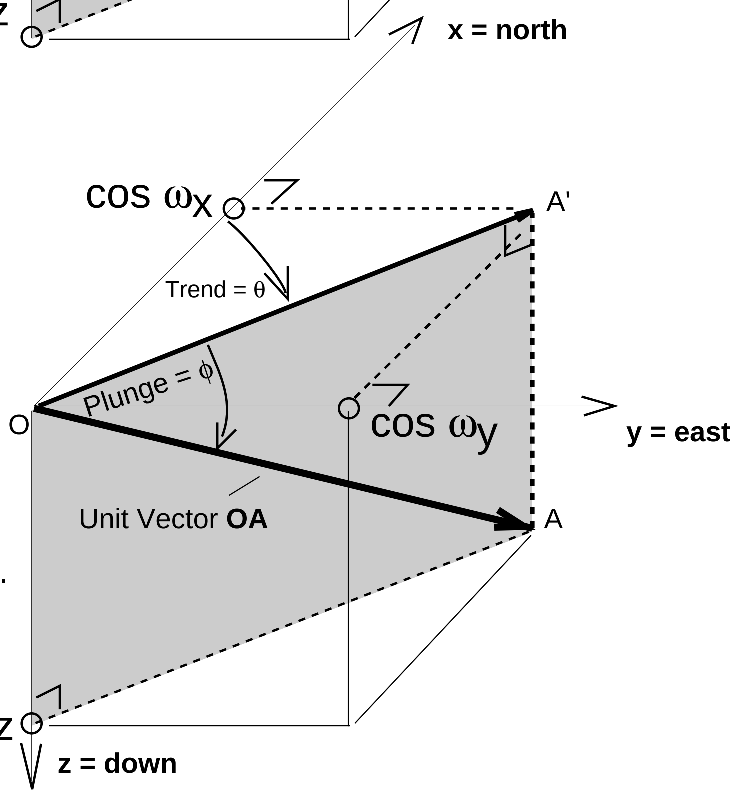
The direction cosines α and β are determined from OA', the length of OA' being |OA'| =|OA| cos φ = cos φ.

$\alpha = \cos\omega_x = (\cos\phi)\,(\cos\theta)$

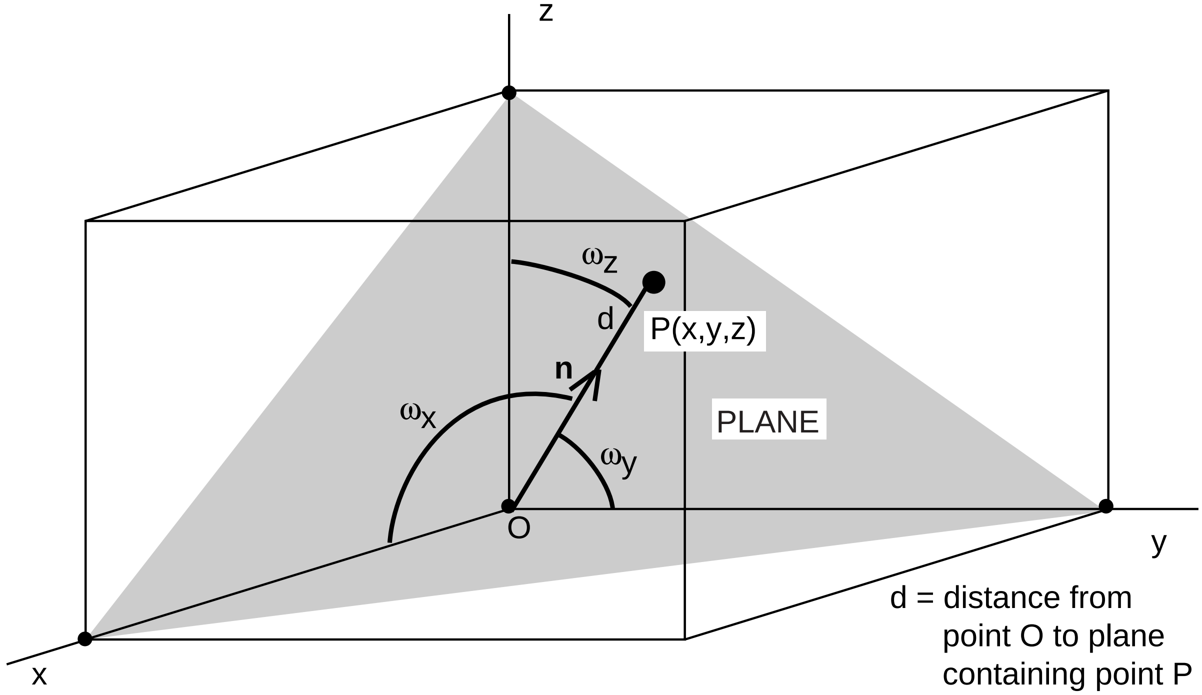
$\beta = \cos\omega_y = (\cos\phi)\,(\sin\theta)$

$\gamma = \cos\omega_z = +(\sin\phi)$

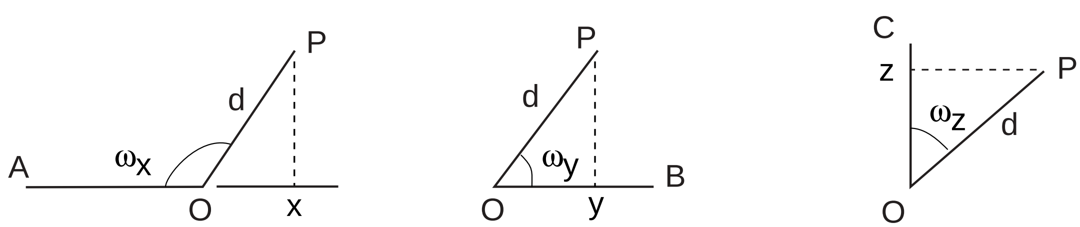
$\alpha^2 + \beta^2 + \gamma^2 = 1$



DIRECTION COSINES AND THE EQUATION OF A PLANE



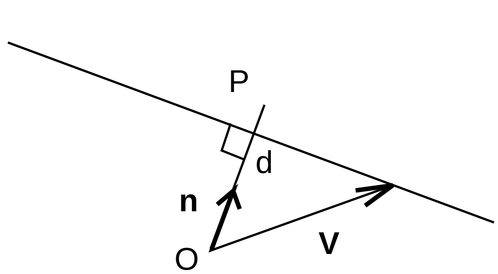
The angle between the x-axis and OP is ω_x. The angle between the y-axis and OP is ω_y. The angle between the z-axis and OP is ω_z.



$\alpha = \cos \omega_x = \dfrac{x}{d}$ $\beta = \cos \omega_y = \dfrac{y}{d}$ $\gamma = \cos \omega_z = \dfrac{z}{d}$

α, β, and γ are the direction cosines of the angles between the normal to the plane and the x-, y-, and z- axes, respectively.

If **n** is a unit vector (**|n|** = 1) normal to the plane through point P, then **n•V** = d



The distance d is positive if the normal points from the reference point to the plane.

The distance d is negative if the normal points from the plane to the reference point.