28. Folds (II)

I Main Topic: Mechanics of folds above intrusions

A Background

B G.K. Gilbert's idealization

C Superposition

D Displacements around an opening-mode crack (sill)

E Dimensional analysis of governing eq. for bending

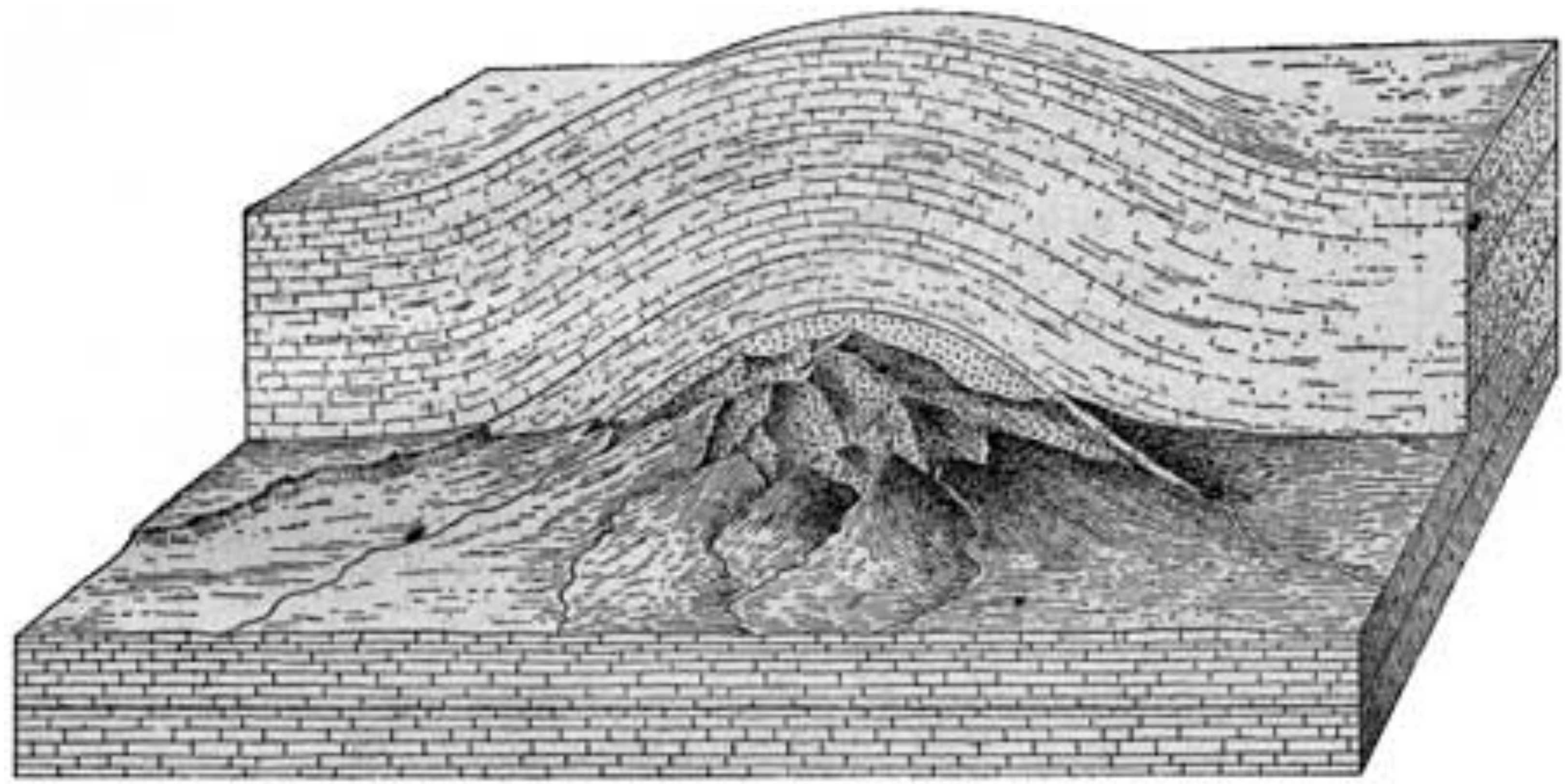
F Idealized form of folds over a laccolith

G Development of laccoliths and saucer-shaped sills

28. Folds (II)
Laccolith, Montana

http://upload.wikimedia.org/wikipedia/en/a/a6/Laccolith_Montana.jpg

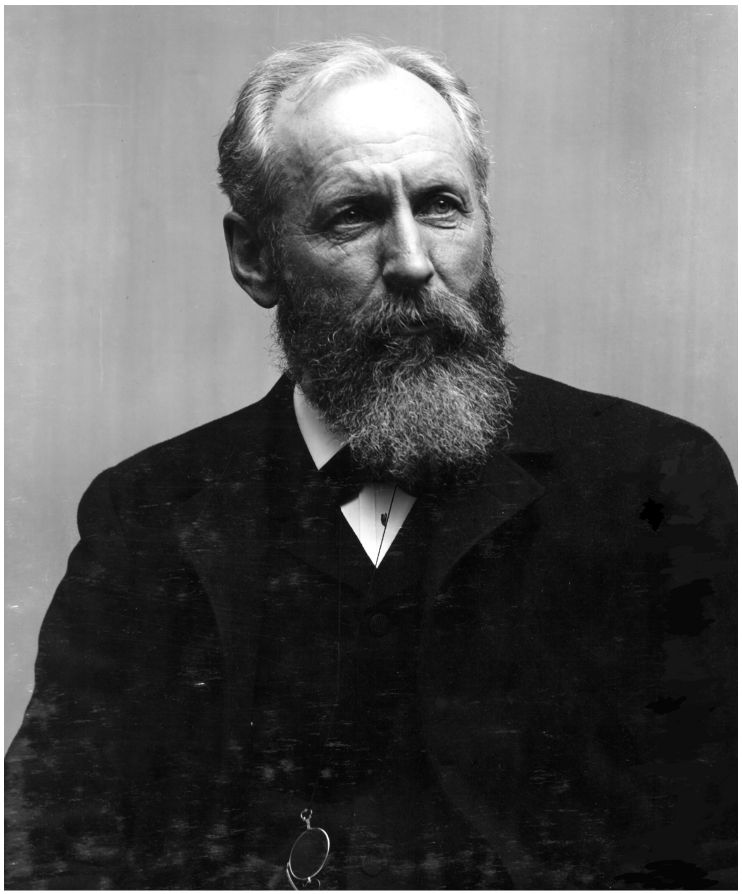
28. Folds (II)
II Mechanics of folds above intrusions



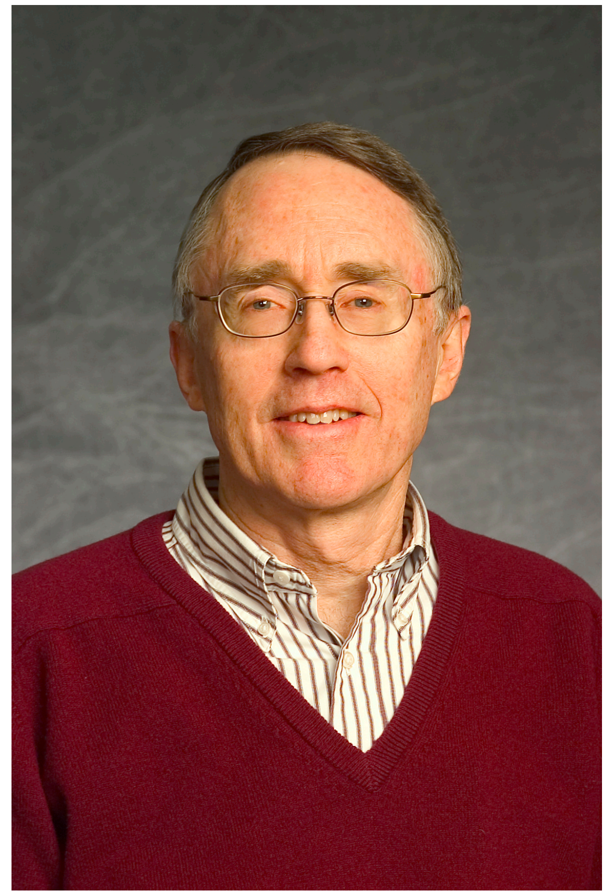
Half-stereogram of Mount Ellsworth
From Gilbert, 1877, Report on the geology of the Henry Mountains
http://www.nps.gov/history/history/online_books/geology/publications/bul/707/images/fig53.jpg

28. Folds (II)
II Mechanics of folds above intrusions

G.K. Gilbert

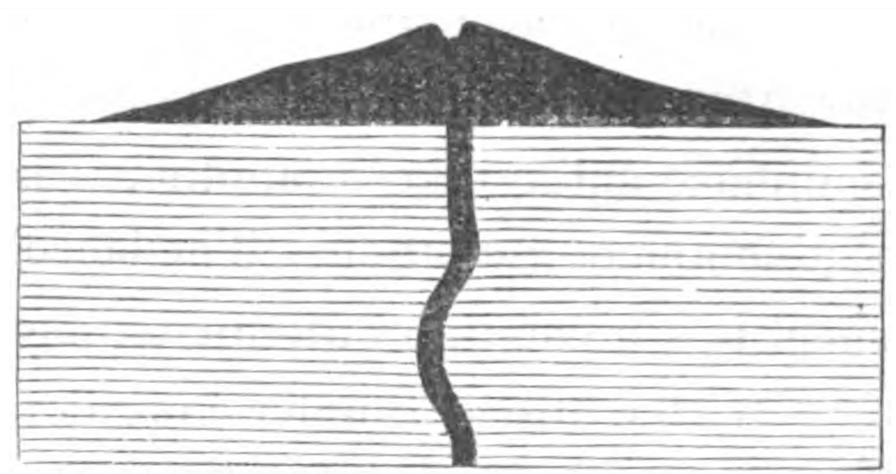


David Pollard

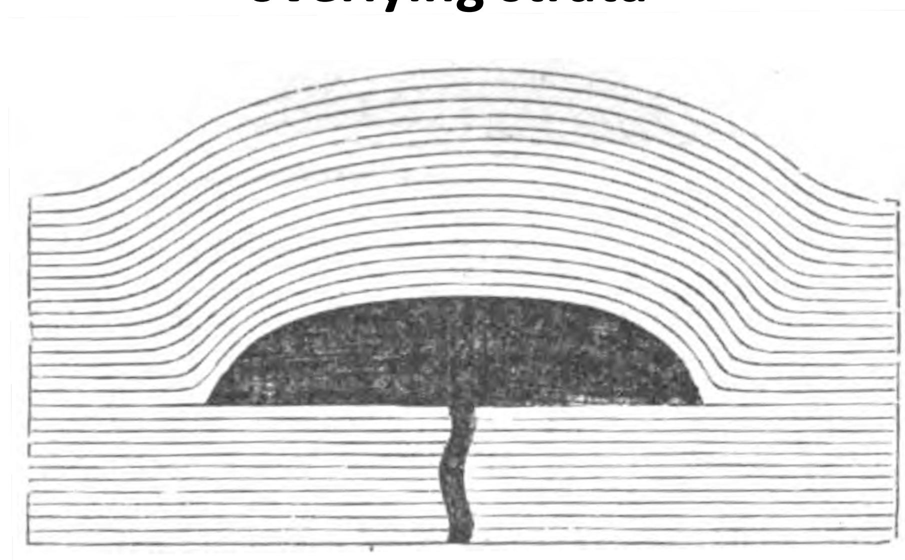


28. Folds (II)
II Mechanics of folds above intrusions (beam theory)

"Ideal Cross-section of a Mountain of Eruption"



"Ideal Cross-section of a Laccolite, showing the typical form and the arching of the overlying strata"

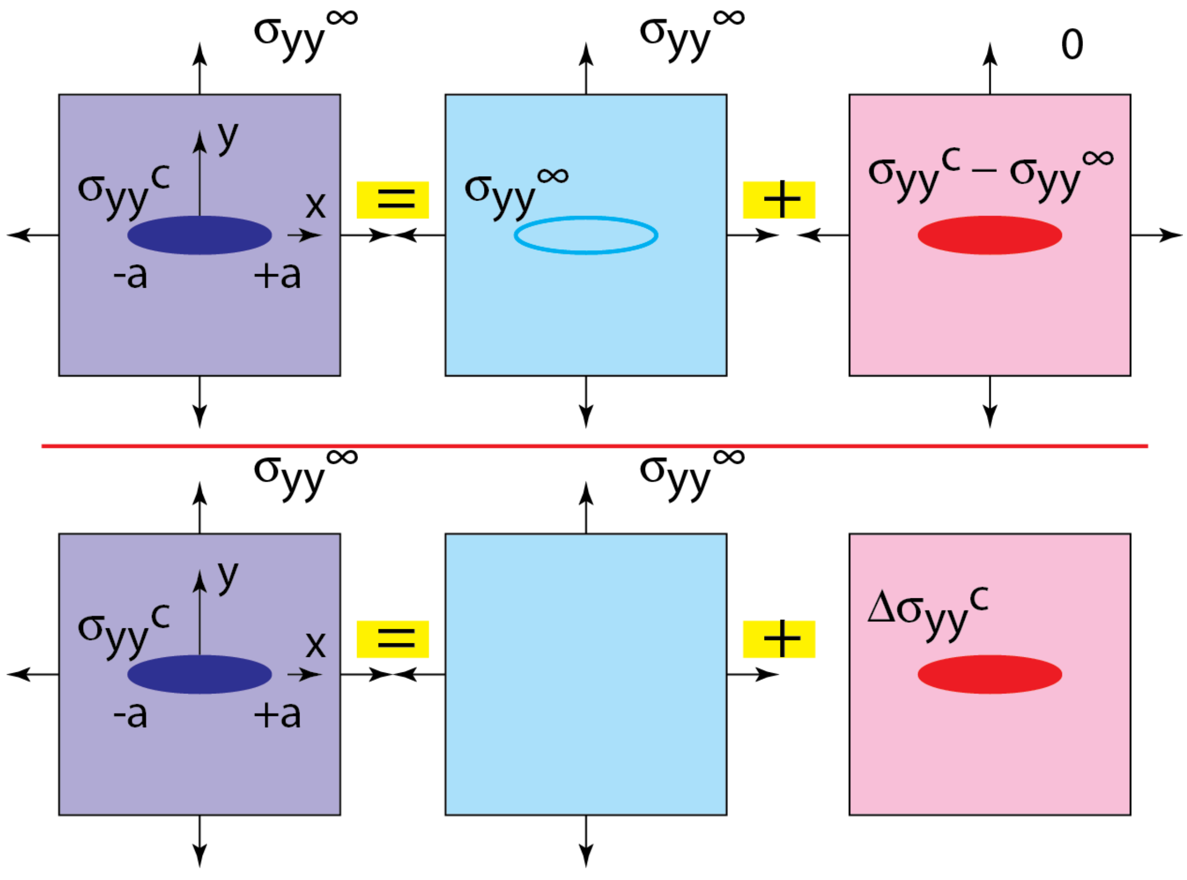


Figures from Gilbert, 1887

28. Folds (II)

II Mechanics of folds above intrusions

Superposition

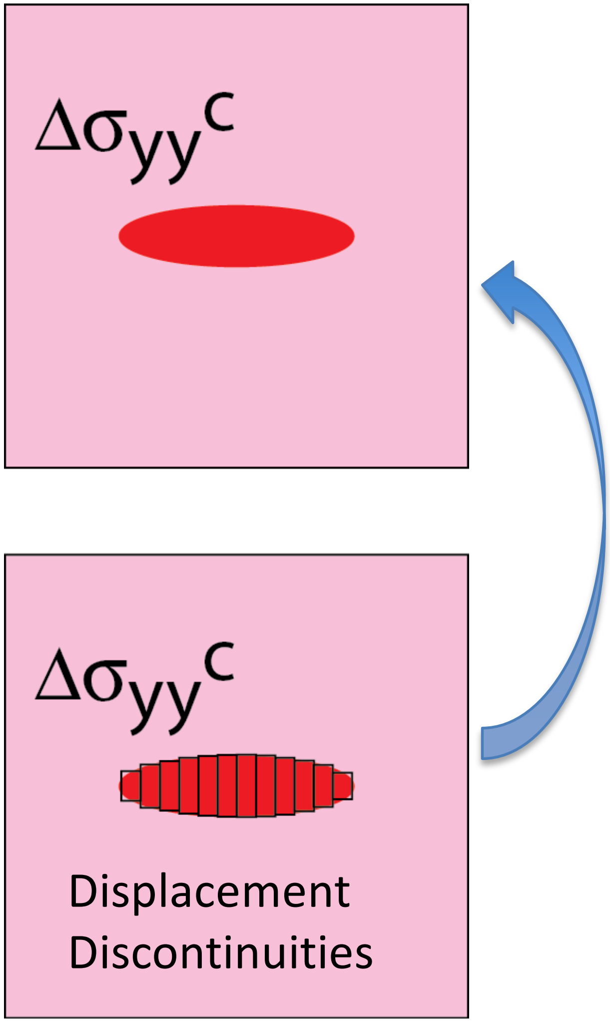


$\sigma_{ij}{}^{T} = \sigma_{ij}{}^{0} + \Delta\sigma_{ij}$

$\mathbf{u}^{T} = \mathbf{u}^{0} + \Delta\mathbf{u}$

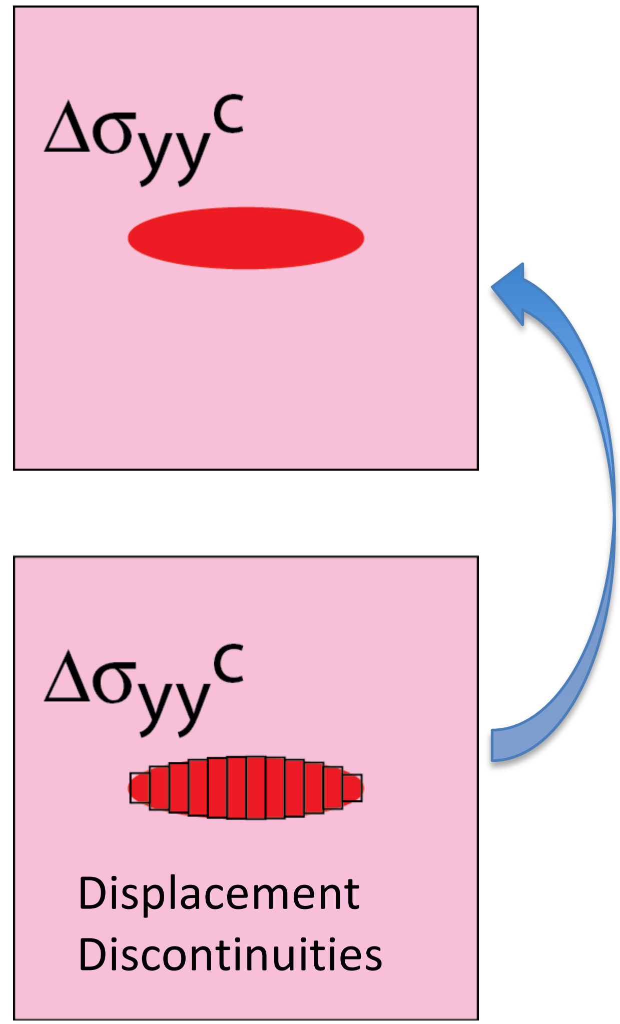
28. Folds (II)
II Mechanics of folds above intrusions
Superposition

- Opening-mode crack modeled by opening-mode displacement discontinuities (dds) of different apertures
- Openings $[X_{(i)}]$ of dds set so that sum of traction changes matches boundary condition $[B_{(j)}]$ on crack walls = $\Delta\sigma_{yy}{}^{c}$
- $[A_{(ij)}][X_{(i)}] = [B_{(j)}]$, where $A_{(ij)}$ is effect of unit opening at element i on tractions at element j



28. Folds (II)
II Mechanics of folds above intrusions
Superposition

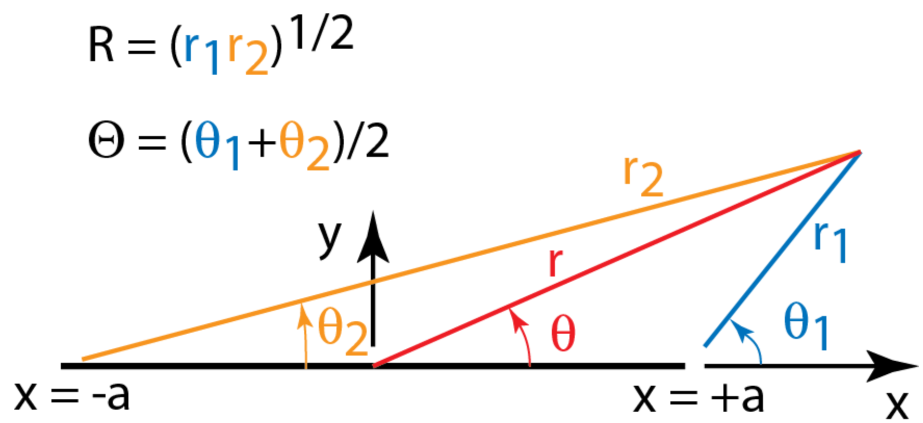
- Total stress field around crack equals sum of stress contributions of all dds:
  $\sigma^t = \Sigma\sigma_i$
- Total displacement field around crack equals sum of displacement contributions of all dds:
  $u^t = \Sigma u_i$



28. Folds (II)

II Mechanics of folds above intrusions

Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)



$$u_x = \frac{\Delta\sigma_I}{2G}\left\{(1-2\nu)(R\cos\Theta - r\cos\theta) - r\underline{\sin\theta}\left[\frac{r}{R}\sin(\theta-\Theta)\right]\right\}$$

"Driving Pressure" (over-pressure)

Note:
rsinθ = y

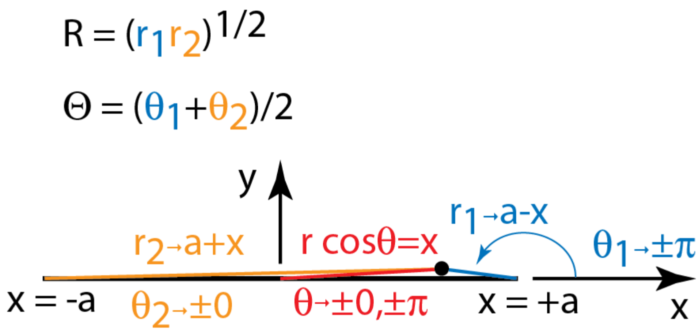
$$u_y = \frac{\Delta\sigma_I}{2G}\left\{2(1-\nu)(R\sin\Theta - r\underline{\sin\theta}) - r\underline{\sin\theta}\left[\frac{r}{R}\cos(\theta-\Theta) - 1\right]\right\}$$

Shear modulus of host rock

28. Folds (II)

II Mechanics of folds above intrusions

Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)



Now specialize to the crack walls (y = ±0).

rsinθ = y ⟶ 0, hence

$$u_x = \frac{\Delta\sigma_I}{2G}\left\{(1-2\nu)(R\cos\Theta - r\cos\theta) - r\sin\theta\left[\frac{r}{R}\sin(\theta-\Theta)\right]\right\} \rightarrow \frac{\Delta\sigma_I}{2G}\{(1-2\nu)(R\cos\Theta - r\cos\theta)\}$$

$$u_y = \frac{\Delta\sigma_I}{2G}\left\{2(1-\nu)(R\sin\Theta - r\sin\theta) - r\sin\theta\left[\frac{r}{R}\cos(\theta-\Theta) - 1\right]\right\} \rightarrow \frac{\Delta\sigma_I}{2G}\{2(1-\nu)(R\sin\Theta)\}$$

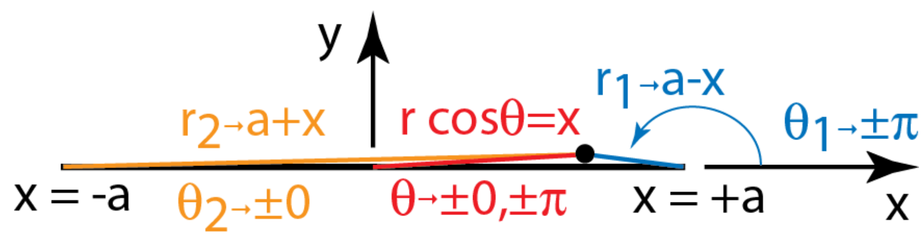
28. Folds (II)

II Mechanics of folds above intrusions

Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)

$R = (r_1 r_2)^{1/2}$

$\Theta = (\theta_1 + \theta_2)/2$



r_1 is distance from right end

r_2 is distance from left end

So the remaining key terms are:
R, cosΘ, sinΘ, and rcosθ.
<u>Along the crack</u>,
these terms are simple:

$$R = [(a - x)(a + x)]^{1/2} = \sqrt{a^2 - x^2}$$

$$\Theta = \frac{\theta_1}{2} = \frac{\pm\pi}{2}, \text{ so } \cos\Theta = 0, \; \sin\Theta = \pm 1$$

$$r\cos\theta = x$$

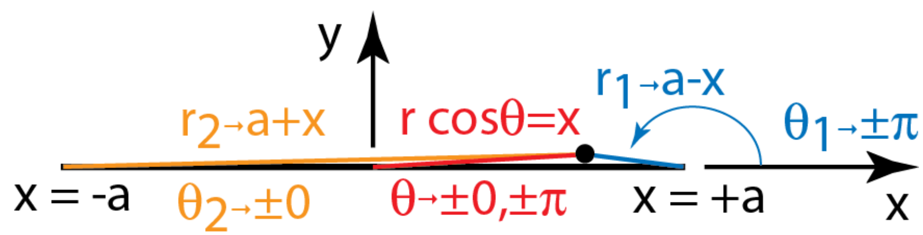
28. Folds (II)

II Mechanics of folds above intrusions

Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)

$R = (r_1 r_2)^{1/2}$

$\Theta = (\theta_1 + \theta_2)/2$



Along the crack, $R=[(a-x)(a+x)]^{1/2}$, and $r\cos\theta = x$

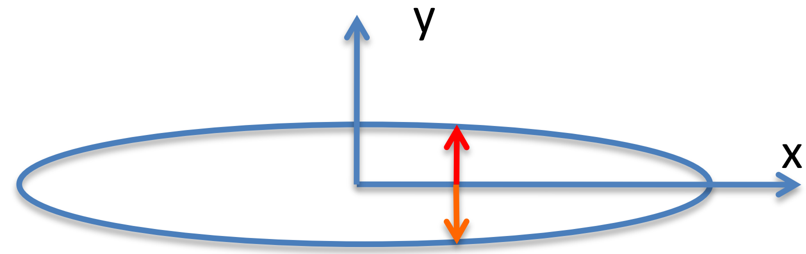
$$u^c_x\left(|x| \le a\right) = \frac{\Delta\sigma_I}{2G}\left\{(1-2\nu)\left(R\cos\Theta - r\cos\theta\right)\right\} \rightarrow u^c_x = \frac{\Delta\sigma_I}{2G}\left\{(1-2\nu)(-x)\right\}$$

$$u^c_y\left(|x| \le a\right) = \frac{\Delta\sigma_I}{2G}\left\{2(1-\nu)\left(R\sin\Theta\right)\right\} \rightarrow u^c_y = \frac{\Delta\sigma_I}{2G}\left\{2(1-\nu)\left(\pm\sqrt{a^2 - x^2}\right)\right\}$$

28. Folds (II)

II Mechanics of folds above intrusions

Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)



Now consider the displacements normal to the crack:

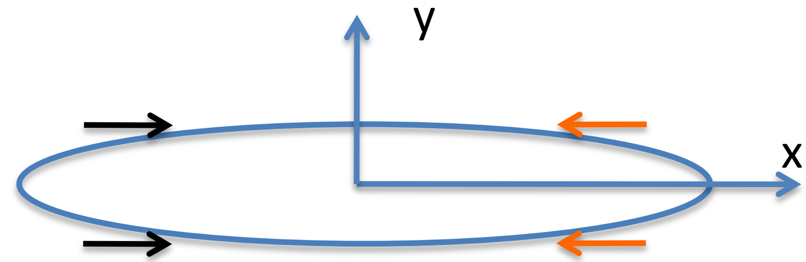
$$u_y^{\,c} = \frac{\pm\Delta\sigma_I}{2G}\left\{2(1-\nu)\left(\sqrt{a^2-x^2}\right)\right\} \rightarrow u^c_{\;y(\max)}(x=0) = \frac{+\Delta\sigma_I}{2G}\left\{2(1-\nu)a\right\}$$

$$\frac{u_y^{\,c}}{u^c_{\;y(\max)}} = \frac{\pm\left(\sqrt{a^2-x^2}\right)}{a} = \pm\left(\sqrt{1-\left(\frac{x}{a}\right)^2}\right)$$

28. Folds (II)

II Mechanics of folds above intrusions

Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)



Now consider the displacements parallel to the crack:

$$u^c{}_x\left(|x|\le a\right)=\frac{\Delta\sigma_I}{2G}\left\{(1-2\nu)(-x)\right\},\ and\ u^c{}_{y(\max)}\left(x=0\right)=\frac{+\Delta\sigma_I}{2G}\left\{2(1-\nu)a\right\}$$

$$\frac{u_x{}^c}{u^c{}_{y(\max)}}=\frac{(1-2\nu)(-x)}{2(1-\nu)a}$$

$$For\ \ \nu=0.25,\ \frac{u_x{}^c}{u^c{}_{y(\max)}}=\frac{(1/2)(-x)}{2(3/4)a}=\frac{(1/2)(-x)}{(3/2)a}=\frac{-1}{3}\frac{x}{a}$$

28. Folds (II)

II Mechanics of folds above intrusions

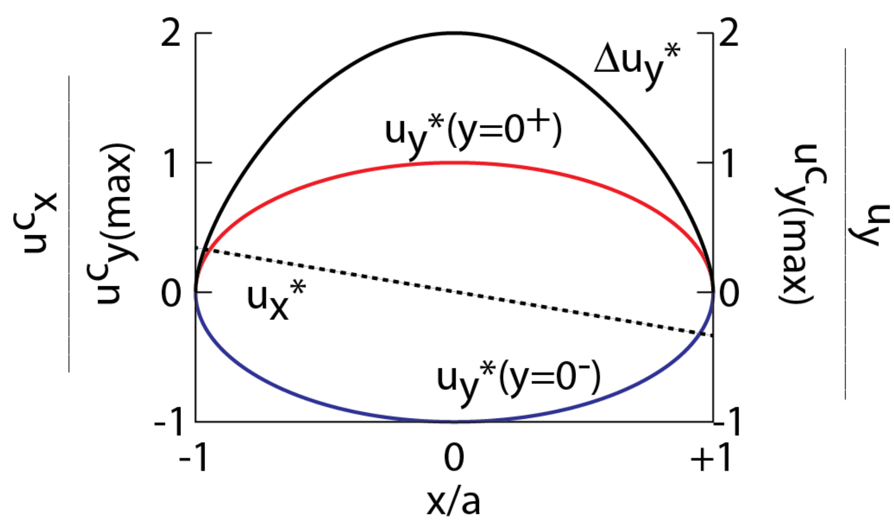
Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)

Along the crack (|x/a| ≤ 1)

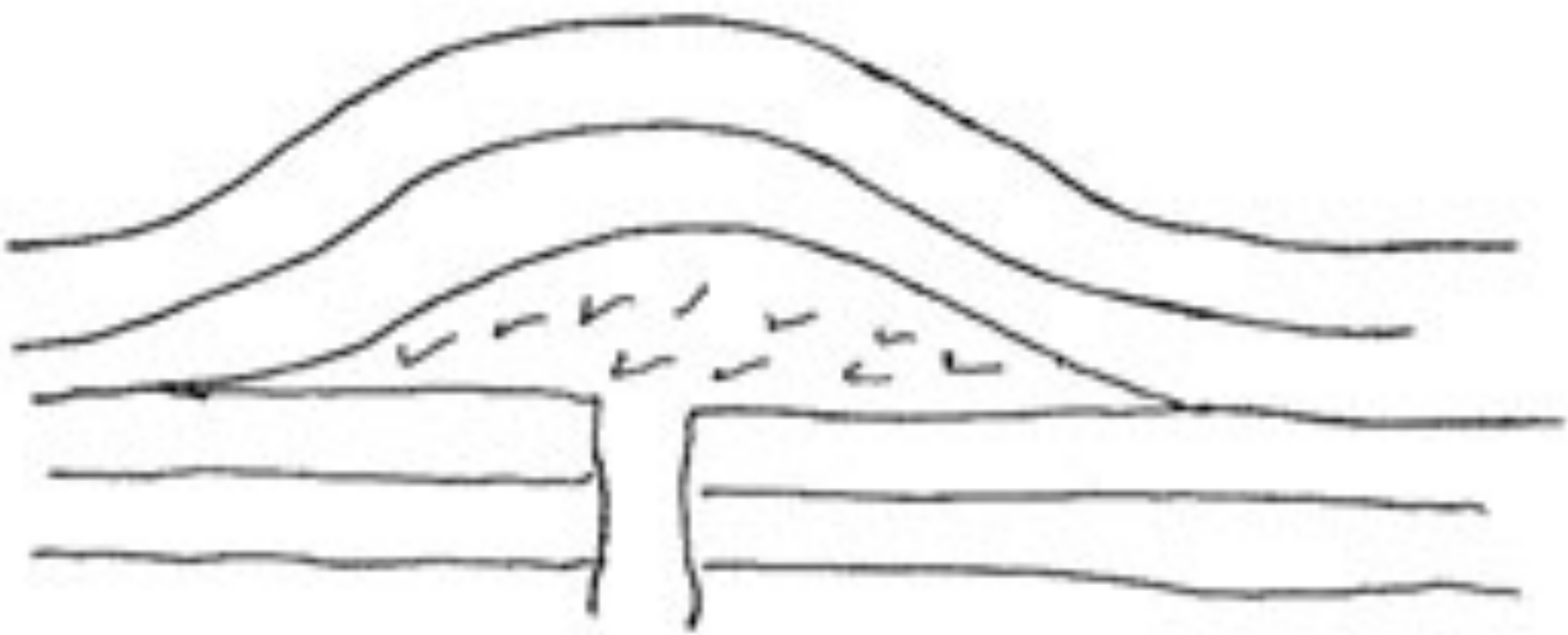
$$\frac{u_y{}^c}{u^c{}_{y(\max)}} = \frac{\pm\left(\sqrt{a^2 - x^2}\right)}{a} = \pm\left(\sqrt{1-\left(\frac{x}{a}\right)^2}\right)$$

$$For\ \ \nu = 0.25,\ \frac{u_x{}^c}{u^c{}_{y(\max)}} = \frac{(1/2)(-x)}{2(3/4)a} = \frac{(1/2)(-x)}{(3/2)a} = \frac{-1}{3}\frac{x}{a}$$

Normalized Crack wall displacements
(ν = 0.25)



28. Folds (II)
II Mechanics of folds above intrusions
Sketch from field notes of Gilbert



http://pangea.stanford.edu/~annegger/images/colorado%20plateau/laccolith_sketch.jpg

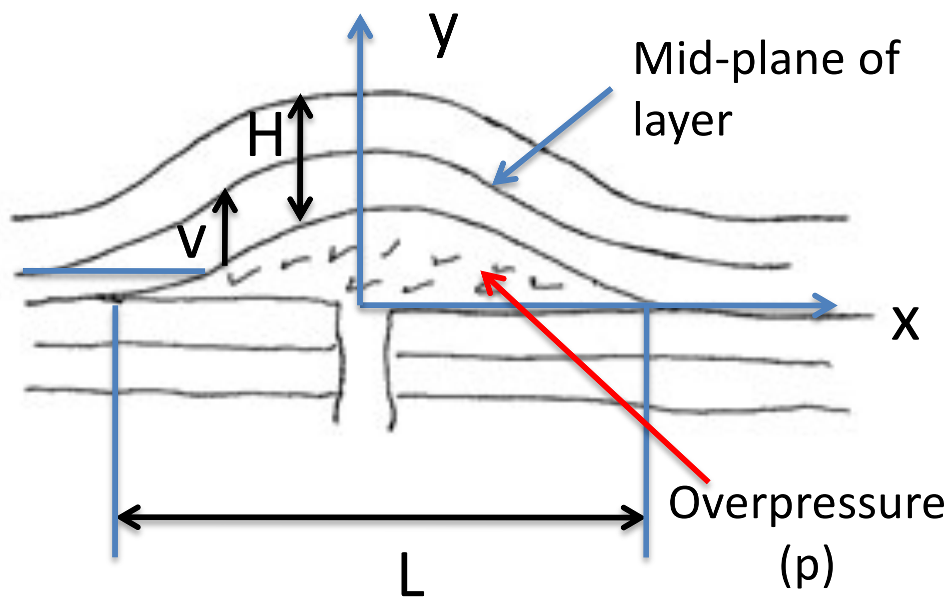
28. Folds (II)

II Mechanics of folds above intrusions

Dimensional analysis of terms in ***governing equation*** for bending of an elastic layer (from Pollard and Fletcher, 2005)

$$\frac{d^4 v}{dx^4} = \frac{12p}{BH^3}$$

v = vertical deflection of mid-plane {Length}
x = horizontal distance {Length}
L = length of flexed part of layer {Length}
p = overpressure {Force/area}
B = stiffness {Force/area}
H = thickness of layer {Length}
Dimensions check



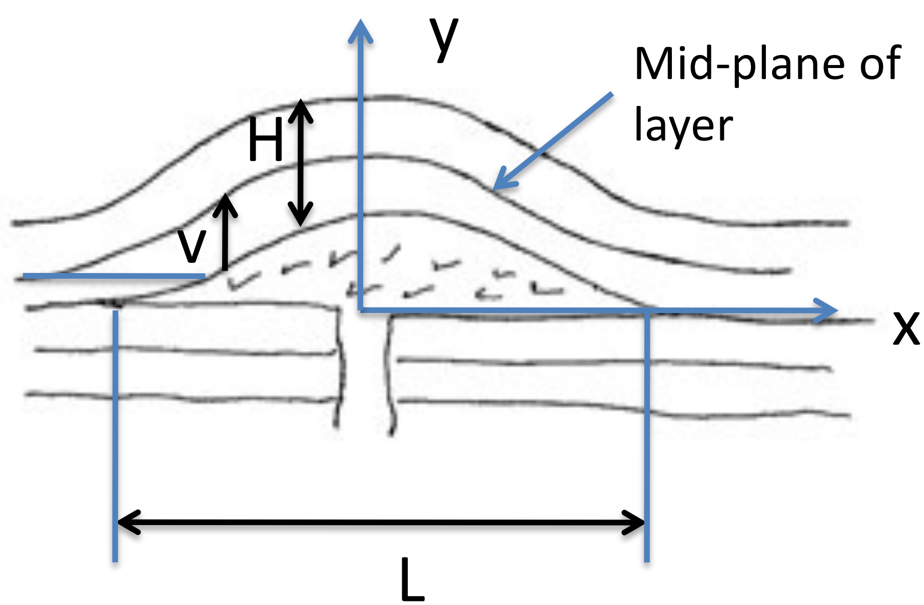
28. Folds (II)

II Mechanics of folds above intrusions

Dimensional analysis of terms in governing equation for bending of an elastic layer (from Pollard and Fletcher, 2005)

Find constant length scales and non-dimensionalize

$$x^* = \frac{x}{L}, v^* = \frac{v}{v_{\max}}$$



28. Folds (II)

II Mechanics of folds above intrusions

Dimensional analysis … (cont.)

Now non-dimensionalize the differential operator

$$x^* = \frac{1}{L}x \rightarrow \frac{dx^*}{dx} = \frac{1}{L}$$

$$\frac{d}{dx} = \frac{d}{dx^*}\frac{dx^*}{dx} = \frac{d}{dx^*}\frac{1}{L}$$

$$\frac{d^2}{dx^2} = \left(\frac{d}{dx}\right)\left(\frac{d}{dx}\right) = \left(\frac{d}{dx^*}\frac{1}{L}\right)\left(\frac{d}{dx^*}\frac{1}{L}\right) = \left(\frac{1}{L}\right)^2\left(\frac{d^2}{dx^{*2}}\right)$$

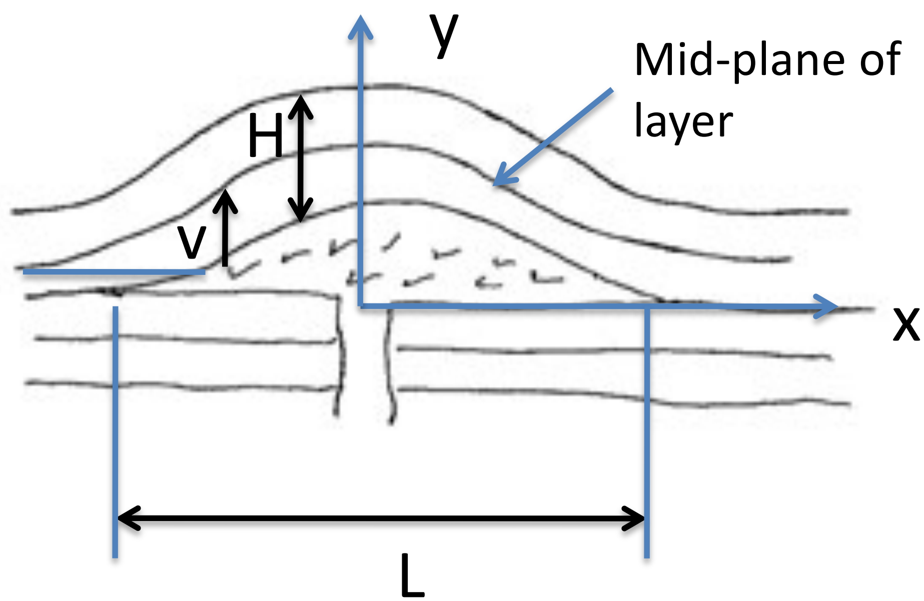
$$\frac{d^3}{dx^3} = \left(\frac{d}{dx}\right)\left(\frac{d^2}{dx^2}\right) = \left(\frac{d}{dx^*}\frac{1}{L}\right)\left(\left(\frac{1}{L}\right)^2\left(\frac{d^2}{dx^{*2}}\right)\right) = \left(\frac{1}{L}\right)^3\left(\frac{d^3}{dx^{*3}}\right)$$

$$\frac{d^4}{dx^4} = \left(\frac{d}{dx}\right)\left(\frac{d^3}{dx^3}\right) = \left(\frac{d}{dx^*}\frac{1}{L}\right)\left(\left(\frac{1}{L}\right)^3\left(\frac{d^3}{dx^{*3}}\right)\right) = \left(\frac{1}{L}\right)^4\left(\frac{d^4}{dx^{*4}}\right), etc.$$

28. Folds (II)

II Mechanics of folds above intrusions

Dimensional analysis of terms in governing equation for bending of an elastic layer (from Pollard and Fletcher, 2005)



Substitute into governing eq.

$$\frac{d^4(v)}{dx^4} = \frac{12p}{BH^3}$$

$$v = v^* v_{max} \qquad \frac{d^4}{dx^4} = \frac{d^4}{dx^{*4}}\frac{1}{L^4}$$

$$\frac{d^4(v)}{dx^4} = \frac{1}{L^4}\frac{d^4}{dx^{*4}}(v^* v_{max}) = \frac{1}{L^4}\frac{(v_{max})d^4(v^*)}{dx^{*4}} = \frac{12p}{BH^3}$$

28. Folds (II)

II Mechanics of folds above intrusions

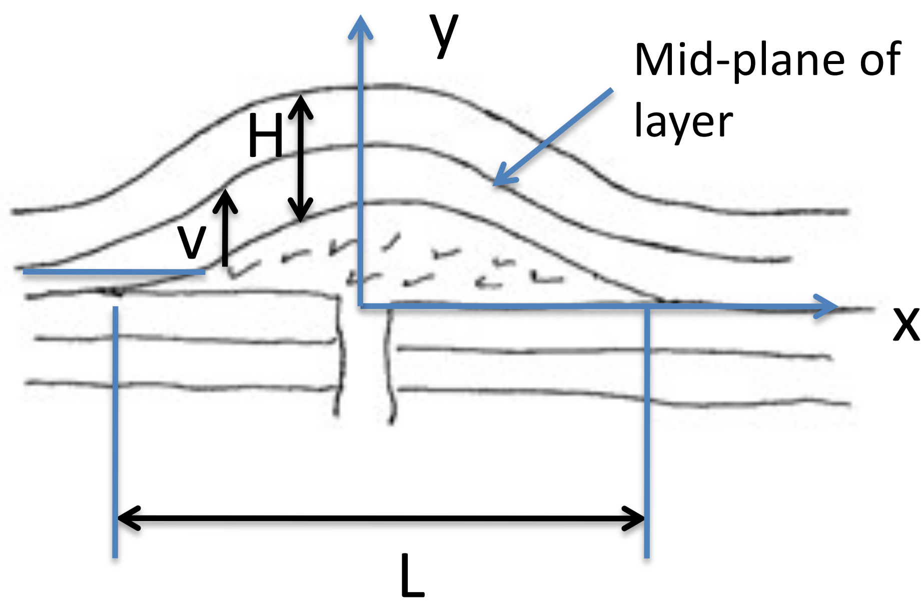
Dimensional analysis of terms in governing equation for bending of an elastic layer (from Pollard and Fletcher, 2005)

$$\frac{d^4(v)}{dx^4} = \frac{12p}{BH^3}$$

$$\frac{d^4(v)}{dx^4} = \frac{1}{L^4}\frac{(v_{max})d^4(v^*)}{dx^{*4}} = \frac{12p}{BH^3}$$

$$\frac{d^4(v^*)}{dx^{*4}} = \frac{12p}{B}\frac{L^4}{v_{max}H^3}$$

Setting up the problem in dimensionless form gives insight into its solution



Right side contains only constants

v* ~ L^4

v* ~ $1/H^3$

Long thin layers will deflect much more than short thick layers

28. Folds (II)

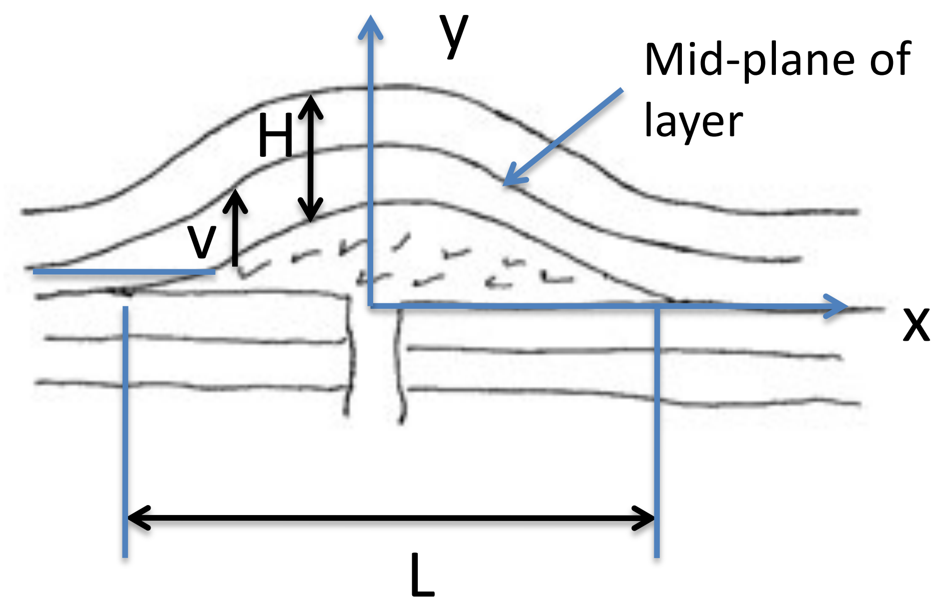
II Mechanics of folds above intrusions

Theoretical form of solution

$$\frac{d^4v}{dx^4} = \frac{12p}{BH^3} = C_4$$

$$\frac{d^3v}{dx^3} = C_4x + C_3$$

$$\frac{d^2v}{dx^2} = \frac{C_4}{2}x^2 + C_3x + C_2$$



$$\frac{dv}{dx} = \frac{C_4}{6}x^3 + \frac{C_3}{2}x^2 + C_2x + C_1 \longrightarrow v = \frac{C_4}{24}x^4 + \frac{C_3}{6}x^3 + \frac{C_2}{2}x^2 + C_1x + C_0$$

$v(x=0) = C_0$

The function v is even: v(-x) = v(x)

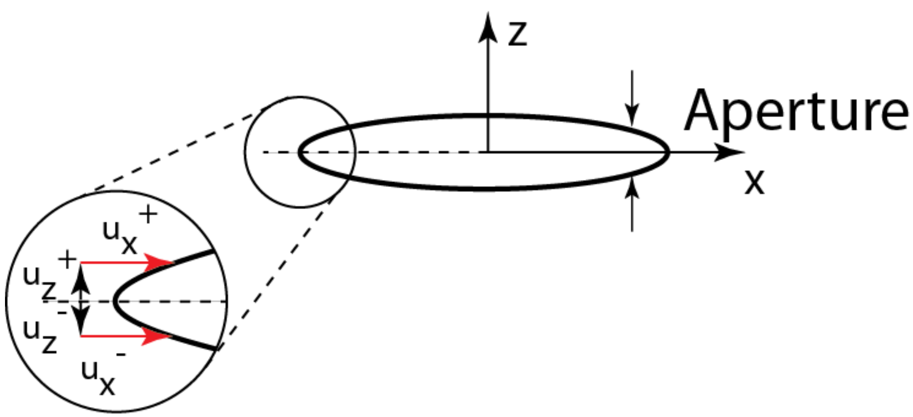
By symmetry, the odd coefficients (C_3 and C_1) must equal zero

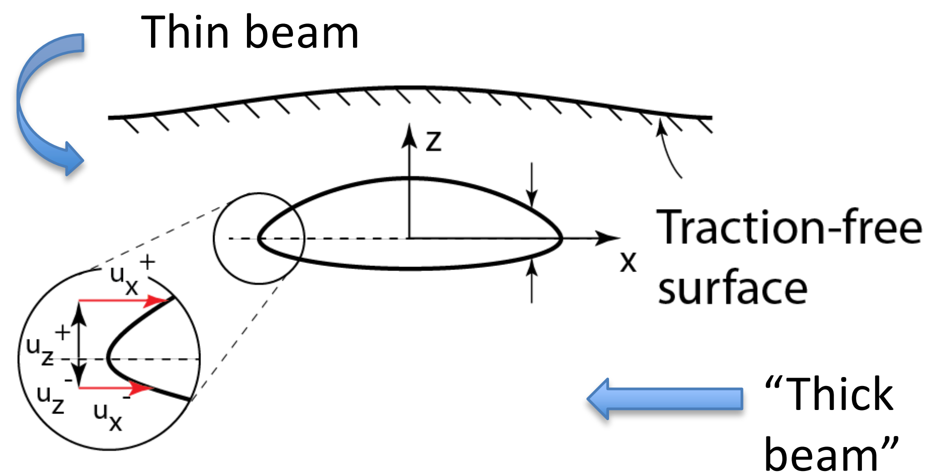
C_2 and C_4 are set so that v = 0 at x = ±L/2 and v' = 0 at x = ±L/2

28. Folds (II)
II Mechanics of folds above intrusions

Symmetric Pressurized Crack in an Infinite Body

Asymmetric Pressurized Crack Parallel to a Surface

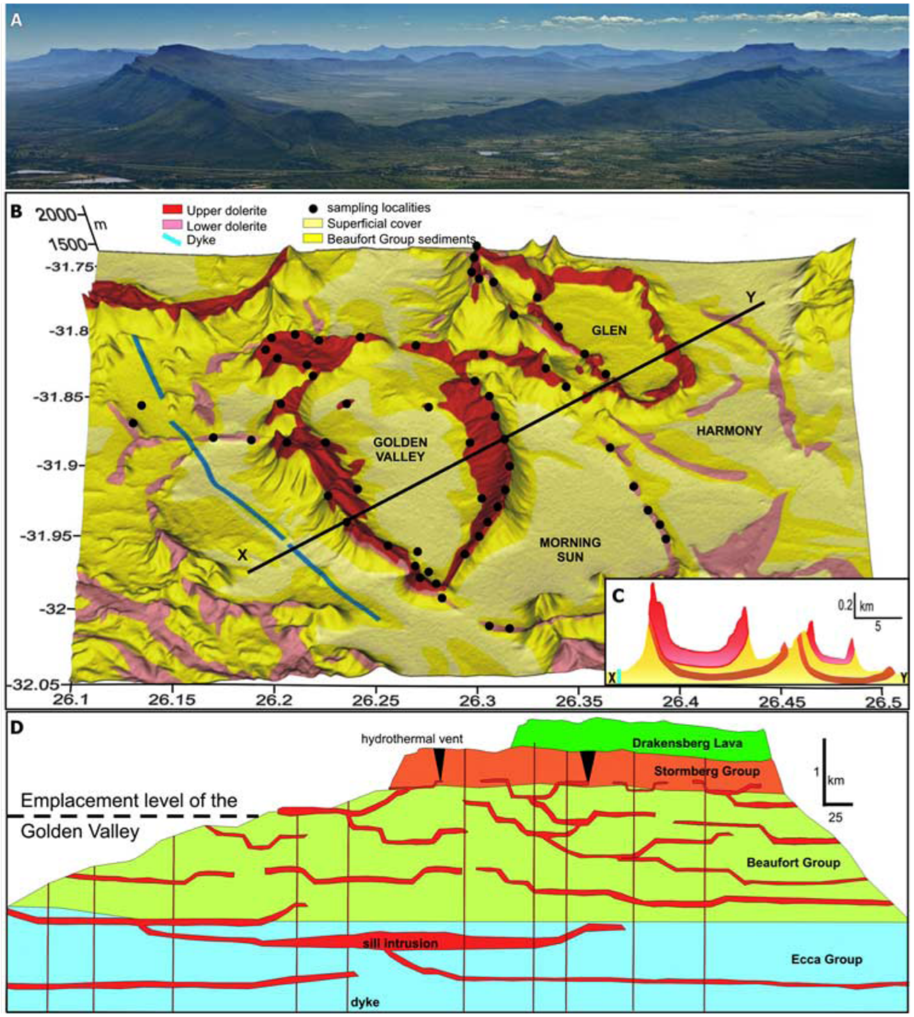




Bending of layer over laccolith should cause shearing at laccolith perimeter. This suggests laccoliths should propagate up towards the surface as they grow.

28. Folds (II)

II Mechanics of folds above intrusions



The Golden Valley Sill, South Africa – a saucer-shaped sill
From Polteau et al., 2008

28. Folds (II)

II Mechanics of folds above intrusions