

## 28. Folds (II)

- I Main Topic: Mechanics of folds above intrusions
  - A Background
  - B G.K. Gilbert's idealization
  - C Superposition
  - D Displacements around an opening-mode crack (sill)
  - E Dimensional analysis of governing eq. for bending
  - F Idealized form of folds over a laccolith
  - G Development of laccoliths and saucer-shaped sills

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## 28. Folds (II) Laccolith, Montana



[http://upload.wikimedia.org/wikipedia/en/a/a6/Laccolith\\_Montana.jpg](http://upload.wikimedia.org/wikipedia/en/a/a6/Laccolith_Montana.jpg)

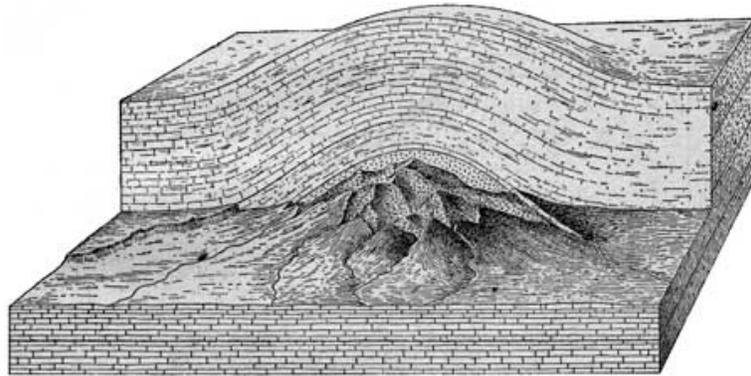
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## 28. Folds (II)

### II Mechanics of folds above intrusions



Half-stereogram of Mount Ellsworth

From Gilbert, 1877, Report on the geology of the Henry Mountains  
[http://www.nps.gov/history/history/online\\_books/geology/publications/bul/707/images/fig53.jpg](http://www.nps.gov/history/history/online_books/geology/publications/bul/707/images/fig53.jpg)

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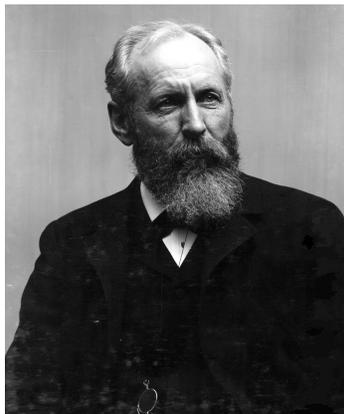
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## 28. Folds (II)

### II Mechanics of folds above intrusions

**G.K. Gilbert**



**David Pollard**



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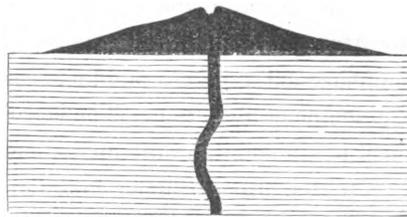
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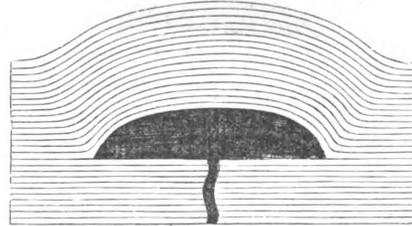
## 28. Folds (II)

### II Mechanics of folds above intrusions (beam theory)

“Ideal Cross-section of a Mountain of Eruption”



“Ideal Cross-section of a Laccolite, showing the typical form and the arching of the overlying strata”



Figures from Gilbert, 1887

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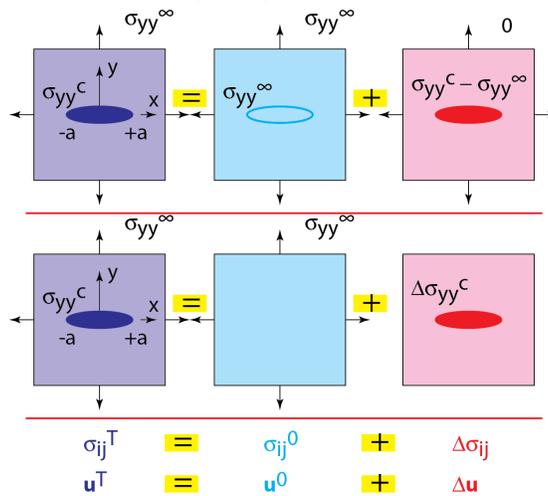
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## 28. Folds (II)

### II Mechanics of folds above intrusions

#### Superposition



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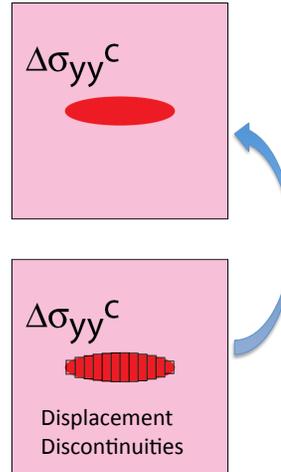
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## 28. Folds (II)

### II Mechanics of folds above intrusions

#### Superposition

- Opening-mode crack modeled by opening-mode displacement discontinuities (dds) of different apertures
- Openings  $[X_{(i)}]$  of dds set so that sum of traction changes matches boundary condition  $[B_{(j)}]$  on crack walls =  $\Delta\sigma_{yy}^c$
- $[A_{(ij)}][X_{(i)}] = [B_{(j)}]$ , where  $A_{(ij)}$  is effect of unit opening at element  $i$  on tractions at element  $j$



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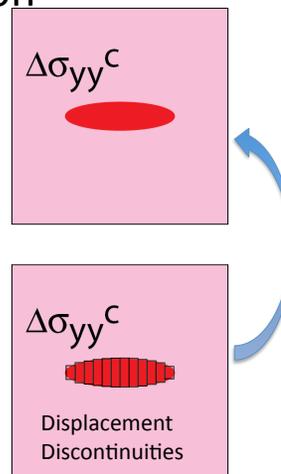
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## 28. Folds (II)

### II Mechanics of folds above intrusions

#### Superposition

- Total stress field around crack equals sum of stress contributions of all dds:  
 $\sigma^t = \sum \sigma_i$
- Total displacement field around crack equals sum of displacement contributions of all dds:  
 $u^t = \sum u_i$



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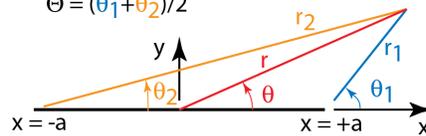
## 28. Folds (II)

### II Mechanics of folds above intrusions

Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)

$$R = (r_1 r_2)^{1/2}$$

$$\Theta = (\theta_1 + \theta_2)/2$$



$$u_x = \frac{\Delta\sigma_I}{2G} \left\{ (1-2\nu)(R\cos\Theta - r\cos\theta) - r\sin\theta \left[ \frac{r}{R} \sin(\theta - \Theta) \right] \right\}$$

“Driving Pressure” (over-pressure)

Note:  
 $r\sin\theta = y$

$$u_y = \frac{\Delta\sigma_I}{2G} \left\{ 2(1-\nu)(R\sin\Theta - r\sin\theta) - r\sin\theta \left[ \frac{r}{R} \cos(\theta - \Theta) - 1 \right] \right\}$$

Shear modulus of host rock

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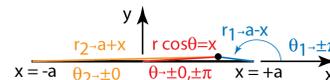
## 28. Folds (II)

### II Mechanics of folds above intrusions

Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)

$$R = (r_1 r_2)^{1/2}$$

$$\Theta = (\theta_1 + \theta_2)/2$$



Now specialize to the crack walls ( $y = \pm 0$ ).

$r\sin\theta = y \rightarrow 0$ , hence

$$u_x = \frac{\Delta\sigma_I}{2G} \left\{ (1-2\nu)(R\cos\Theta - r\cos\theta) - r\sin\theta \left[ \frac{r}{R} \sin(\theta - \Theta) \right] \right\} \rightarrow \frac{\Delta\sigma_I}{2G} \left\{ (1-2\nu)(R\cos\Theta - r\cos\theta) \right\}$$

$$u_y = \frac{\Delta\sigma_I}{2G} \left\{ 2(1-\nu)(R\sin\Theta - r\sin\theta) - r\sin\theta \left[ \frac{r}{R} \cos(\theta - \Theta) - 1 \right] \right\} \rightarrow \frac{\Delta\sigma_I}{2G} \left\{ 2(1-\nu)(R\sin\Theta) \right\}$$

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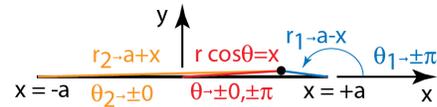
## 28. Folds (II)

### II Mechanics of folds above intrusions

Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)

$$R = (r_1 r_2)^{1/2}$$

$$\Theta = (\theta_1 + \theta_2)/2$$



So the remaining key terms are:  
 $R$ ,  $\cos\Theta$ ,  $\sin\Theta$ , and  $r\cos\theta$ .  
**Along the crack**,  
 these terms are simple:

$r_1$  is distance from right end  
 $r_2$  is distance from left end

$$R = [(a-x)(a+x)]^{1/2} = \sqrt{a^2 - x^2}$$

$$\Theta = \frac{\theta_1}{2} = \frac{\pm\pi}{2}, \text{ so } \cos\Theta = 0, \sin\Theta = \pm 1$$

$$r\cos\theta = x$$

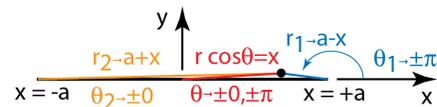
## 28. Folds (II)

### II Mechanics of folds above intrusions

Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)

$$R = (r_1 r_2)^{1/2}$$

$$\Theta = (\theta_1 + \theta_2)/2$$



Along the crack,  $R = [(a-x)(a+x)]^{1/2}$ , and  $r\cos\theta = x$

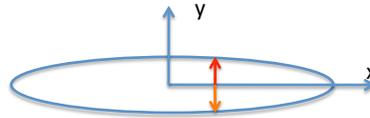
$$u_x^c(|x| \leq a) = \frac{\Delta\sigma_I}{2G} \{(1-2\nu)(R\cos\Theta - r\cos\theta)\} \rightarrow u_x^c = \frac{\Delta\sigma_I}{2G} \{(1-2\nu)(-x)\}$$

$$u_y^c(|x| \leq a) = \frac{\Delta\sigma_I}{2G} \{2(1-\nu)(R\sin\Theta)\} \rightarrow u_y^c = \frac{\Delta\sigma_I}{2G} \{2(1-\nu)(\pm\sqrt{a^2 - x^2})\}$$

## 28. Folds (II)

### II Mechanics of folds above intrusions

Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)



Now consider the displacements normal to the crack:

$$u_y^c = \frac{\pm \Delta \sigma_I}{2G} \left\{ 2(1-\nu) \left( \sqrt{a^2 - x^2} \right) \right\} \rightarrow u_{y(\max)}^c (x=0) = \frac{+\Delta \sigma_I}{2G} \{ 2(1-\nu)a \}$$

$$\frac{u_y^c}{u_{y(\max)}^c} = \frac{\pm \left( \sqrt{a^2 - x^2} \right)}{a} = \pm \left( \sqrt{1 - \left( \frac{x}{a} \right)^2} \right)$$

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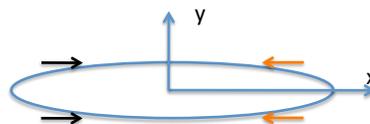
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## 28. Folds (II)

### II Mechanics of folds above intrusions

Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)



Now consider the displacements parallel to the crack:

$$u_x^c (|x| \leq a) = \frac{\Delta \sigma_I}{2G} \{ (1-2\nu)(-x) \}, \text{ and } u_{y(\max)}^c (x=0) = \frac{+\Delta \sigma_I}{2G} \{ 2(1-\nu)a \}$$

$$\frac{u_x^c}{u_{y(\max)}^c} = \frac{(1-2\nu)(-x)}{2(1-\nu)a}$$

$$\text{For } \nu = 0.25, \frac{u_x^c}{u_{y(\max)}^c} = \frac{(1/2)(-x)}{2(3/4)a} = \frac{(1/2)(-x)}{(3/2)a} = \frac{-1}{3} \frac{x}{a}$$

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## 28. Folds (II)

### II Mechanics of folds above intrusions

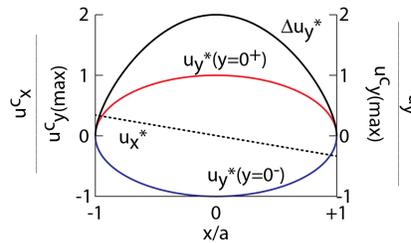
Displacements arising from opening of a mode-I crack, 2D elastic model (from Pollard and Segall, 1987)

Along the crack ( $|x/a| \leq 1$ )

$$\frac{u_y^c}{u_{y(\max)}^c} = \frac{\pm(\sqrt{a^2 - x^2})}{a} = \pm \left( \sqrt{1 - \left(\frac{x}{a}\right)^2} \right)$$

$$\text{For } \nu = 0.25, \frac{u_x^c}{u_{y(\max)}^c} = \frac{(1/2)(-x)}{2(3/4)a} = \frac{(1/2)(-x)}{(3/2)a} = \frac{-1}{3} \frac{x}{a}$$

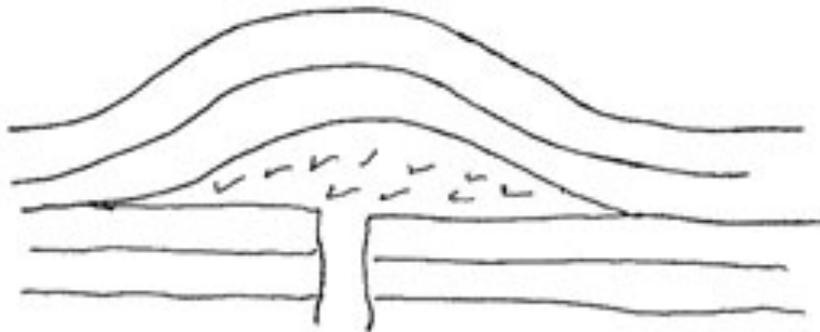
Normalized Crack wall displacements ( $\nu = 0.25$ )



## 28. Folds (II)

### II Mechanics of folds above intrusions

Sketch from field notes of Gilbert



[http://pangea.stanford.edu/~annegger/images/colorado%20plateau/laccolith\\_sketch.jpg](http://pangea.stanford.edu/~annegger/images/colorado%20plateau/laccolith_sketch.jpg)

## 28. Folds (II)

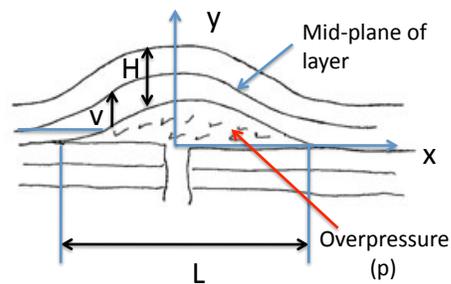
### II Mechanics of folds above intrusions

Dimensional analysis of terms in **governing equation** for bending of an elastic layer (from Pollard and Fletcher, 2005)

$$\frac{d^4 v}{dx^4} = \frac{12p}{BH^3}$$

$v$  = vertical deflection of mid-plane {Length}  
 $x$  = horizontal distance {Length}  
 $L$  = length of flexed part of layer {Length}  
 $p$  = overpressure {Force/area}  
 $B$  = stiffness {Force/area}  
 $H$  = thickness of layer {Length}

**Dimensions check**



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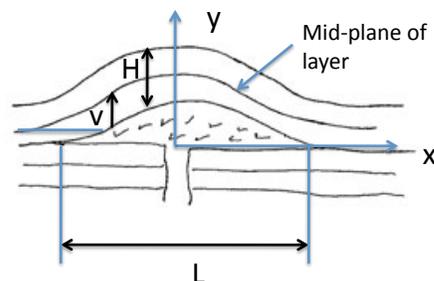
## 28. Folds (II)

### II Mechanics of folds above intrusions

Dimensional analysis of terms in governing equation for bending of an elastic layer (from Pollard and Fletcher, 2005)

Find constant length scales and non-dimensionalize

$$x^* = \frac{x}{L}, v^* = \frac{v}{v_{\max}}$$



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## 28. Folds (II)

### II Mechanics of folds above intrusions

Dimensional analysis ... (cont.)

Now non-dimensionalize the differential operator

$$x^* = \frac{1}{L}x \rightarrow \frac{dx^*}{dx} = \frac{1}{L}$$

$$\frac{d}{dx} = \frac{d}{dx^*} \frac{dx^*}{dx} = \frac{d}{dx^*} \frac{1}{L}$$

$$\frac{d^2}{dx^2} = \left(\frac{d}{dx}\right)\left(\frac{d}{dx}\right) = \left(\frac{d}{dx^*} \frac{1}{L}\right)\left(\frac{d}{dx^*} \frac{1}{L}\right) = \left(\frac{1}{L}\right)^2 \left(\frac{d^2}{dx^{*2}}\right)$$

$$\frac{d^3}{dx^3} = \left(\frac{d}{dx}\right)\left(\frac{d^2}{dx^2}\right) = \left(\frac{d}{dx^*} \frac{1}{L}\right)\left(\left(\frac{1}{L}\right)^2 \left(\frac{d^2}{dx^{*2}}\right)\right) = \left(\frac{1}{L}\right)^3 \left(\frac{d^3}{dx^{*3}}\right)$$

$$\frac{d^4}{dx^4} = \left(\frac{d}{dx}\right)\left(\frac{d^3}{dx^3}\right) = \left(\frac{d}{dx^*} \frac{1}{L}\right)\left(\left(\frac{1}{L}\right)^3 \left(\frac{d^3}{dx^{*3}}\right)\right) = \left(\frac{1}{L}\right)^4 \left(\frac{d^4}{dx^{*4}}\right), \text{etc.}$$

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## 28. Folds (II)

### II Mechanics of folds above intrusions

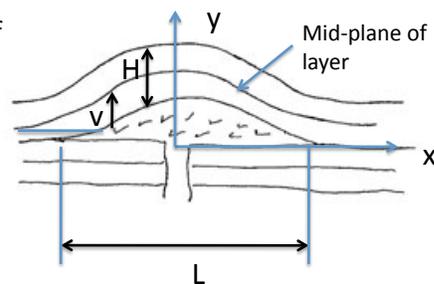
Dimensional analysis of terms in governing equation for bending of an elastic layer (from Pollard and Fletcher, 2005)

Substitute into governing eq.

$$\frac{d^4(v)}{dx^4} = \frac{12p}{BH^3}$$

$$v = v^* v_{\max} \quad \frac{d^4}{dx^4} = \frac{d^4}{dx^{*4}} \frac{1}{L^4}$$

$$\frac{d^4(v)}{dx^4} = \frac{1}{L^4} \frac{d^4}{dx^{*4}} (v^* v_{\max}) = \frac{1}{L^4} (v_{\max}) \frac{d^4(v^*)}{dx^{*4}} = \frac{12p}{BH^3}$$



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## 28. Folds (II)

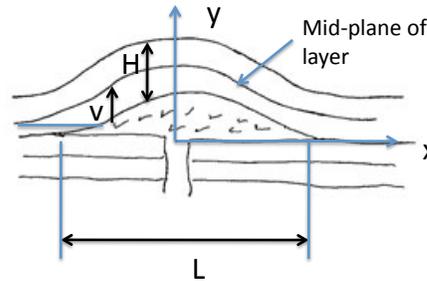
### II Mechanics of folds above intrusions

Dimensional analysis of terms in governing equation for bending of an elastic layer (from Pollard and Fletcher, 2005)

$$\frac{d^4(v)}{dx^4} = \frac{12p}{BH^3}$$

$$\frac{d^4(v)}{dx^4} = \frac{1}{L^4} (v_{\max}) \frac{d^4(v^*)}{dx^{*4}} = \frac{12p}{BH^3}$$

$$\frac{d^4(v^*)}{dx^{*4}} = \frac{12p}{B} \frac{L^4}{v_{\max} H^3}$$



Right side contains only constants  
 $v^* \sim L^4$   
 $v^* \sim 1/H^3$   
 Long thin layers will deflect much more than short thick layers

Setting up the problem in dimensionless form gives insight into its solution

## 28. Folds (II)

### II Mechanics of folds above intrusions

Theoretical form of solution

$$\frac{d^4v}{dx^4} = \frac{12p}{BH^3} = C_4$$

$$\frac{d^3v}{dx^3} = C_4x + C_3$$

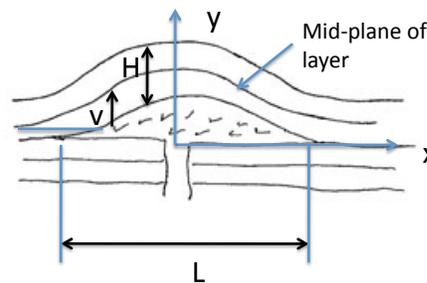
$$\frac{d^2v}{dx^2} = \frac{C_4}{2}x^2 + C_3x + C_2$$

$$\frac{dv}{dx} = \frac{C_4}{6}x^3 + \frac{C_3}{2}x^2 + C_2x + C_1 \quad \rightarrow \quad v = \frac{C_4}{24}x^4 + \frac{C_3}{6}x^3 + \frac{C_2}{2}x^2 + C_1x + C_0$$

$$v(x=0) = C_0$$

The function v is even:  $v(-x) = v(x)$

By symmetry, the odd coefficients ( $C_3$  and  $C_1$ ) must equal zero  
 $C_2$  and  $C_4$  are set so that  $v = 0$  at  $x = \pm L/2$  and  $v' = 0$  at  $x = \pm L/2$

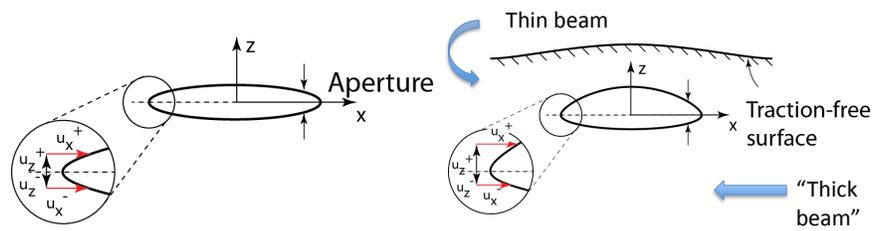


## 28. Folds (II)

### II Mechanics of folds above intrusions

**Symmetric Pressurized Crack in an Infinite Body**

**Asymmetric Pressurized Crack Parallel to a Surface**



Bending of layer over laccolith should cause shearing at laccolith perimeter. This suggests laccoliths should propagate up towards the surface as they grow.

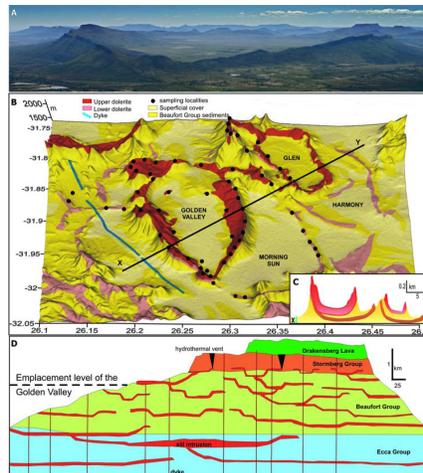
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## 28. Folds (II)

### II Mechanics of folds above intrusions



The Golden Valley Sill, South Africa – a saucer-shaped sill  
From Polteau et al., 2008

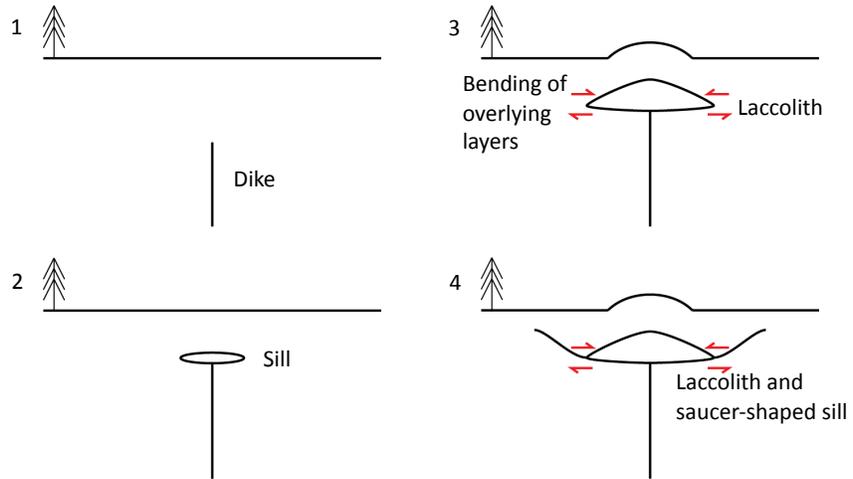
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## 28. Folds (II)

### II Mechanics of folds above intrusions



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