27. Folds (I)

I Main Topics

A What is a fold?

B Curvature of a plane curve

C Curvature of a surface

27. Folds (I)

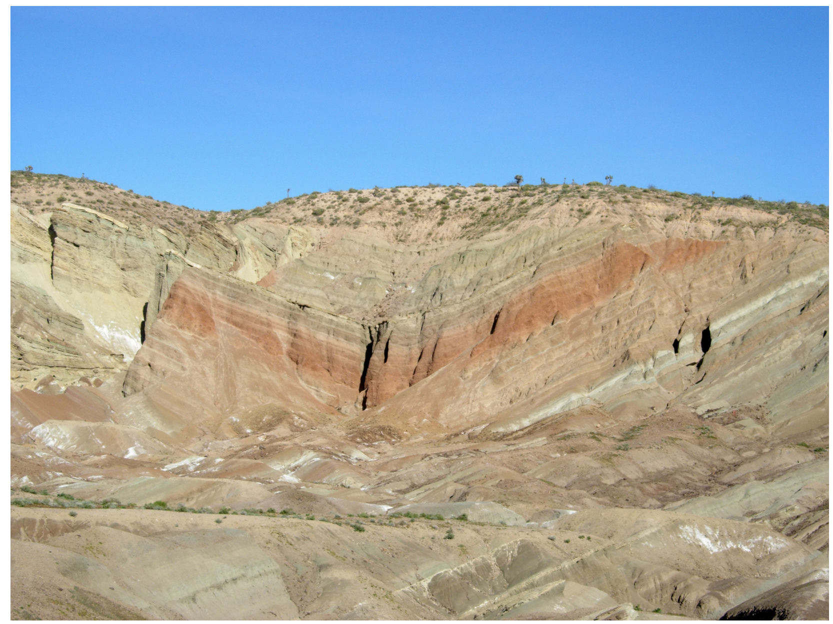
http://upload.wikimedia.org/wikipedia/commons/a/ae/Caledonian_orogeny_fold_in_King_Oscar_Fjord.jpg

27. Folds (I)

Anticline, New Jersey

http://en.wikipedia.org/wiki/File:NJ_Route_23_anticline.jpg

Syncline, Rainbow Basin, California



http://en.wikipedia.org/wiki/File:Rainbow_Basin.JPG

27. Folds (I)

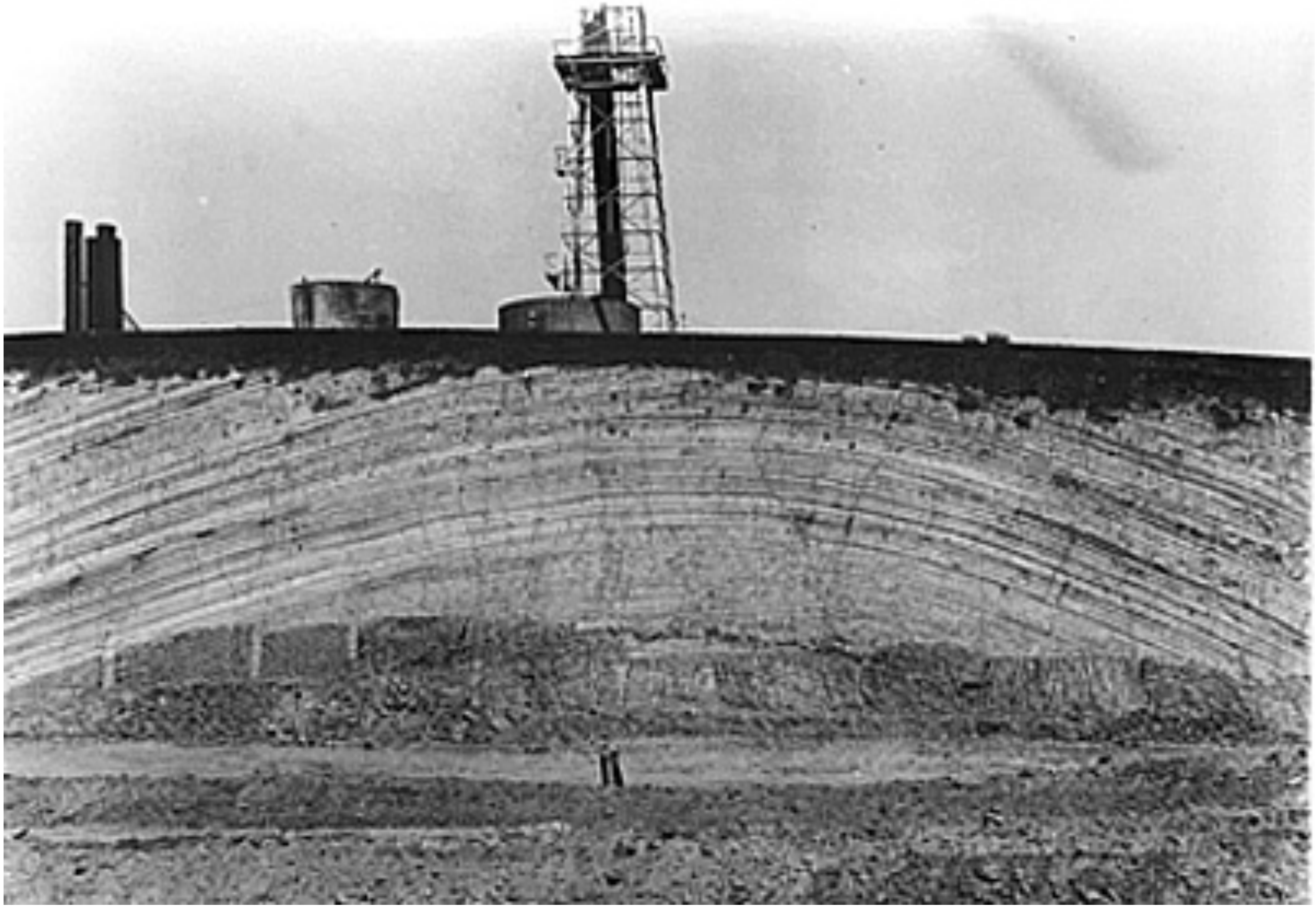
Folds, New South Wales, Australia

http://en.wikipedia.org/wiki/File:Folded_Rock.jpg

Folds in granite, Sierra Nevada, California

27. Folds (I)

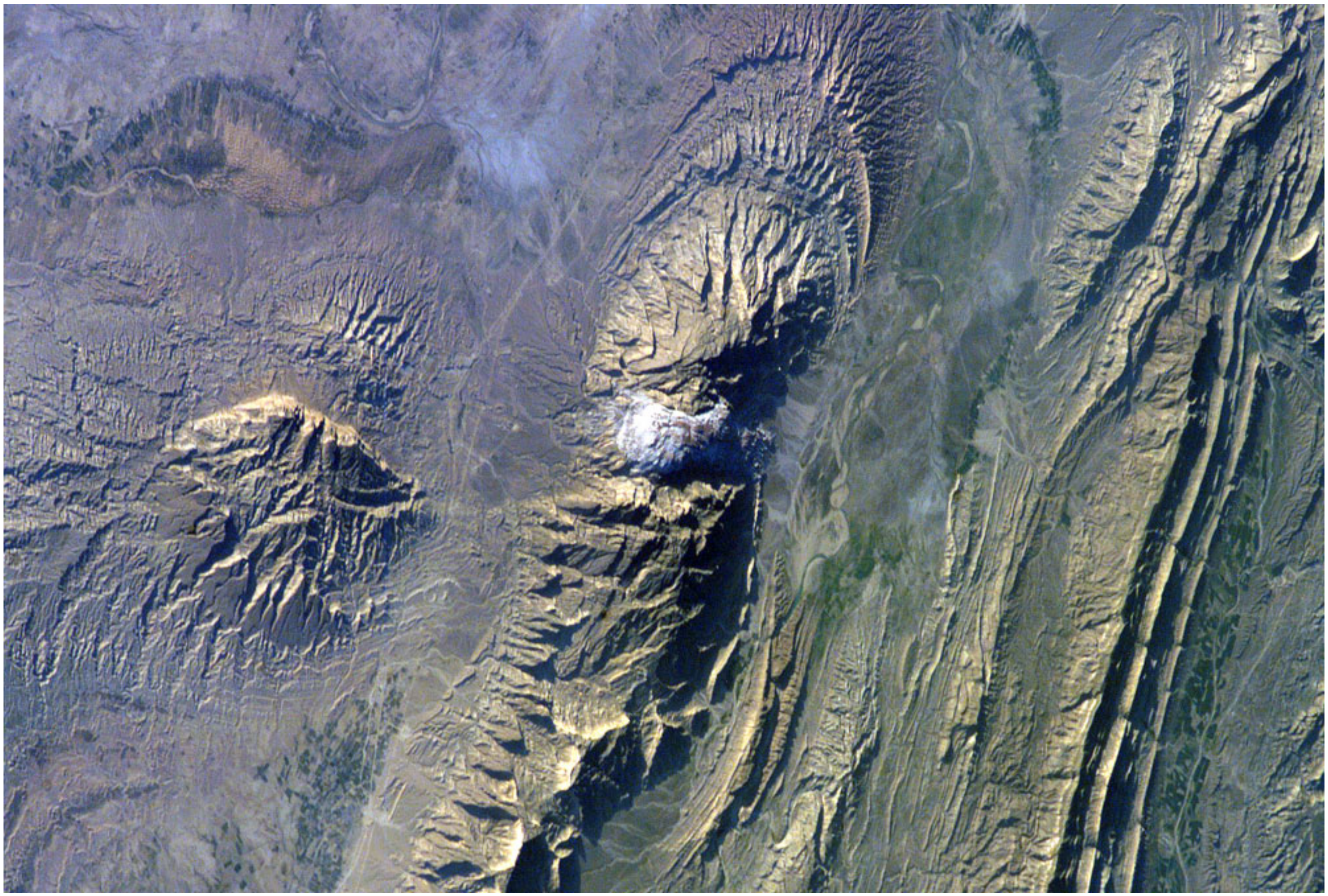
Energy Resources and an Anticline



http://www.wou.edu/las/physci/Energy/graphics/OilAnticline.jpg

27. Folds (I)

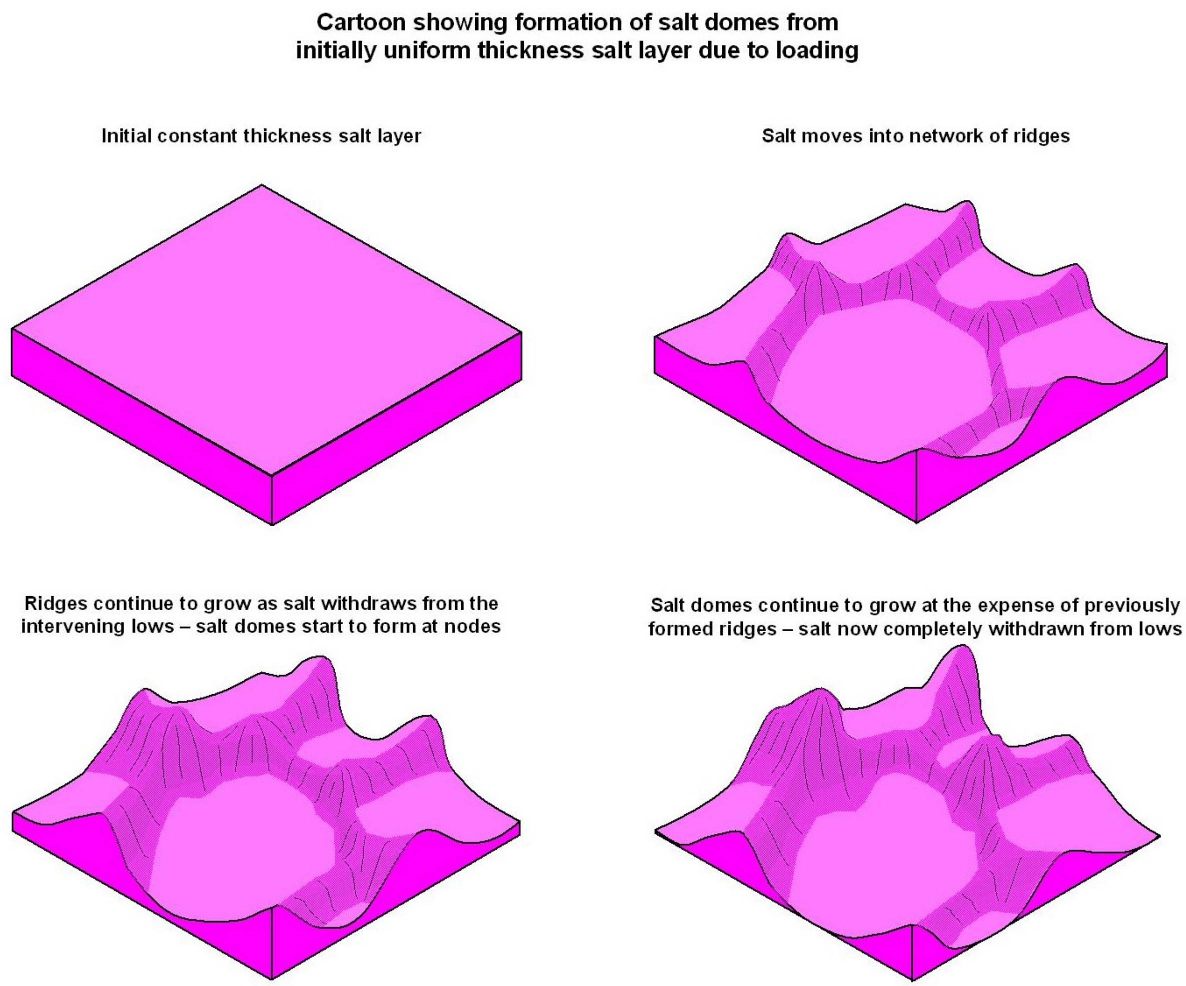
Three-dimensional Fold, Salt Dome, Zagros Mountains



http://upload.wikimedia.org/wikipedia/commons/2/2c/ZagrosMtns_SaltDome_ISS012-E-18774.jpg

27. Folds (I)

Complex Folds



http://upload.wikimedia.org/wikipedia/en/2/2d/SaltTectonics1.jpg

27. Folds (I)

II What is a fold?

- A Definition: a surface (in a rock body) that has undergone a change in its curvature (at least locally)
- B All kinds of rocks can be folded, even granites
- C Consider a folded piece of paper…

Folded dike in granite near fault
Sierra Nevada, California

27. Folds (I)

III Curvature of a plane curve

D Tangents

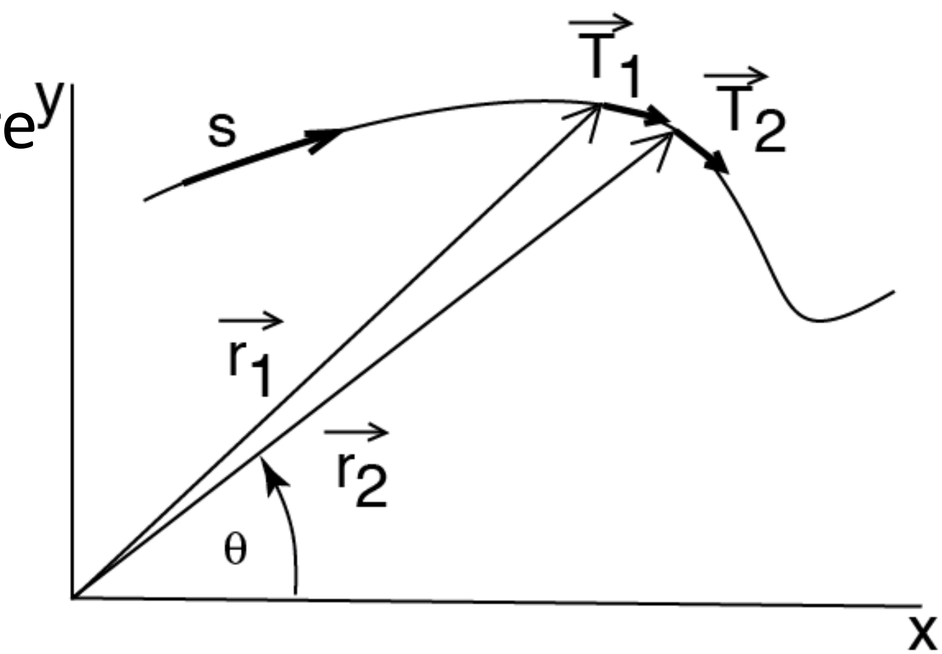
Consider a curve ***r***(t), where ***r*** is a vector function that gives points on the curve, and t is *any* parameter

1 Tangent vector: $\mathbf{r}' = \frac{d\mathbf{r}}{dt}$

2 Unit tangent vector: $\mathbf{T} = \frac{\mathbf{r}'}{|\mathbf{r}'|}$

3 <u>Tangent gives the slope</u>

Tangents Fig. 27.1a



27. Folds (I)

D Tangents (cont.)

5 Example 1: parabola

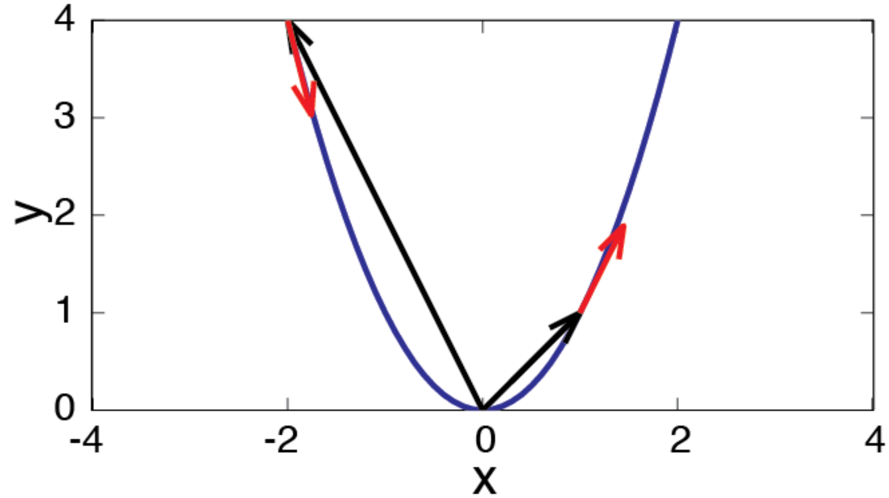
$$y = x^2 \rightarrow \vec{r}(x) = x\vec{i} + x^2\vec{j}$$

$$\mathbf{r}'(x) = \frac{d\vec{r}}{dx} = \frac{d\left(x\vec{i} + x^2\vec{j}\right)}{dx} = \vec{i} + 2x\vec{j}$$

$$\vec{T}(x) = \frac{\mathbf{r}'}{|\mathbf{r}'|} = \frac{1\vec{i} + 2x\vec{j}}{\sqrt{1^2 + (2x)^2}} = \frac{1\vec{i} + 2x\vec{j}}{\sqrt{1 + 4x^2}}$$

$$At\ x = 1, \vec{T} = \frac{\vec{i} + 2\vec{j}}{\sqrt{5}}$$

Positions and Unit Tangents Along a Parabolic Curve (Fig. 27.2a)



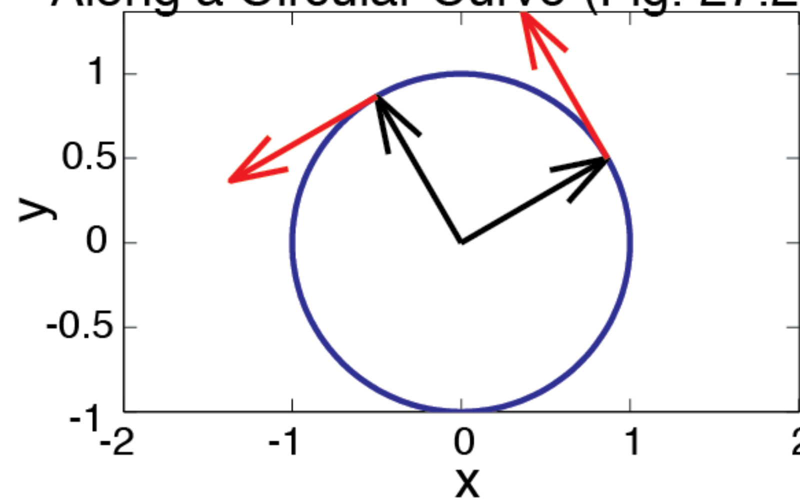
Position vectors in black
Unit tangent vectors in red

27. Folds (I)

D Tangents (cont.)

4 Example 2: unit circle

$$\vec{r}(\theta) = \cos\theta\vec{i} + \sin\theta\vec{j}$$



$$\mathbf{r}' = \frac{d\vec{r}}{d\theta} = \frac{d\left(\cos\theta\vec{i} + \sin\theta\vec{j}\right)}{d\theta} = -\sin\theta\vec{i} + \cos\theta\vec{j}$$

Position vectors in black
Unit tangent vectors in red

$$\vec{T} = \frac{\mathbf{r}'}{|\mathbf{r}'|} = \frac{-\sin\theta\vec{i} + \cos\theta\vec{j}}{\sqrt{(-\sin\theta)^2 + (\cos\theta)^2}} = -\sin\theta\vec{i} + \cos\theta\vec{j}$$

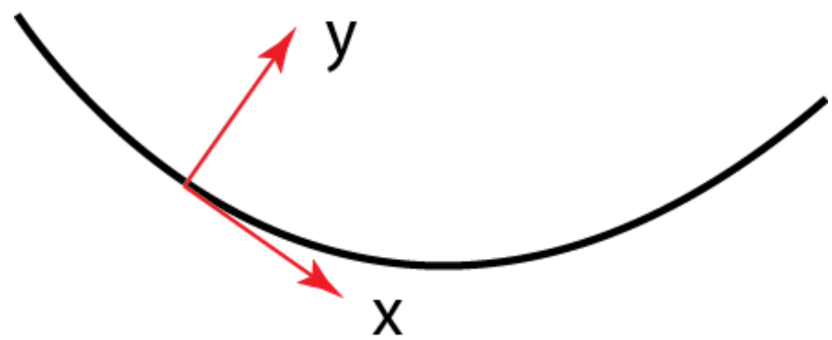
Note that **T•r** = 0 here

27. Folds (I)

III Curvature of a plane curve

A Tangents (cont.)

6 If origin is on curve and reference axis is tangent to curve, then local slope = 0



27. Folds (I)

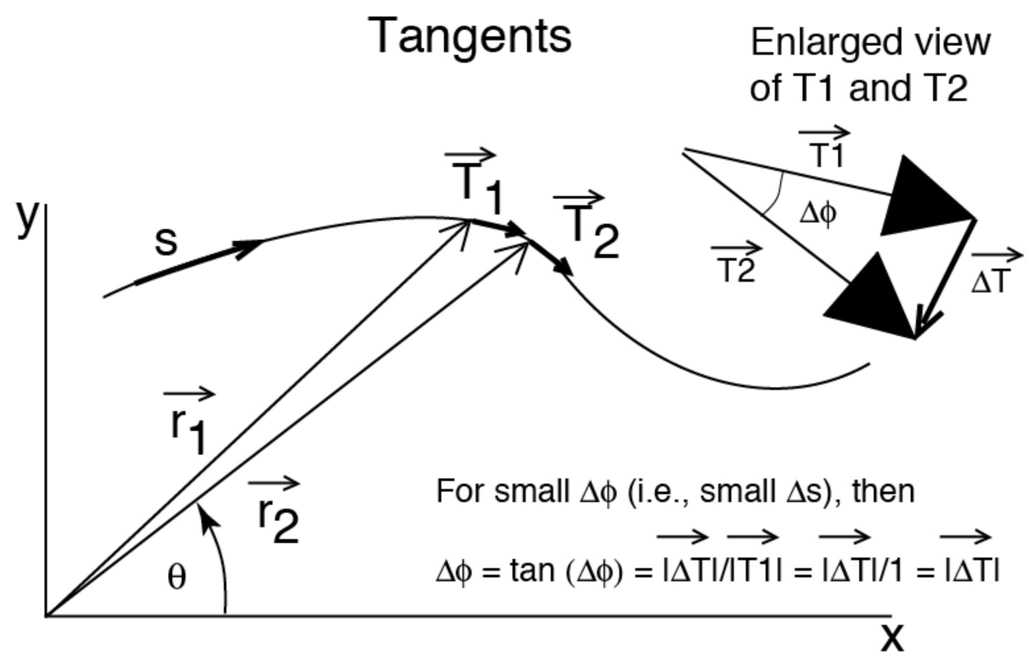
III Curvature of a plane curve

B Curvature = deviation from a straight line

1 Curvature is the first derivative (i.e., rate of change) of the unit tangent (i.e., slope) with respect to distance (s) along the curve

2 Curvature vector is normal to tangent vector

Fig. 27.1



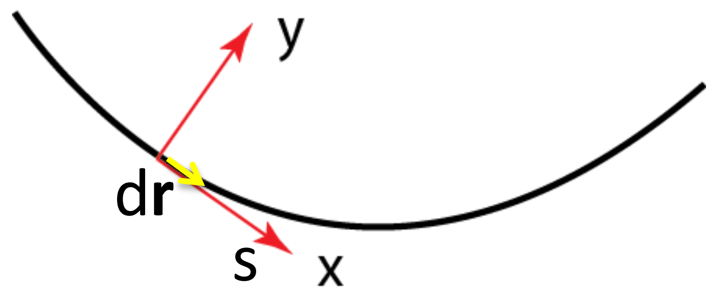
$$\lim_{s\to 0} \Delta\phi = \tan(\Delta\phi) = |\Delta\vec{T}| / |\vec{T}_1| = |\Delta\vec{T}| / 1 = |\Delta\vec{T}|$$

27. Folds (I)

III Curvature of a plane curve

B Curvature (cont.)

3 $$K(s) = |\mathbf{T}'(s)| = \left|\frac{d\mathbf{T}}{ds}\right|$$



4 $$\mathbf{T}(s) = \frac{d\mathbf{r}}{ds} \Big/ \left|\frac{d\mathbf{r}}{ds}\right|$$

In a local tangential reference frame,

dr is in the direction of ds, |dr| = ds ,
and |dr/ds|=1

5 $$K(s) = |\mathbf{T}'(s)| = \left|\frac{d\mathbf{T}}{ds}\right| = \left|\frac{d\left(\frac{d\mathbf{r}}{ds}\right)}{ds}\right| = \left|\frac{d^2\mathbf{r}}{ds^2}\right| = |\mathbf{r}''(s)|$$

27. Folds (I)

B Curvature of a plane curve(cont.)

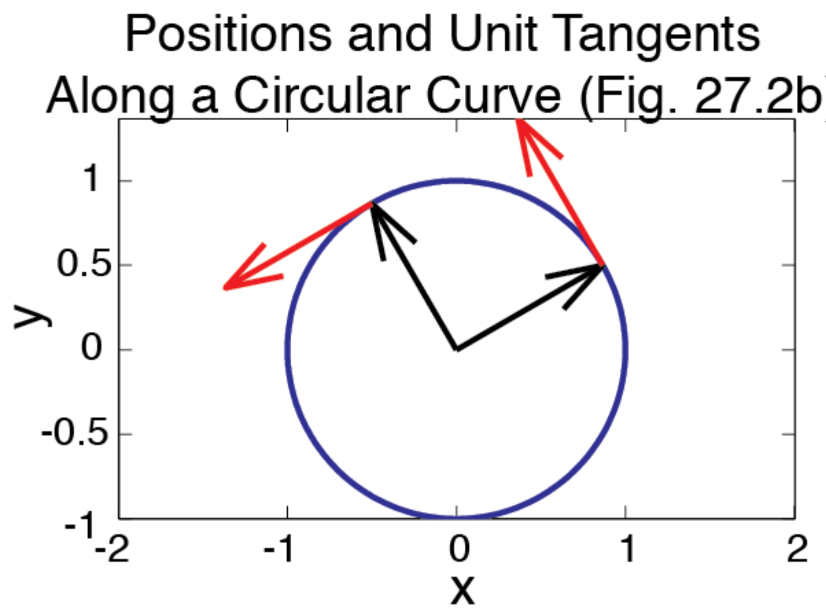
6 Curvature vector (**K**)

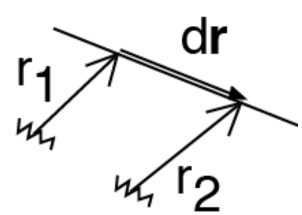
a $\vec{\mathbf{K}}(t) = d\vec{\mathbf{T}} / |d\vec{\mathbf{r}}|$

b $\vec{\mathbf{K}}(t) = \dfrac{d\vec{\mathbf{T}}}{dt} \Big/ \left|\dfrac{d\vec{\mathbf{r}}}{dt}\right|$

7 Curvature magnitude

$$K(t) = |\mathbf{K}| = \left|\dfrac{d\vec{\mathbf{T}}}{dt} \Big/ \left|\dfrac{d\vec{\mathbf{r}}}{dt}\right|\right|$$



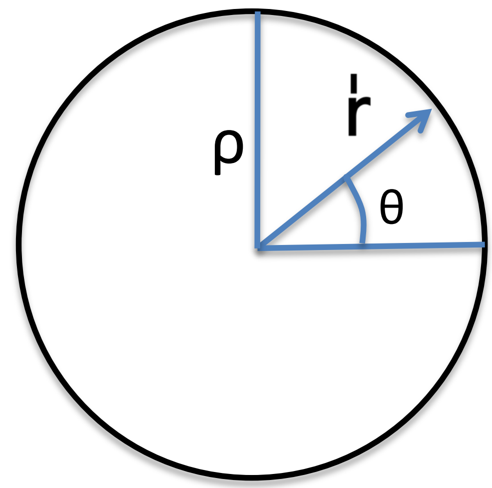


27. Folds (I)

B Curvature of a plane curve (cont.)

8 Example: circle of radius ρ

$$\vec{\mathrm{r}}(\theta) = \rho\cos\theta\vec{\mathrm{i}} + \rho\sin\theta\vec{\mathrm{j}}$$



$$\vec{\mathbf{K}}(\theta) = \frac{\mathrm{d}\vec{\mathbf{T}}}{\mathrm{d}\theta} \Big/ \left|\frac{\mathrm{d}\vec{\mathbf{r}}}{\mathrm{d}\theta}\right|$$

$$\frac{d\vec{\mathbf{r}}}{d\theta} = \frac{d\left(\rho\cos\theta\vec{\mathbf{i}} + \rho\sin\theta\vec{\mathbf{j}}\right)}{d\theta} = -\rho\sin\theta\vec{\mathbf{i}} + \rho\cos\theta\vec{\mathbf{j}}$$

$$\vec{T} = \frac{\mathbf{r}'}{|\mathbf{r}'|} = \frac{-\rho\sin\theta\vec{\mathbf{i}} + \rho\cos\theta\vec{\mathbf{j}}}{\sqrt{(-\rho\sin\theta)^2 + (\rho\cos\theta)^2}} = -\sin\theta\vec{\mathbf{i}} + \cos\theta\vec{\mathbf{j}}$$

$$\vec{\mathbf{K}}(\theta) = \left(-\cos\theta\vec{\mathbf{i}} - \sin\theta\vec{\mathbf{j}}\right) \Big/ \rho = \frac{(-\vec{\mathbf{r}}/\rho)}{\rho} = \frac{-\vec{\mathbf{r}}}{\rho^2}$$

$$\left|\vec{\mathbf{K}}\right| = \left|\frac{-\vec{\mathbf{r}}}{\rho^2}\right| = \frac{1}{\rho}$$

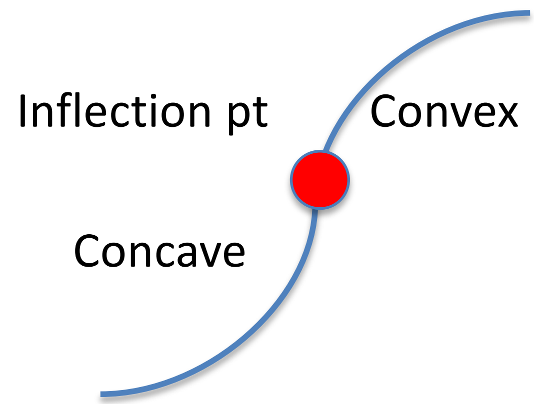
27. Folds (I)

B Curvature of a plane curve (cont.)

9 One can assign a sign to the curvature

a Positive = concave (curve opens up)

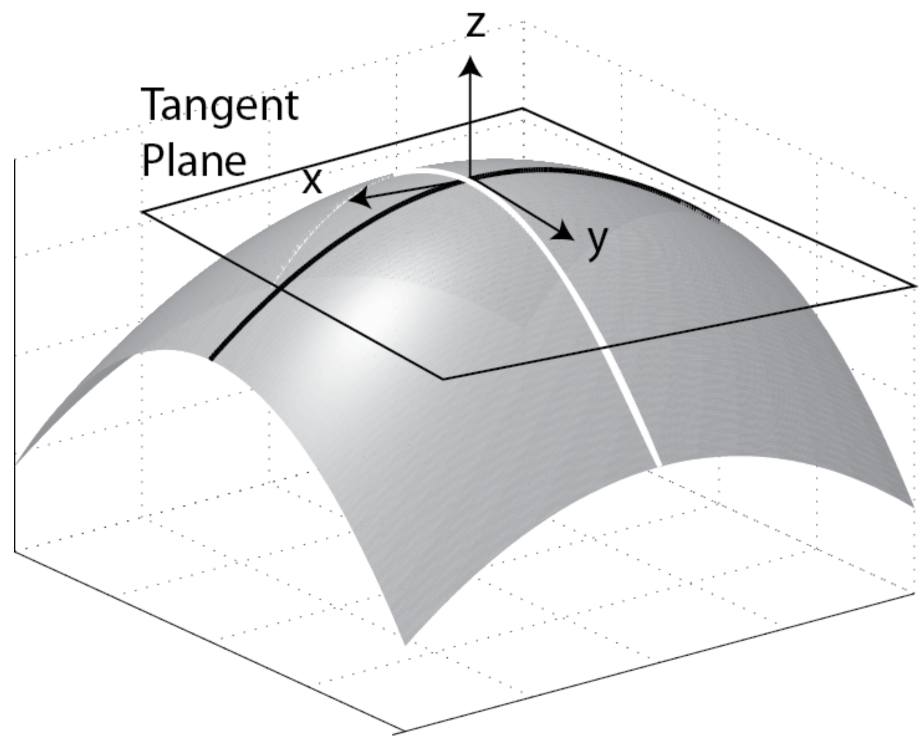
b Negative = convex (curve opens down)



27. Folds (I)

IV Curvature of a surface

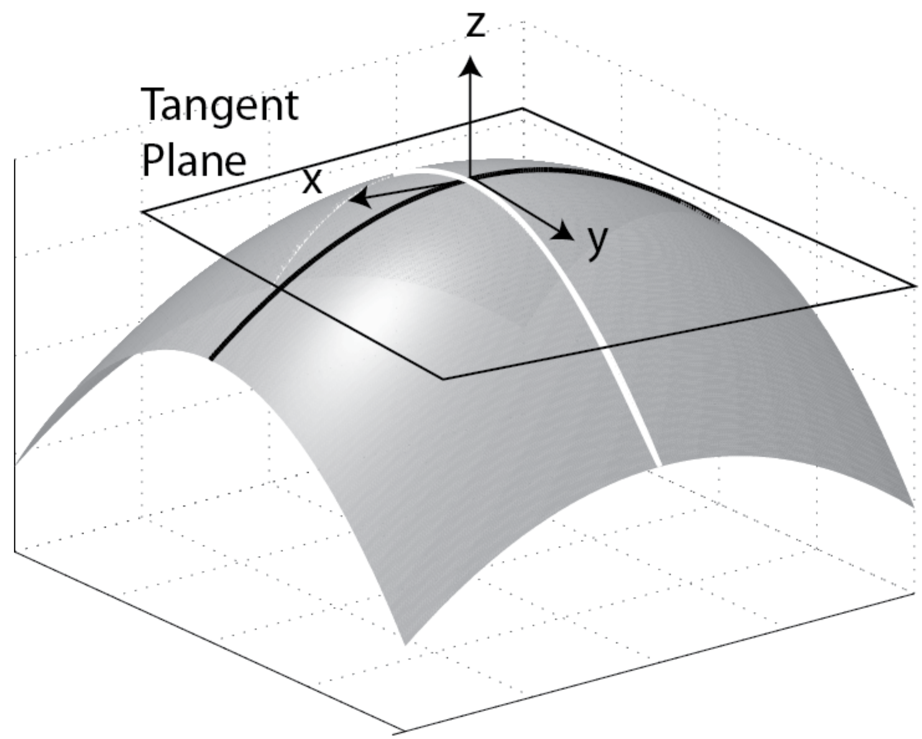
- A Consider a local x,y,z “tangential” reference frame, where the x and y axes are tangent to the surface and z is perpendicular to a folded surface that was originally planar



27. Folds (I)

IV Curvature of a surface

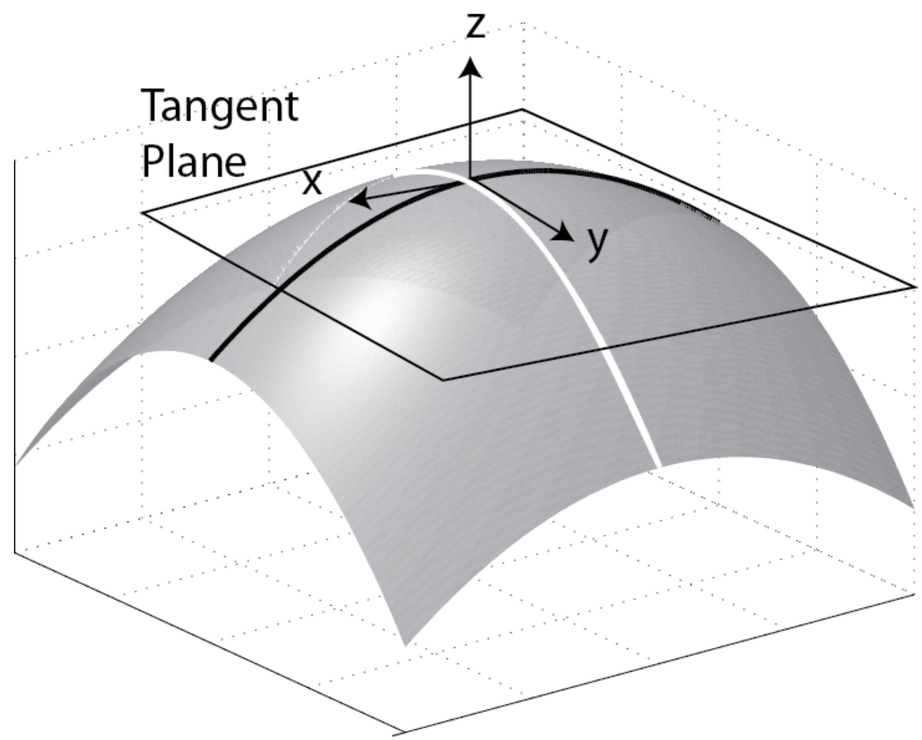
B The first partial derivatives ∂z/∂x and ∂z/∂y are the slopes of the curves formed by intersecting the surface with xz-plane (black curve) and the yz-plane (white curve), respectively. At the local origin, these derivatives equal zero.



27. Folds (I)

IV Curvature of a surface

C The second partial derivatives $\partial^2 z/\partial x^2$, $\partial^2 z/\partial x \partial y$, $\partial^2 z/\partial y \partial x$, and $\partial^2 z/\partial y^2$ can be arranged in a *symmetric* matrix (a *Hessian* matrix).



$$H = \begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial x \partial y} \\ \frac{\partial^2 z}{\partial y \partial x} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix} \quad If\,[X] = \begin{bmatrix} dx \\ dy \end{bmatrix}, [H][X] = \begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial x \partial y} \\ \frac{\partial^2 z}{\partial y \partial x} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{bmatrix}$$

27. Folds (I)

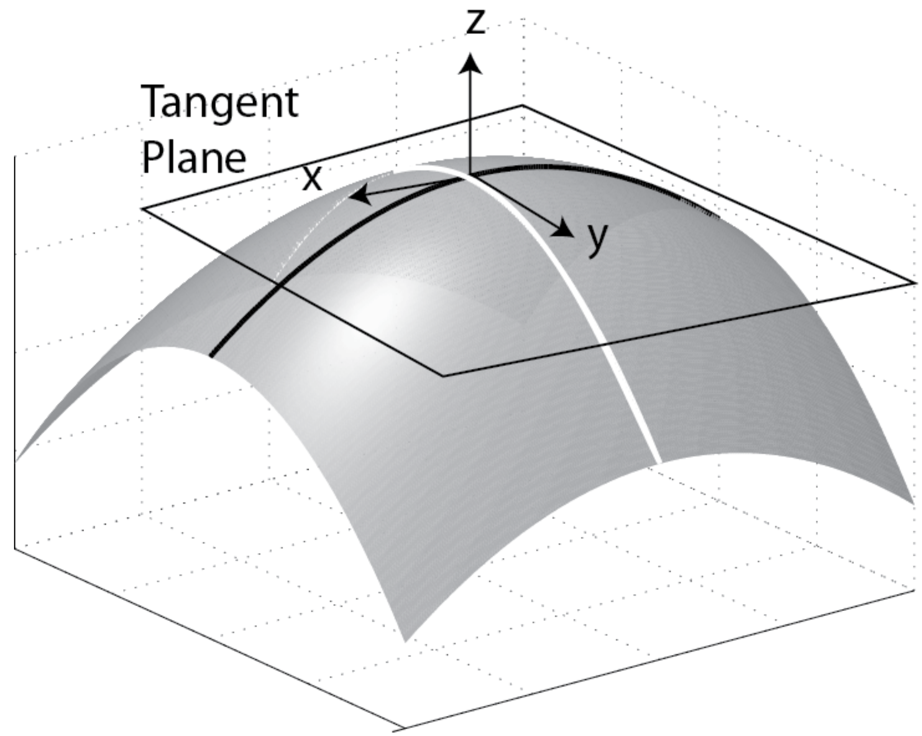
IV Curvature of a surface

D The principal values of the *symmetric Hessian* matrix are the greatest and least normal curvatures

E The principal directions **X** of the symmetric Hessian matrix are the directions of the principal curvatures; these directions are perpendicular

$$H = \begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial x \partial y} \\ \frac{\partial^2 z}{\partial y \partial x} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix} \rightarrow \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$$

Analogous to principal stresses



$$[H][X] = k[X]$$

X gives directions in which slope increases or decreases most rapidly

27. Folds (I)

Curvature-based Three-dimensional Fold Classification Scheme