23. Dislocations

I Main Topics

A Dislocations and other defects in solids

B Significance of dislocations

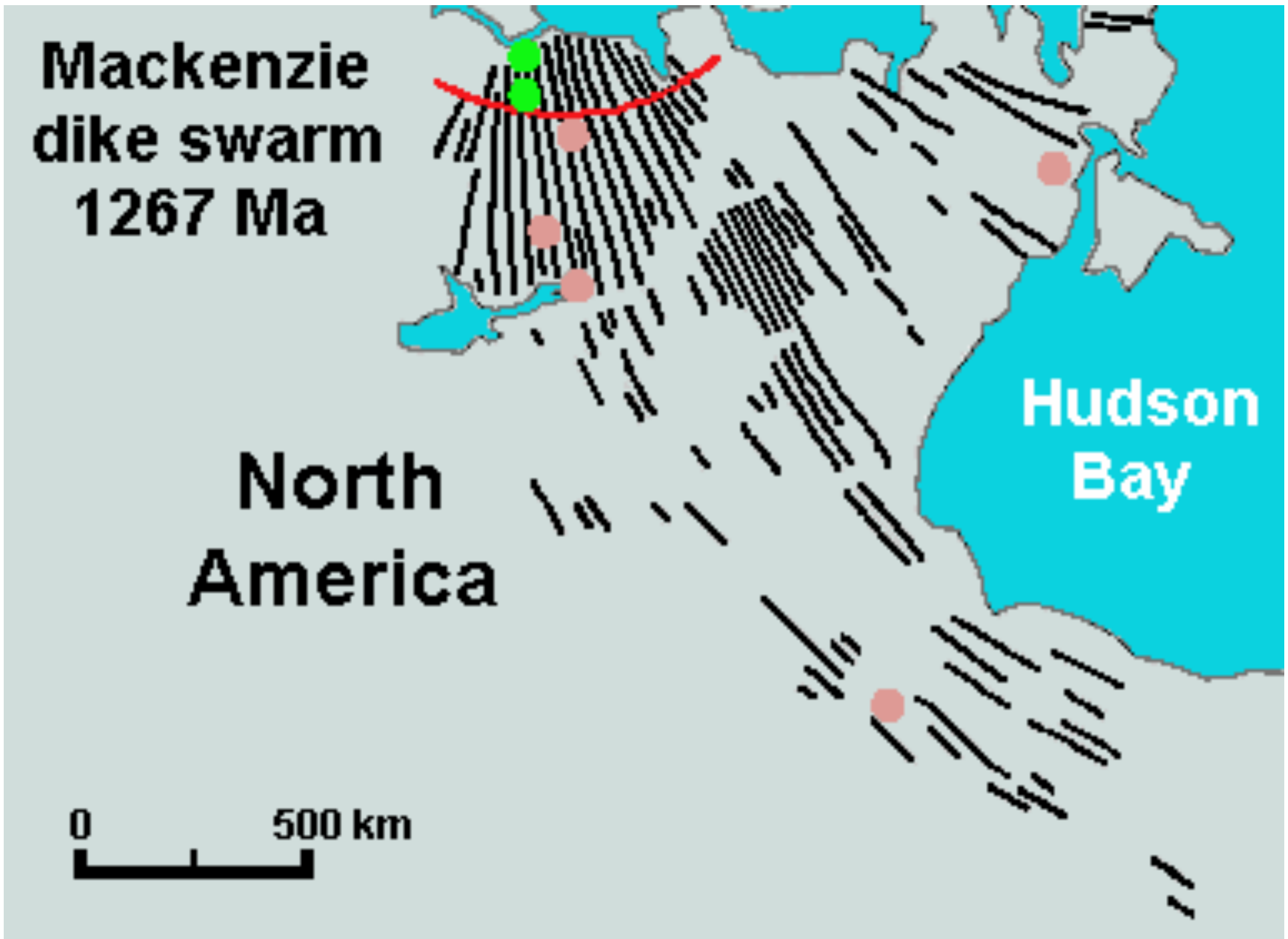
C Planar dislocations

D Displacement and stress fields for a screw dislocation (mode III)

23. Dislocations

http://volcanoes.usgs.gov/Imgs/Jpg/Photoglossary/fissure4_large.JPG

23. Dislocations



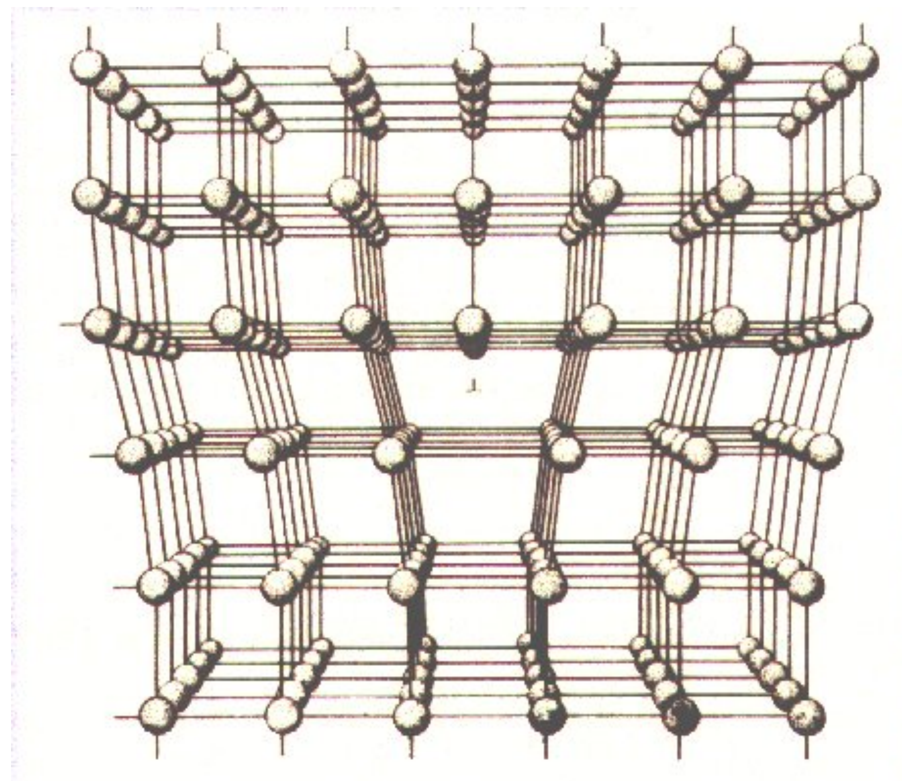
http://upload.wikimedia.org/wikipedia/commons/4/43/Mackenzie_dike_swarm.png

23. Dislocations

II Dislocations and other defects in solids

A Dislocations

1 Originally, extra (or missing) planes or partial planes of material (e.g., atoms)



http://www.tf.uni-kiel.de/matwis/amat/def_ge/kap_5/backbone/r5_2_2.html

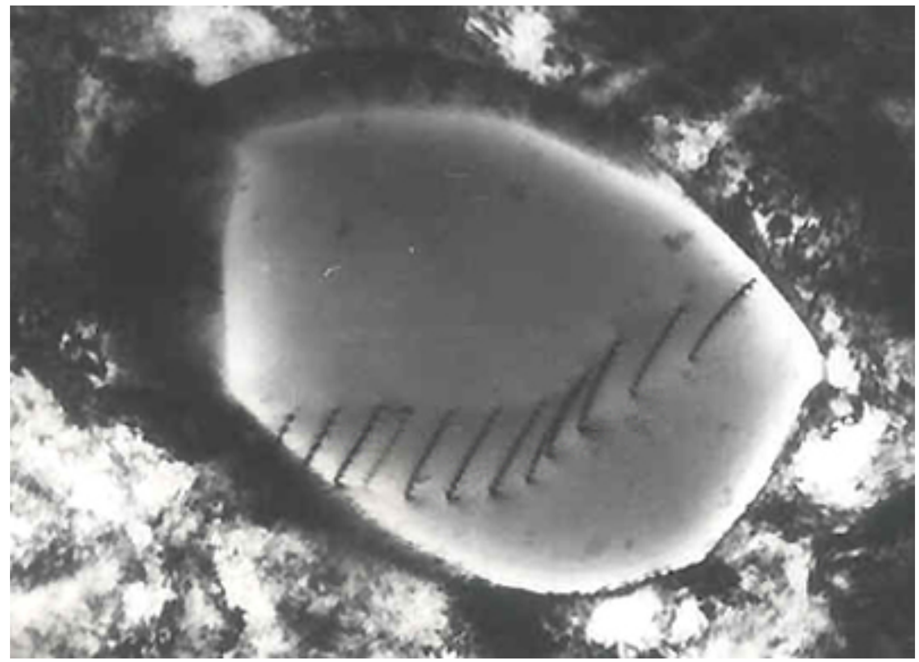
23. Dislocations

II Dislocations and other defects in solids

A Dislocations (cont.)

2 Evidence for dislocations from electron microscopy

Transmission Electron Micrograph
Of Dislocations



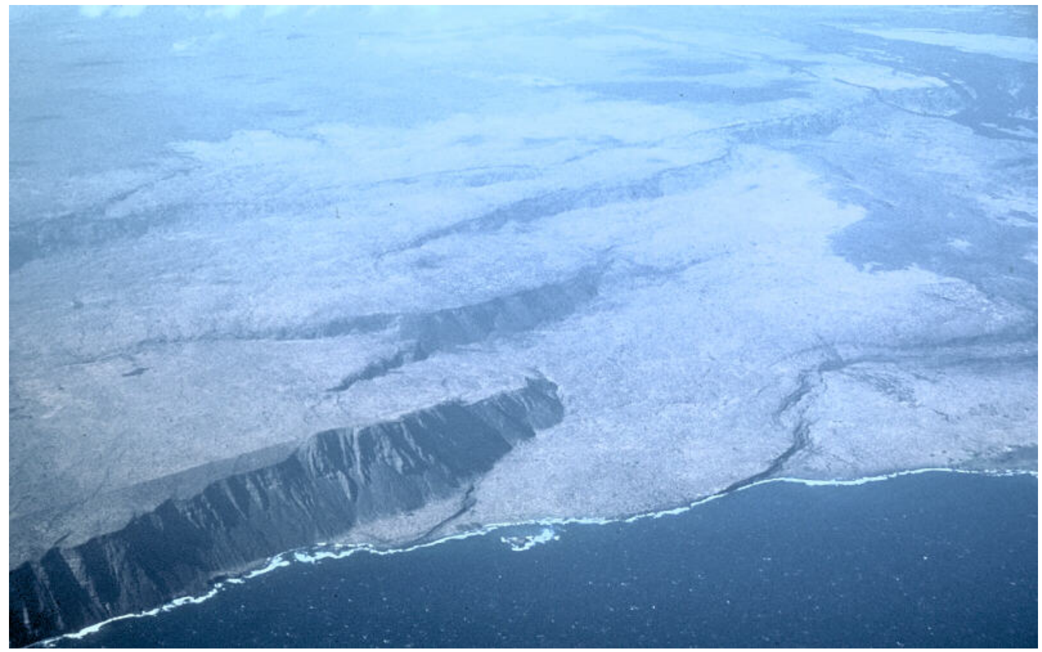
http://en.wikipedia.org/wiki/Dislocation

23. Dislocations

II Dislocations and other defects in solids

A Dislocations (cont.)

3 Continuum mechanics usage: surfaces across which displacements are discontinuous



http://volcanoes.usgs.gov/Imgs/Jpg/Photoglossary/fault2_large.jpg

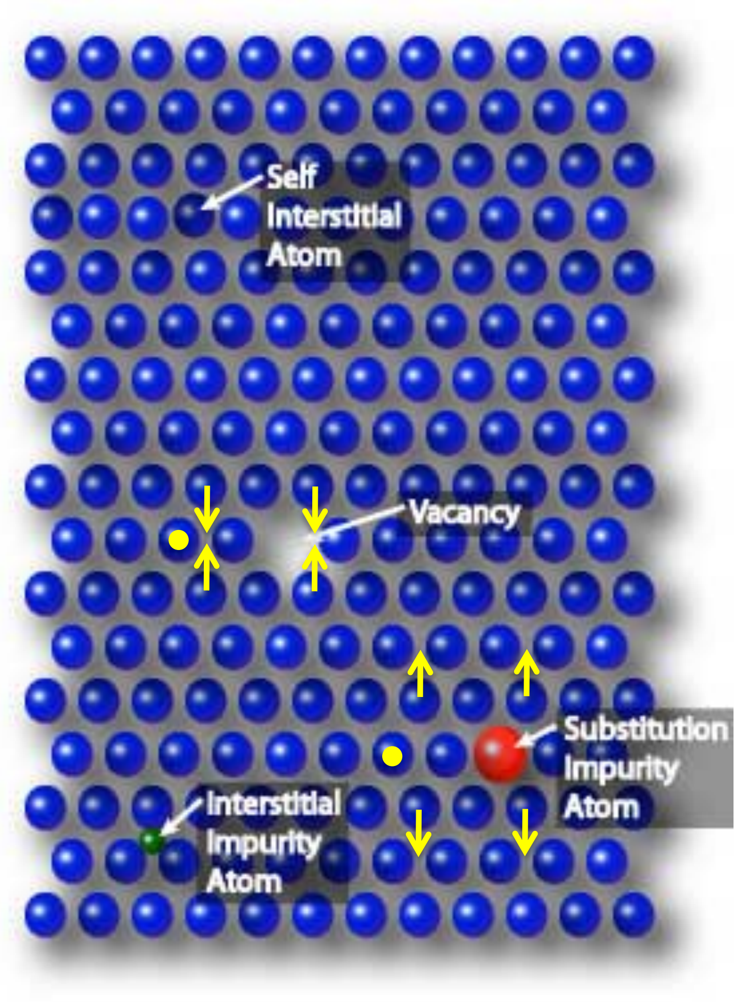
23. Dislocations

II Dislocations and other defects in solids (cont.)

B Point defects

1 Originally, extra (or missing) volumes (e.g., atoms)

2 Displacements are discontinuous across point defects



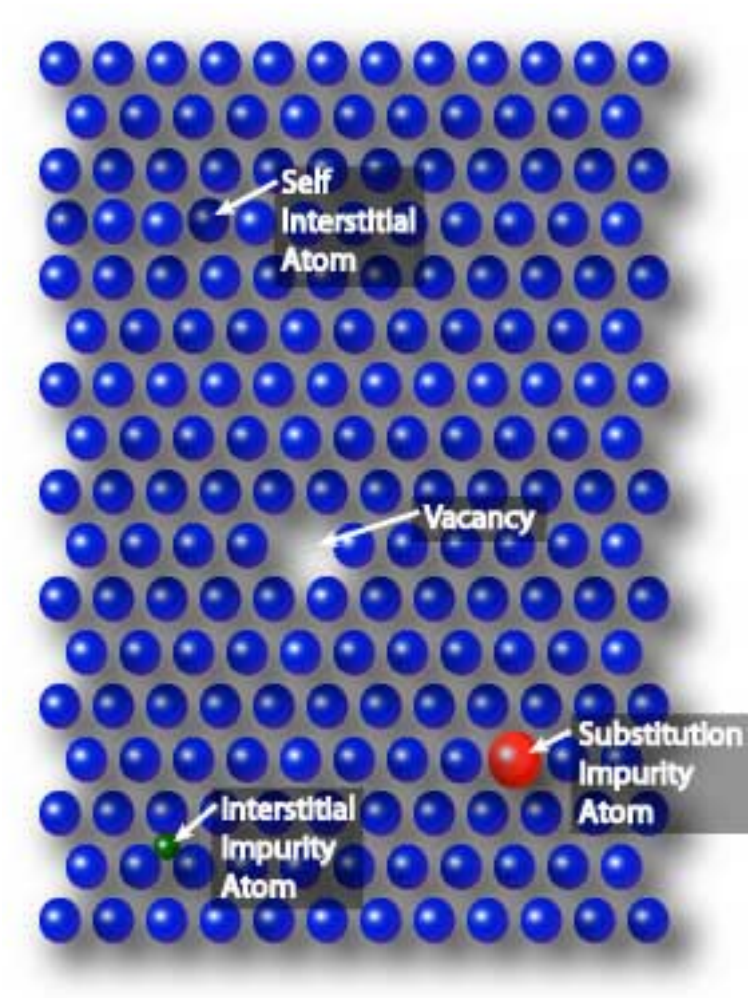
http://www.ndt-ed.org/EducationResources/CommunityCollege/Materials/Graphics/Chrystal-Defects.jpg

23. Dislocations

III Significance of dislocations

A Account for permanent plastic deformation in crystals

B Account for the low observed strength of crystals relative to theoretical predictions



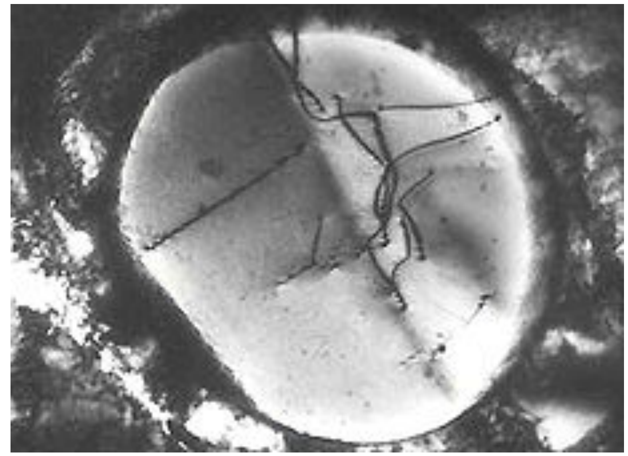
http://www.ndt-ed.org/EducationResources/CommunityCollege/Materials/Graphics/Chrystal-Defects.jpg

23. Dislocations

III Significance of dislocations

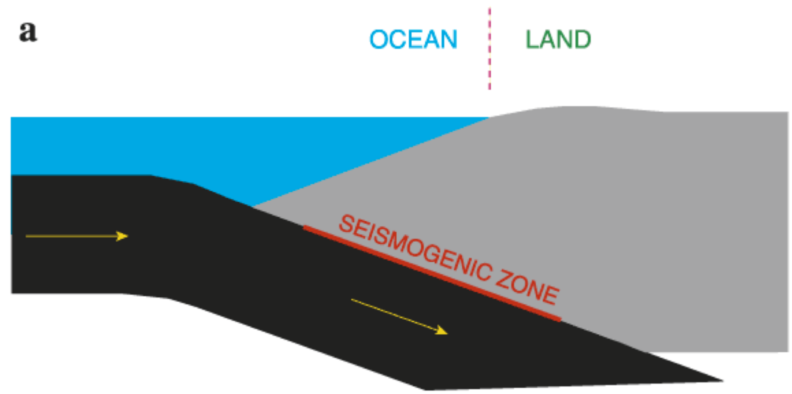
C They provide useful quantitative description of relative motions across surfaces across a broad range of scale (crystals [10^{-6} m] to plate boundaries [10^{6} m]) – ~12 orders of magnitude!

Transmission Electron Micrograph



http://en.wikipedia.org/wiki/Dislocation

Dislocation model of subduction zone

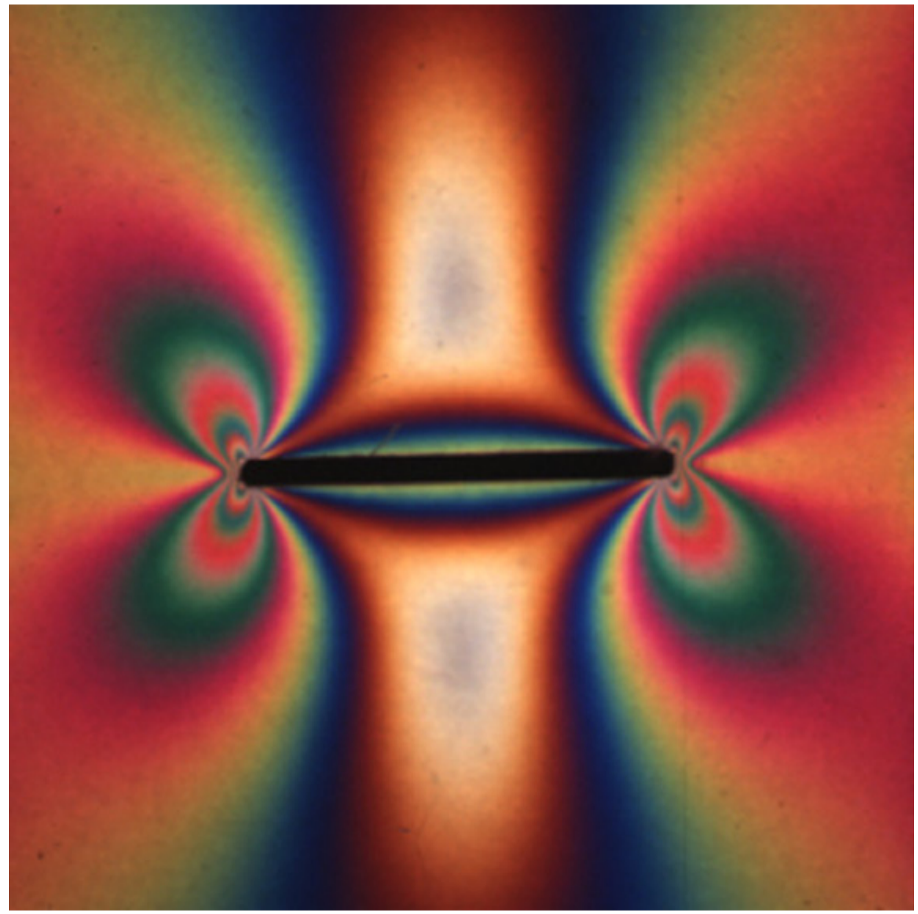


From Bevis and Martel, 2001

23. Dislocations

III Significance of dislocations

D Dislocations induce tremendous stress concentrations and account for “large” deformations under small “average” stresses



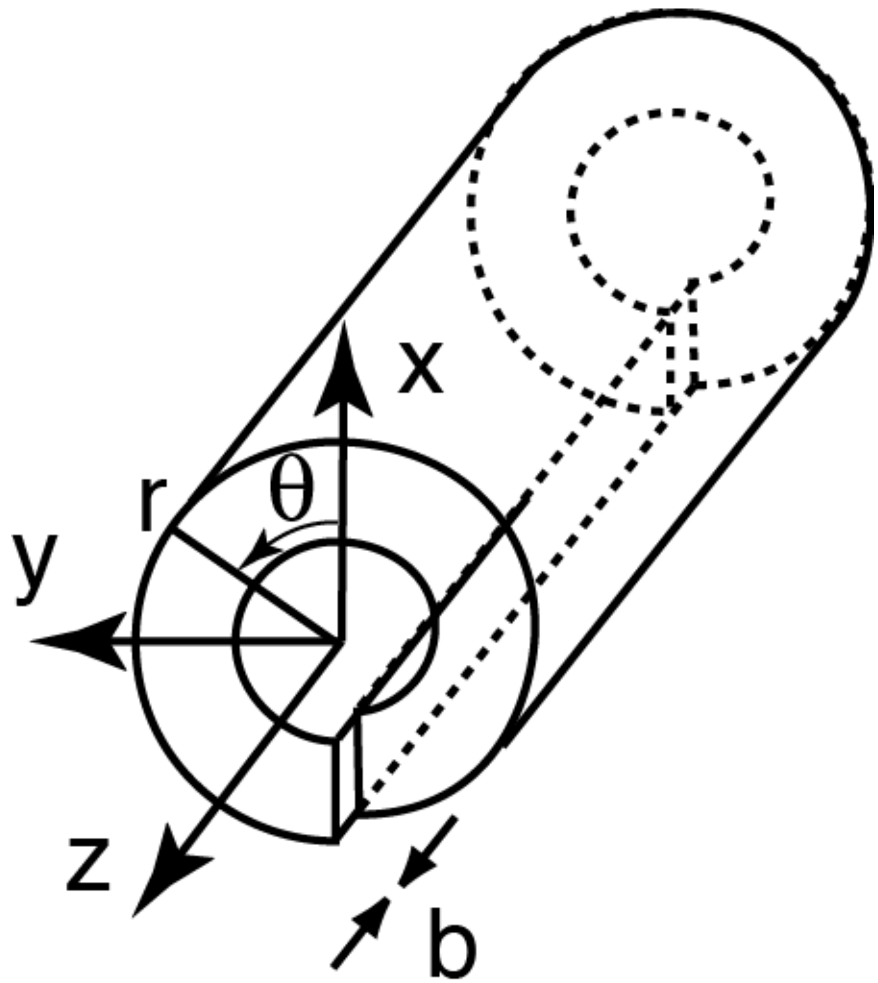
http://pangea.stanford.edu/research/geomech/Faculty/crack.html

23. Dislocations

IV Planar dislocations

A Represented mathematically as infinitely long cut with a straight edge

B Relative displacement (of one side of the dislocation relative to the other) across a dislocation is called the Burger's vector b.



23. Dislocations

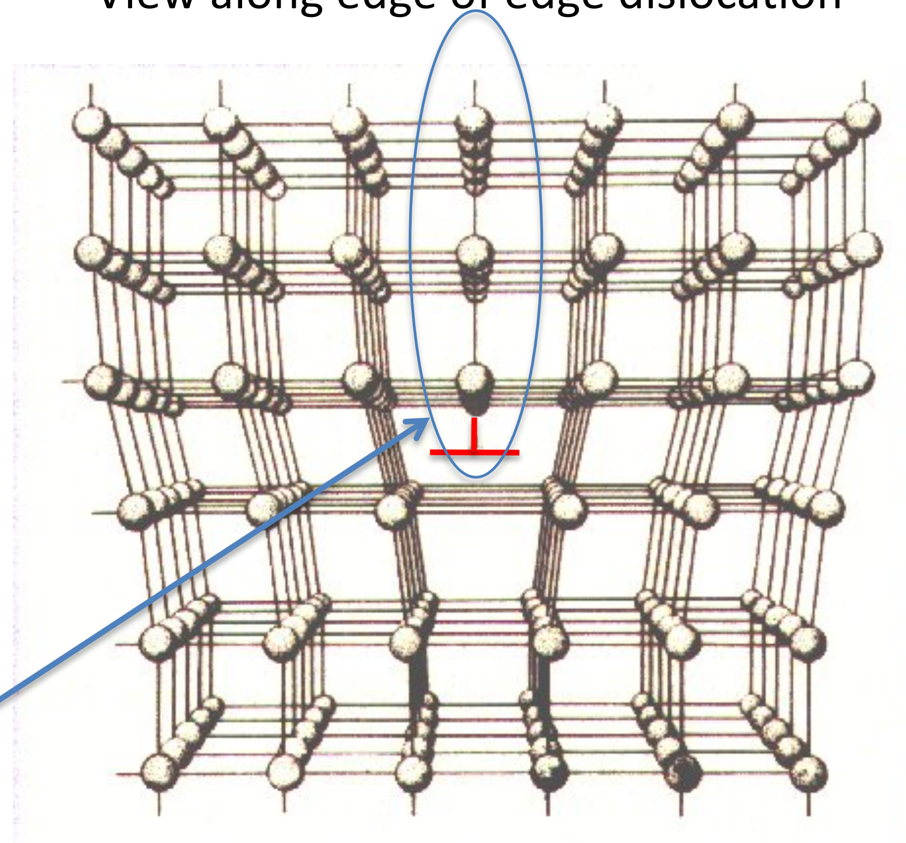
D Edge dislocation

1 Accommodate opening or sliding motions

2 Displacement is exclusively perpendicular to the dislocation edge

3 Displacement can be parallel or perpendicular to the dislocation plane

View along edge of edge dislocation



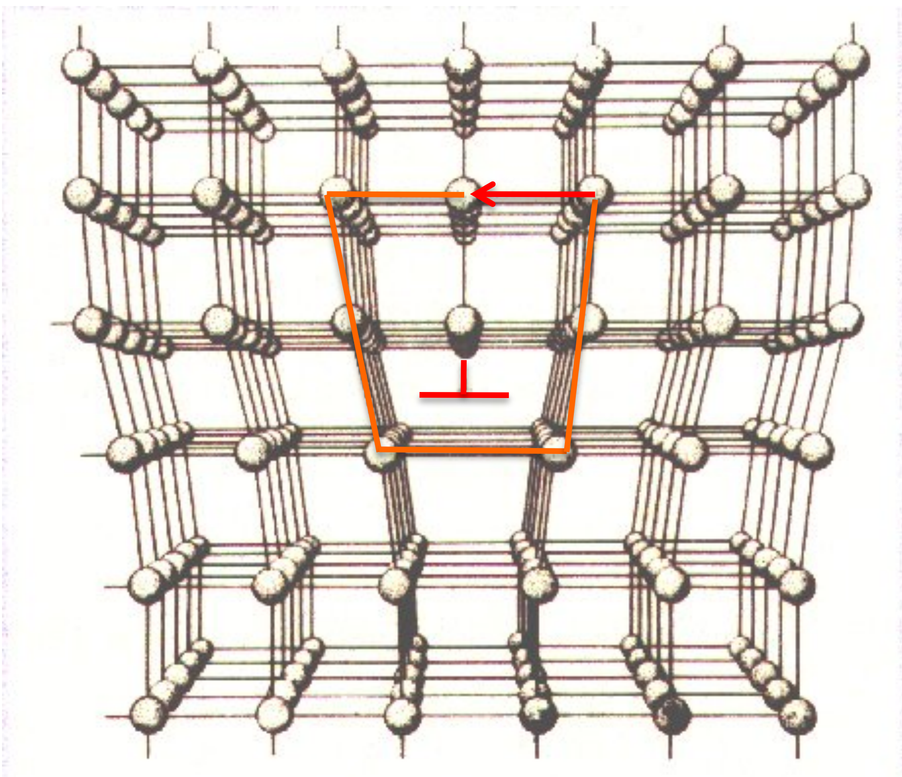
http://www.tf.uni-kiel.de/matwis/amat/def_ge/kap_5/backbone/r5_2_2.html

23. Dislocations

D Edge dislocation (cont.)

4 Analogy: an extra row of corn kernels on a cob of corn

Extra half-plane of atoms in lattice



http://www.tf.uni-kiel.de/matwis/amat/def_ge/kap_5/backbone/r5_2_2.html

Extra row of kernels in an ear of corn

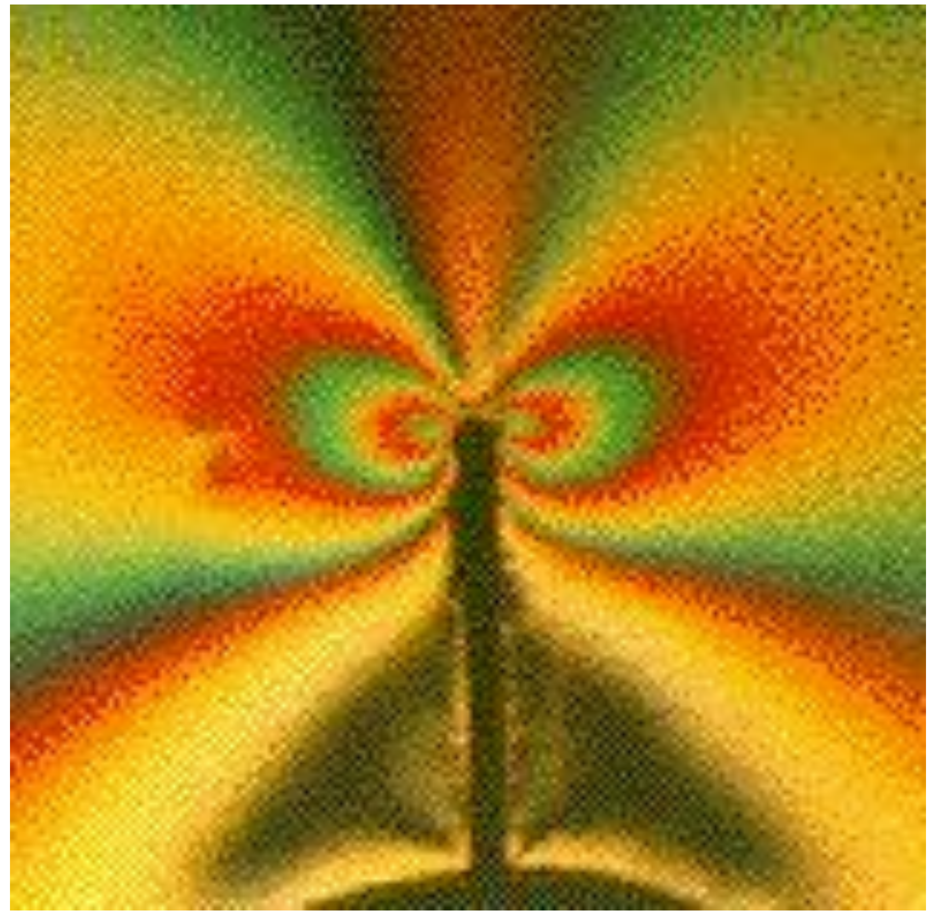
blog.fatfreevegan.com

23. Dislocations

D Edge dislocation (cont.)

5 Macroscopic geologic use: modeling dikes or faults

http://hvo.wr.usgs.gov/gallery/kilauea/erupt/24ds182_caption.html



http://www.experimentalstress.com/topic/reflect.jpg

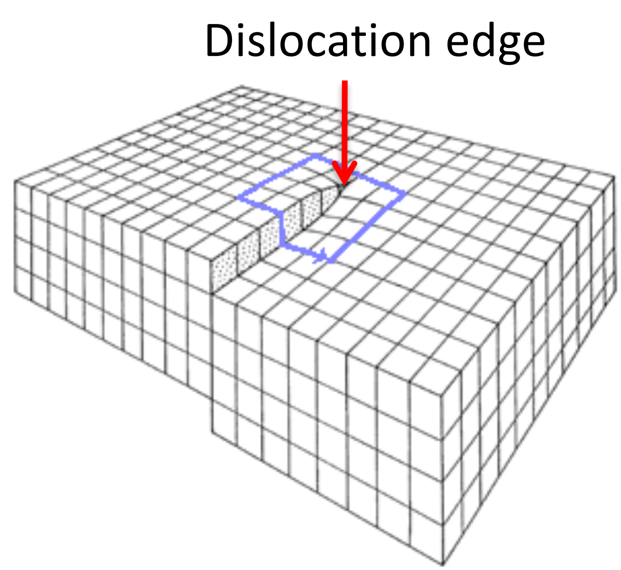
23. Dislocations

C Screw dislocation

1 Accommodate a tearing motion

2 Displacement is exclusively parallel to the dislocation edge

3 Analogy: a lock washer or a 360° spiral staircase



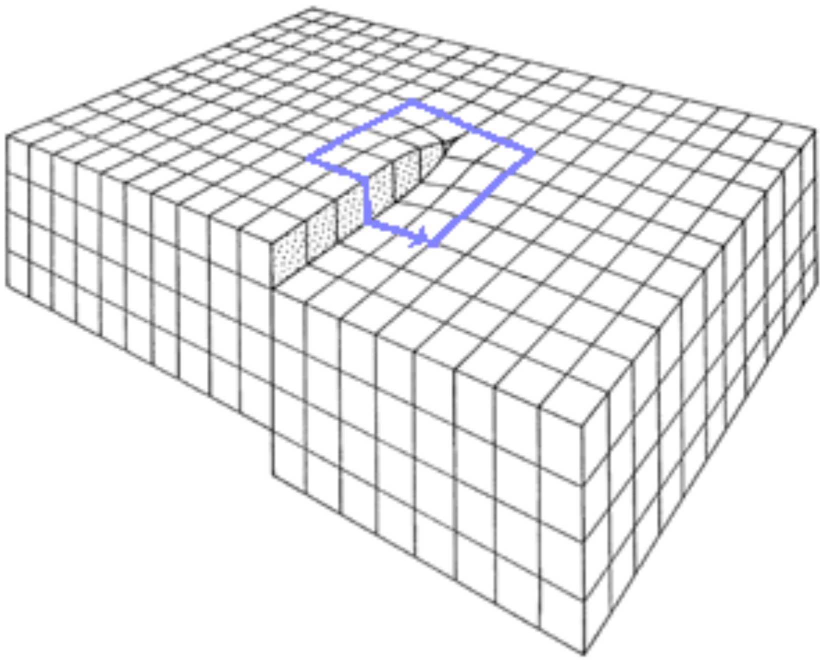
http://www.tf.uni-kiel.de/matwis/amat/def_ge/kap_5/backbone/r5_2_2.html

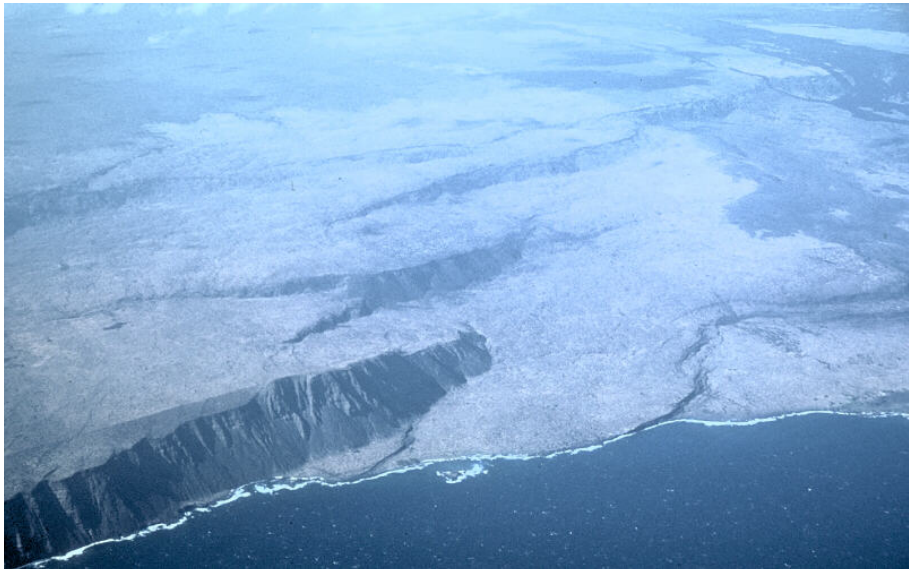
http:///www.boltsplus.com

23. Dislocations

C Screw dislocation

4 Macroscopic geologic use: modeling faults





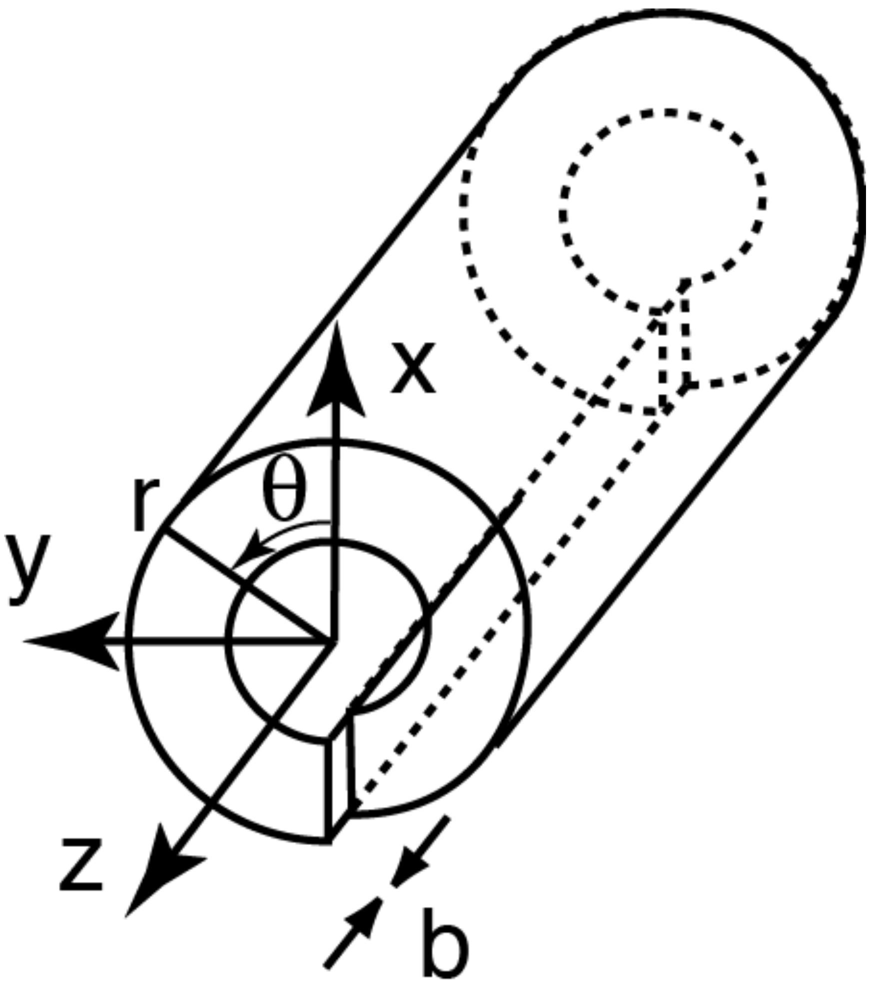
http://volcanoes.usgs.gov/Imgs/Jpg/Photoglossary/fault2_large.jpg

http://www.tf.uni-kiel.de/matwis/amat/def_ge/kap_5/backbone/r5_2_2.html

23. Dislocations

V Displacement and stress fields for a screw dislocation (mode III)

- A Displacement parallel to the dislocation edge (w) increases uniformly along a spiral-like circuit from one side of the dislocation to the other (for a right-handed screw dislocation, point your right thumb along the dislocation edge; displacement parallel to the edge increases in the direction your fingers curl.
- B Angular position: $\theta = \tan^{-1}(y/x)$
- C $w = b\theta/2\pi$
- D Anti-plane strain ($u,v = 0$)



23. Dislocations

Cylindrical Reference Frame

$u_r = 0$, $u_\theta = 0$, $w = b\theta/2\pi$

Normal Strains	$\varepsilon_{rr} = \frac{1}{2}\left(\frac{\partial u_r}{\partial r} + \frac{\partial u_r}{\partial r}\right)$ $\varepsilon_{rr} = 0$	$\varepsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}$ $\varepsilon_{\theta\theta} = 0$	$\varepsilon_{zz} = \frac{1}{2}\left(\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z}\right)$ $\varepsilon_{zz} = 0$
Shear Strains	$\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)$ $\varepsilon_{r\theta} = \varepsilon_{\theta r} = 0$	$\varepsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta}\right)$ $\varepsilon_{\theta z} = \varepsilon_{z\theta} = \frac{b}{4\pi r}$	$\varepsilon_{zr} = \frac{1}{2}\left(\frac{\partial w}{\partial r} + \frac{\partial u_r}{\partial z}\right)$ $\varepsilon_{zr} = \varepsilon_{rz} = 0$

* The derivatives are *very* easy to do. Check them to verify the solutions.

23. Dislocations

Cartesian Reference Frame

u = 0, v = 0, w = b[tan^{-1}(y/x)]/2π

Normal Strains	$\varepsilon_{xx} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}\right)$ $\varepsilon_{xx} = 0$	$\varepsilon_{yy} = \frac{1}{2}\left(\frac{\partial v}{\partial y} + \frac{\partial y}{\partial y}\right)$ $\varepsilon_{yy} = 0$	$\varepsilon_{zz} = \frac{1}{2}\left(\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z}\right)$ $\varepsilon_{zz} = 0$
Shear Strains	$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$ $\varepsilon_{xy} = 0$	$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$ $\varepsilon_{yz} = \frac{b}{4\pi}\frac{x}{x^2 + y^2}$ $\varepsilon_{yz} = \frac{b}{4\pi}\frac{x}{r^2}$	$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$ $\varepsilon_{xz} = \frac{-b}{4\pi}\frac{y}{x^2 + y^2}$ $\varepsilon_{xz} = \frac{-b}{4\pi}\frac{y}{r^2}$

* The derivatives are still rather easy to do. Check them to verify the solutions.

23. Dislocations

Strains	Stresses
$\varepsilon_{r\theta} = \varepsilon_{\theta r} = 0$	$\sigma_{r\theta} = \sigma_{\theta r} = 2G\varepsilon_{r\theta} = 0$
$\varepsilon_{\theta z} = \varepsilon_{z\theta} = \dfrac{b}{4\pi r}$	$\sigma_{\theta z} = \sigma_{z\theta} = 2G\varepsilon_{\theta z} = \dfrac{Gb}{2\pi r}$
$\varepsilon_{r\theta} = \varepsilon_{\theta r} = 0$	$\sigma_{r\theta} = \sigma_{\theta r} = 2G\varepsilon_{r\theta} = 0$
$\varepsilon_{rr} = 0$	$\sigma_{rr} = 0$
$\varepsilon_{\theta\theta} = 0$	$\sigma_{\theta\theta} = 0$
$\varepsilon_{zz} = 0$	$\sigma_{zz} = 0$

23. Dislocations

Strains	Stresses
$\varepsilon_{xy} = \varepsilon_{yx} = 0$	$\sigma_{xy} = \sigma_{yx} = 2G\varepsilon_{xy} = 0$
$\varepsilon_{yz} = \frac{b}{4\pi}\frac{x}{x^2 + y^2}$	$\sigma_{yz} = \sigma_{zy} = 2G\varepsilon_{yz} = \frac{Gb}{2\pi}\frac{x}{x^2 + y^2} = \frac{Gb}{2\pi}\frac{x}{r^2}$
$\varepsilon_{xz} = \frac{-b}{4\pi}\frac{y}{x^2 + y^2}$	$\sigma_{xz} = \sigma_{zx} = 2G\varepsilon_{xz} = \frac{-Gb}{2\pi}\frac{y}{x^2 + y^2} = \frac{-Gb}{2\pi}\frac{y}{r^2}$
$\varepsilon_{xx} = 0$	$\sigma_{xx} = 0$
$\varepsilon_{yy} = 0$	$\sigma_{yy} = 0$
$\varepsilon_{zz} = 0$	$\sigma_{zz} = 0$

23. Dislocations

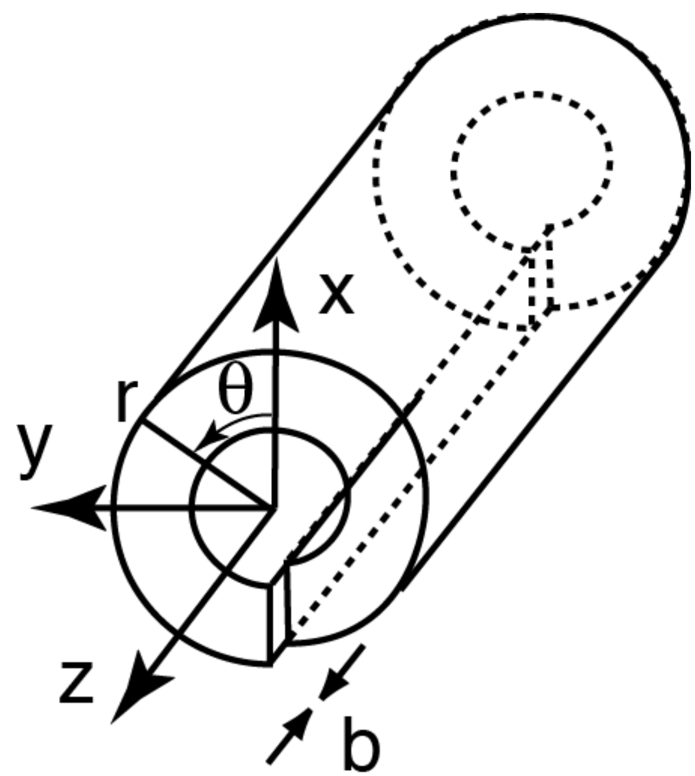
Key points

a Only the shear stresses acting on a plane normal to the z-direction or in the z-direction are non-zero

b The stresses are singular (i.e., go to infinity) near the dislocation edge: a powerful stress concentration exists there.

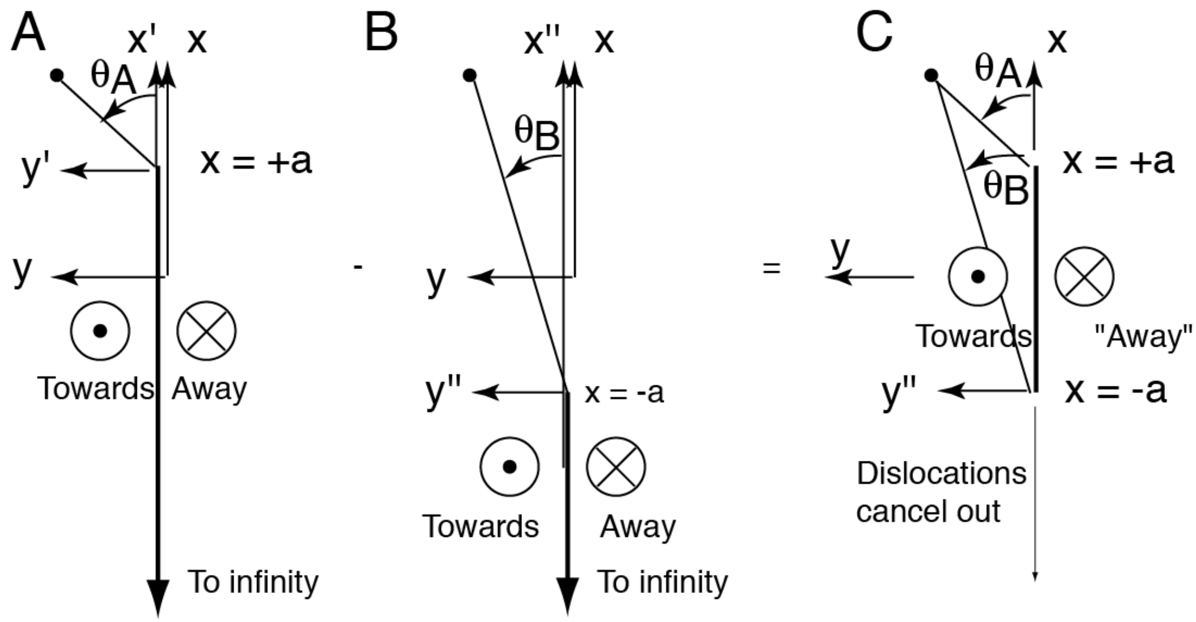
c This theoretical singular stress concentration exists no matter how small *G* or the relative displacement *b* is, provided *G* > 0 and *b* > 0.

$$\sigma_{\theta z} = \sigma_{z\theta} = \frac{Gb}{2\pi r}$$



23. Dislocations

SUPERPOSTION OF TWO (INFINITE) SCREW DISLOCATIONS (A,B) TO FORM A FINITE DISPLACEMENT DISCONTINTUITY (C)



$w_A = b\theta_A/(2\pi)$

$= (b/2\pi)\tan^{-1}(y'/x')$

$= (b/2\pi)\tan^{-1}(y/[x-a])$

$-\pi \le \theta \le \pi$

$w_B = b\theta_B/(2\pi)$

$= (b/2\pi)\tan^{-1}(y''/x'')$

$= (b/2\pi)\tan^{-1}(y/[x+a])$

$w_C = w_A - w_B$

$= b(\theta_A - \theta_B)/(2\pi)$

$w_C\ (\theta_A = -\pi, \theta_B = 0) = -B/2$
$w_C\ (\theta_A = 0, \theta_B = 0) = 0$
$w_C\ (\theta_A = \pi, \theta_B = 0) = B/2$
$w_C\ (\theta_A = \pi, \theta_B = \pi) = 0$