20. Rheology & Linear Elasticity

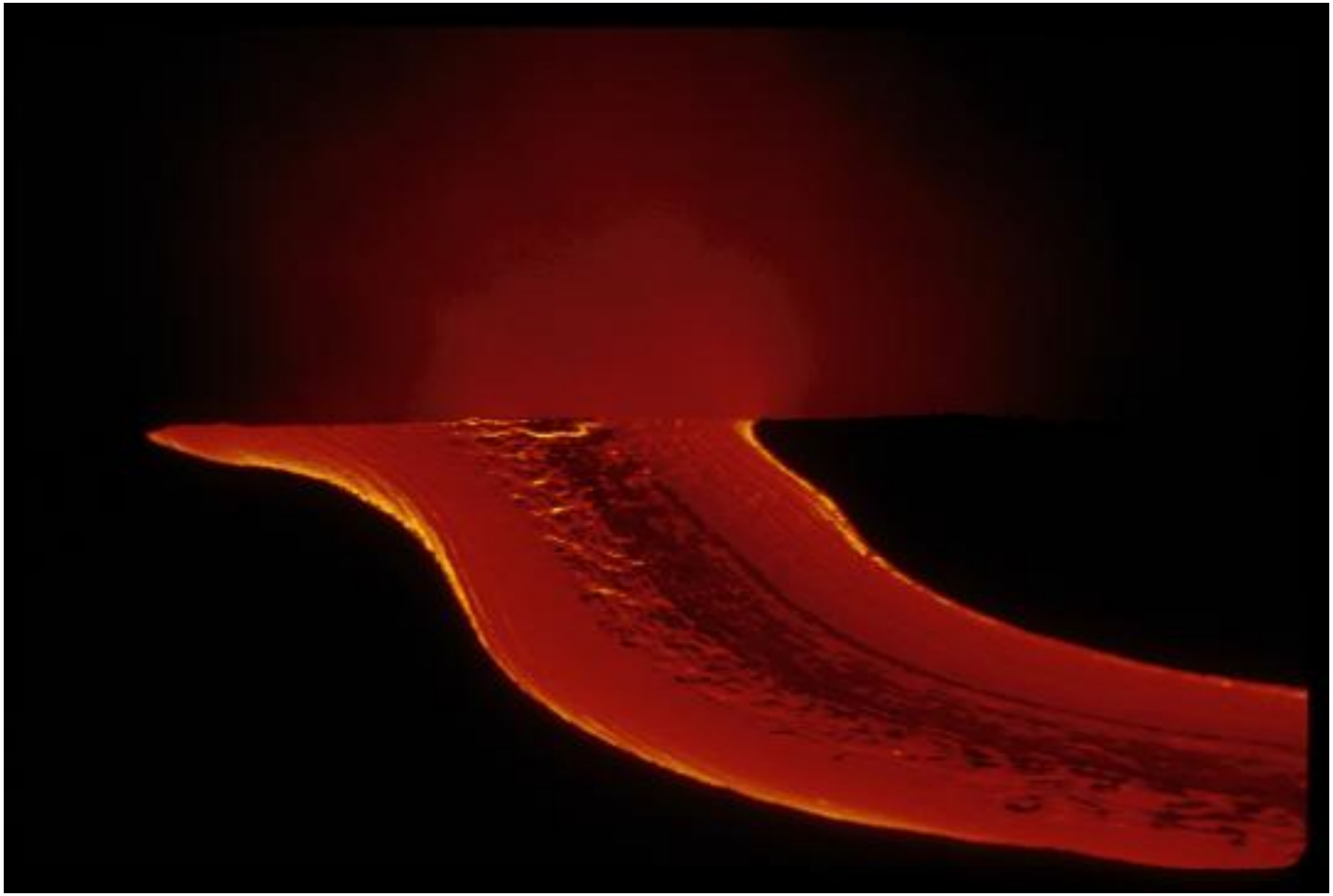
I Main Topics

A Rheology: Macroscopic deformation behavior

B Linear elasticity for homogeneous isotropic materials

20. Rheology & Linear Elasticity

Viscous (fluid) Behavior



http://manoa.hawaii.edu/graduate/content/slide-lava

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http://hvo.wr.usgs.gov/kilauea/update/images.html

Ductile (plastic) Behavior

http://www.hilo.hawaii.edu/~csav/gallery/scientists/LavaHammerL.jpg

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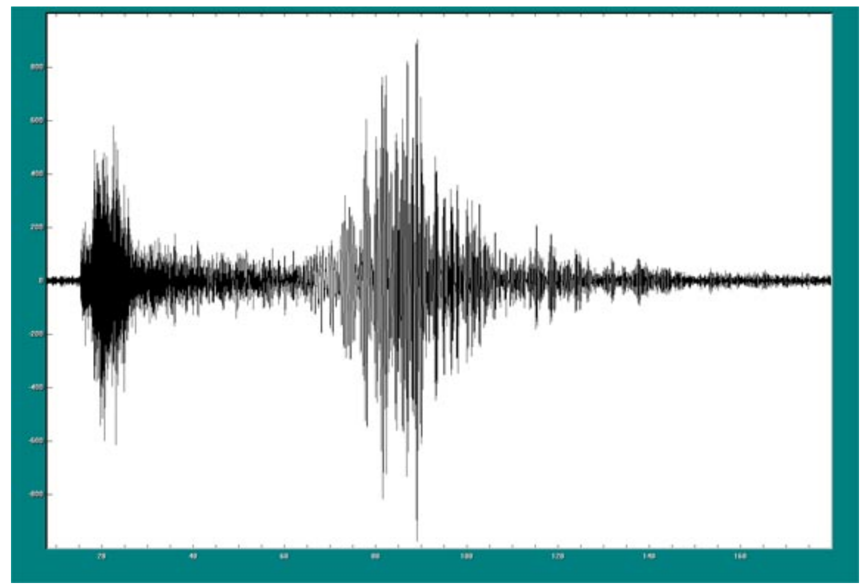
http://www.earth.ox.ac.uk/__data/assets/image/0006/3021/seismic_hammer.jpg

Elastic Behavior



https://thegeosphere.pbworks.com/w/page/24663884/Sumatra

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Brittle Behavior (fracture)

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II Rheology: Macroscopic deformation behavior

A Elasticity

1 Deformation is reversible when load is removed

2 Stress (σ) is related to strain (ε)

3 Deformation *is not time dependent if load is constant*

4 Examples: Seismic (acoustic) waves, rubber ball

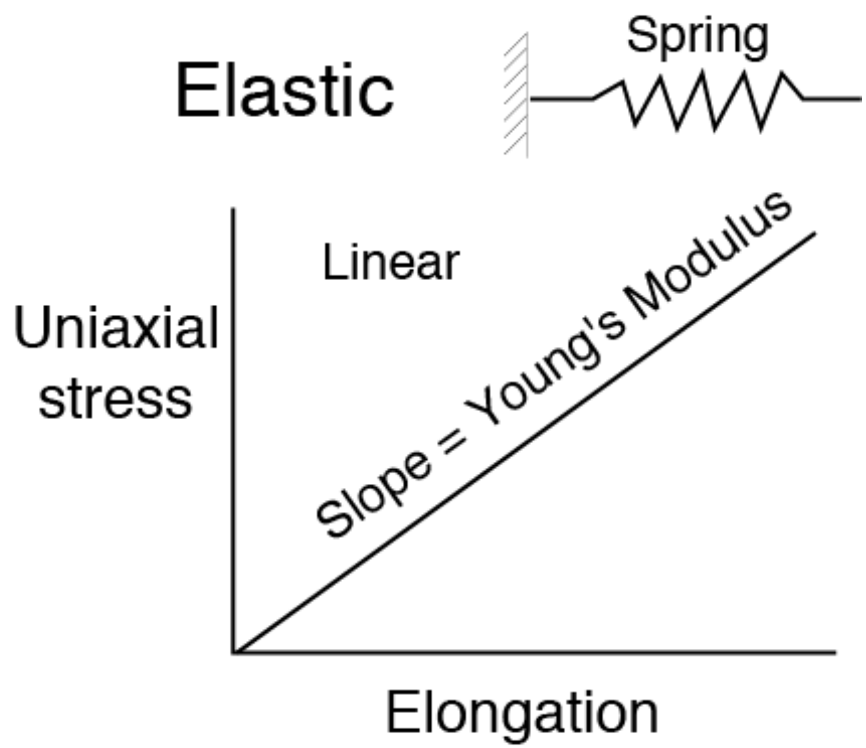
http://www.fordogtrainers.com

20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior

A Elasticity

1. Deformation is reversible when load is removed
2. Stress (σ) is related to strain (ε)
3. Deformation *is not time dependent if load is constant*
4. *Examples: Seismic (acoustic) waves, rubber ball*



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II Rheology: Macroscopic deformation behavior

B Viscosity

1. Deformation is irreversible when load is removed
2. Stress (σ) is related to strain rate ($\dot{\varepsilon}$)
3. Deformation is time dependent if load is constant
4. Examples: Lava flows, corn syrup

http://wholefoodrecipes.net

20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior

B Viscosity

1. Deformation is irreversible when load is removed
2. Stress (σ) is related to strain rate ($\dot{\varepsilon}$)
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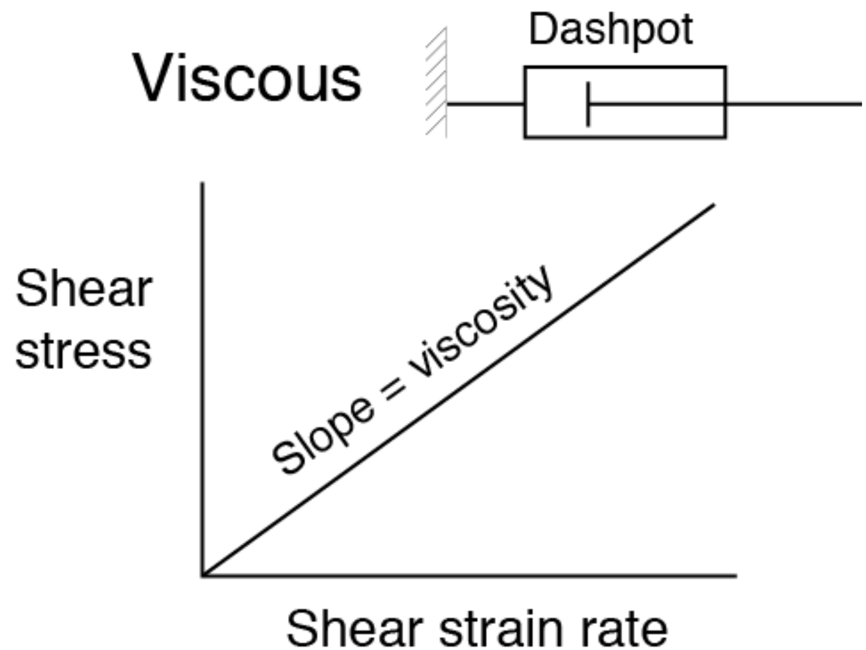
4. Examples: Lava flows, corn syrup



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II Rheology: Macroscopic deformation behavior

C Plasticity

1 No deformation until yield strength is locally exceeded; then irreversible deformation occurs under a constant load

2 Deformation can increase with time under a constant load

3 Examples: plastics, soils

http://www.therapyputty.com/images/stretch6.jpg

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II Rheology: Macroscopic deformation behavior

C Brittle Deformation

1 Discontinuous deformation

2 Failure surfaces separate

http://www.thefeeherytheory.com

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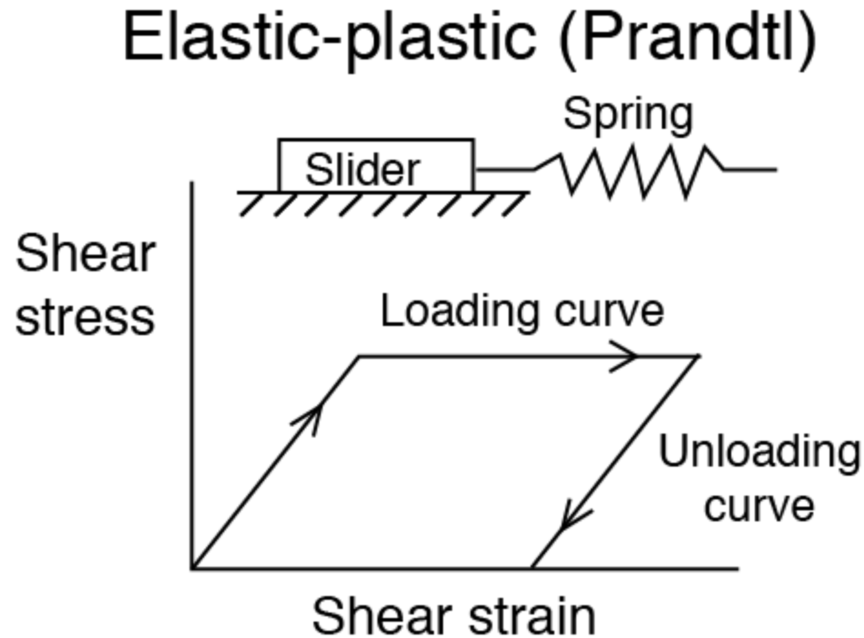
II Rheology: Macroscopic deformation behavior

D Elasto-plastic rheology

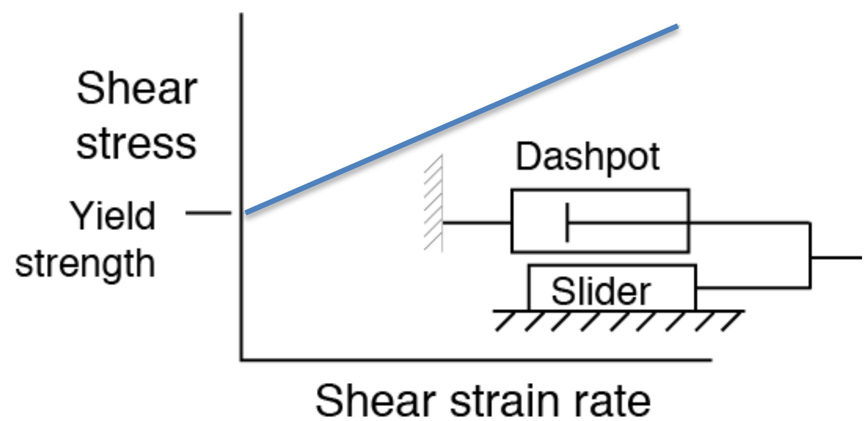


20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior

E Visco-plastic rheology

Visco-plastic (Bingham)



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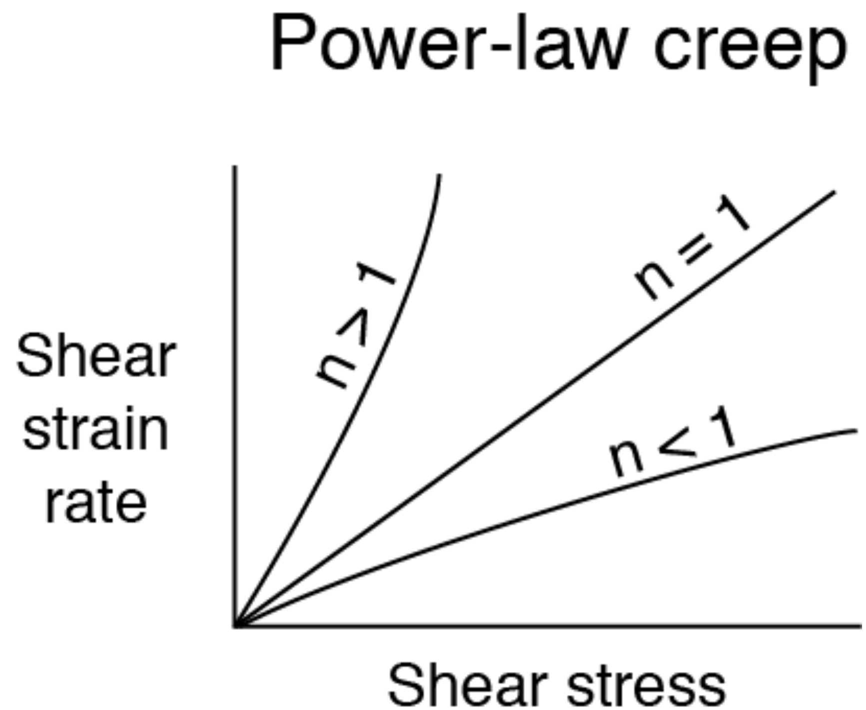
II Rheology: Macroscopic deformation behavior

F Power-law creep

1 $\dot{\mathcal{E}} = (\sigma_1 - \sigma_3)^n e^{(-Q/RT)}$

2 Example: rock salt



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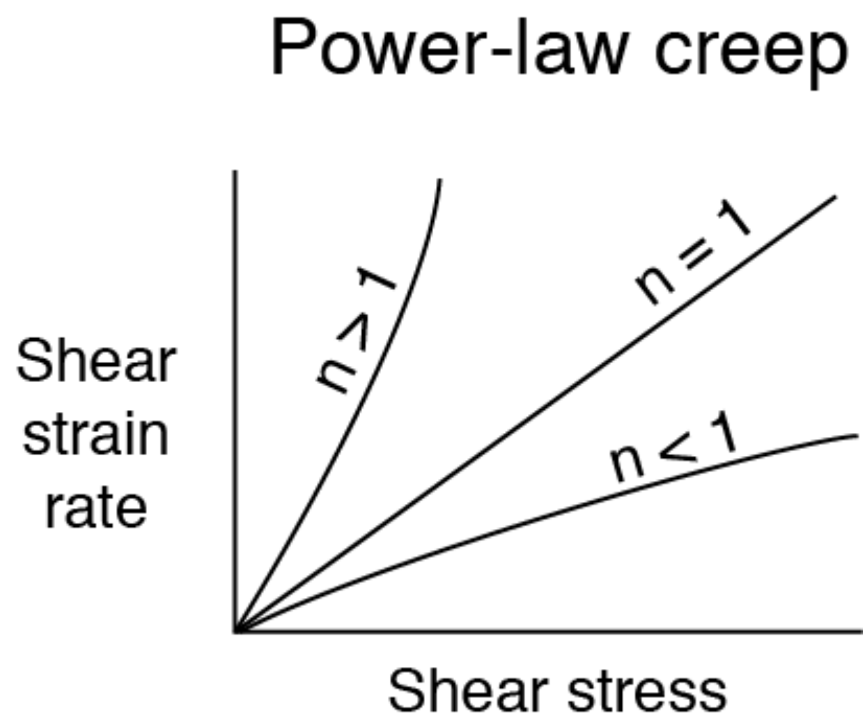
II Rheology: Macroscopic deformation behavior

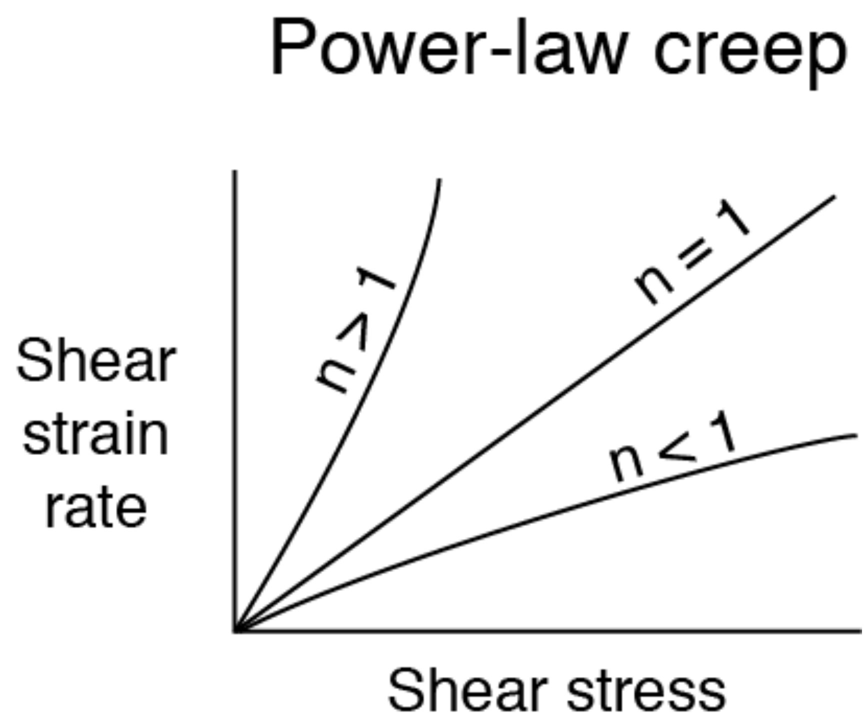
G Linear vs. nonlinear behavior



20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior

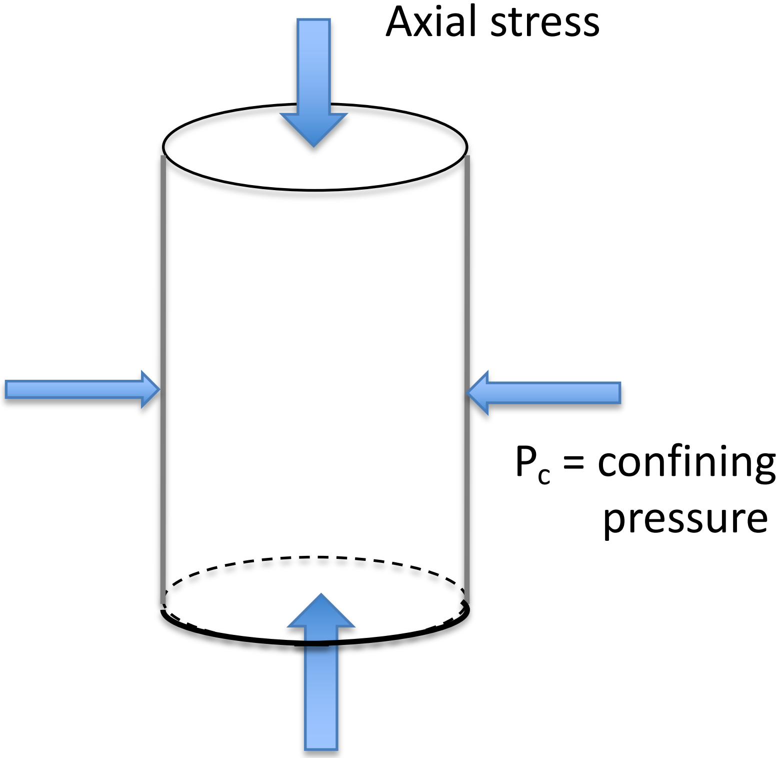
- H Rheology=f (σ_{ij} ,fluid pressure, strain rate, chemistry, temperature)
- I Rheologic equation of real rocks = ?

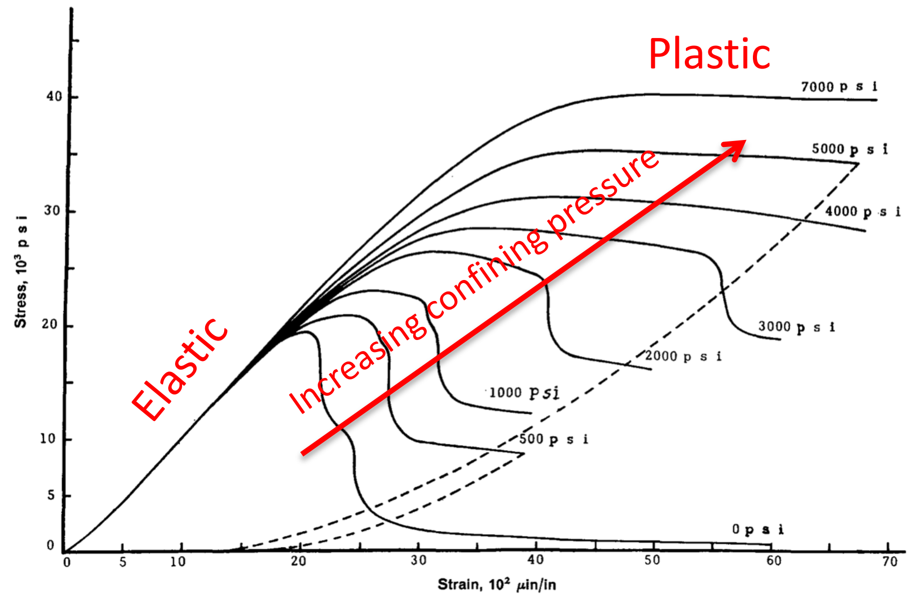


20. Rheology & Linear Elasticity

II Rheology (cont.)

J Experimental results





Compression test data on Tennessee marble II from Wawersik and Fairhurst, 1970

20. Rheology & Linear Elasticity

III Linear elasticity

A Force and displacement of a spring (from Hooke, 1676): F= kx

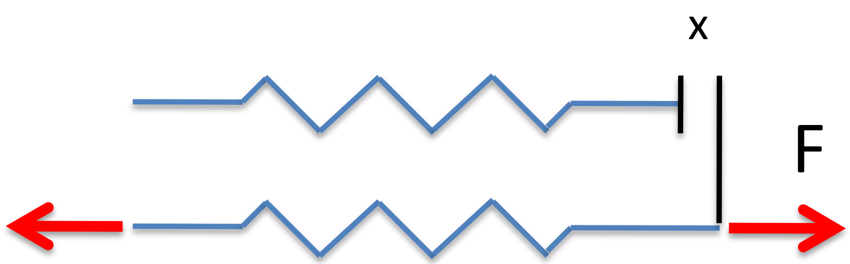
1 F = force

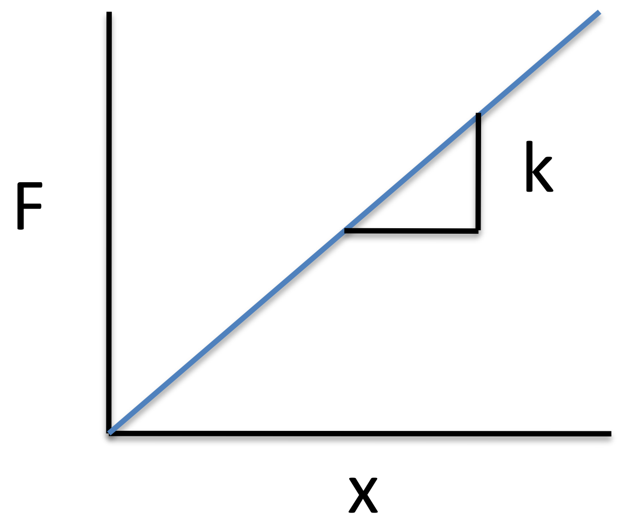
2 k = spring constant

Dimensions:F/L

3 x = displacement

Dimensions: length L)





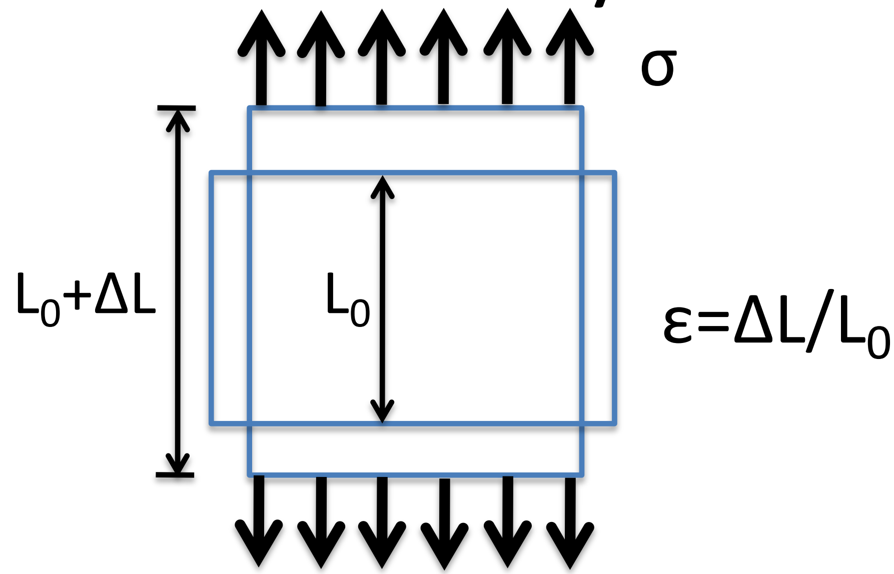
20. Rheology & Linear Elasticity

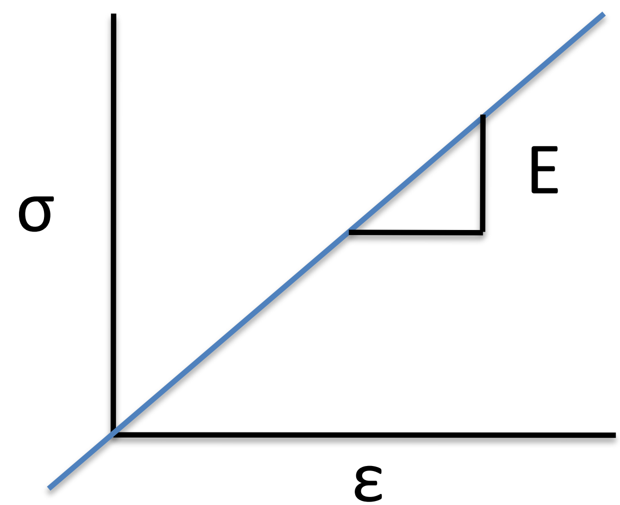
III Linear elasticity (cont.)

- B Hooke's Law for uniaxial stress: $\sigma = E\varepsilon$
  - 1 σ = uniaxial stress
  - 2 E = Young's modulus

    Dimensions: stress
  - 3 ε = strain (elongation)

    Dimensionless





20. Rheology & Linear Elasticity Typical rock moduli and strengths

Rock type	E_{min}	E_{max}	ν_{min}	ν_{max}
	Young's Modulus (GPa)		Poisson's Ratio	
Quartzite	70	105	0.11	0.25
Gneiss	16	103	0.10	0.40
Basalt	16	101	0.13	0.38
Granite	10	74	0.10	0.39
Limestone	1	92	0.08	0.39
Sandstone	10	46	0.10	0.40
Shale	10	44	0.10	0.19
Coal	1.5	3.7	0.33	0.37

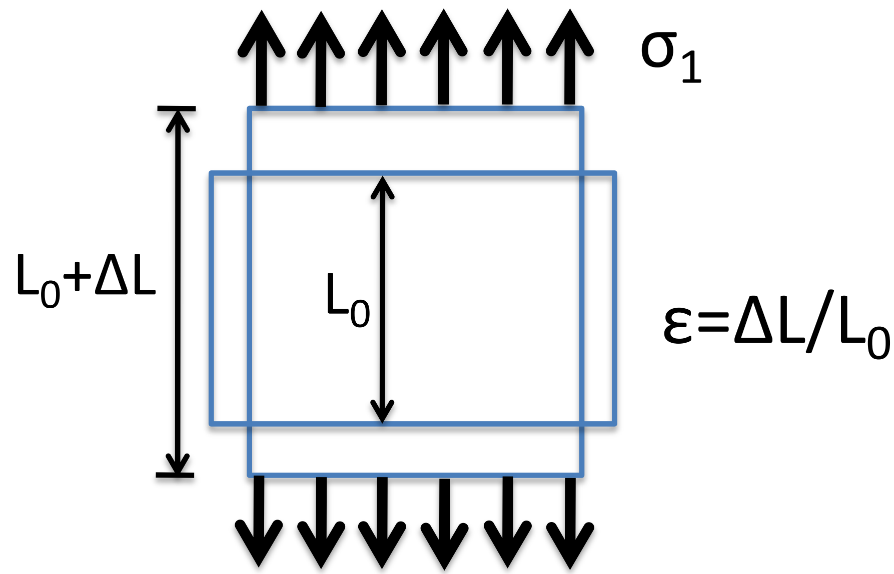
Uniaxial Strengths (MPa)

Rock type	Tensile (low)	Tensile (high)	Comp. (low)	Comp. (high)
Quartzite	17	28	200	304
Gneiss	3	21	73	340
Basalt	2	28	42	355
Granite	3	39	30	324
Limestone	2	40	48	210
Sandstone	3	7	40	179
Shale	2	5	36	172
Coal	1.9	3.2	14	30

20. Rheology & Linear Elasticity

III Linear elasticity (cont.)

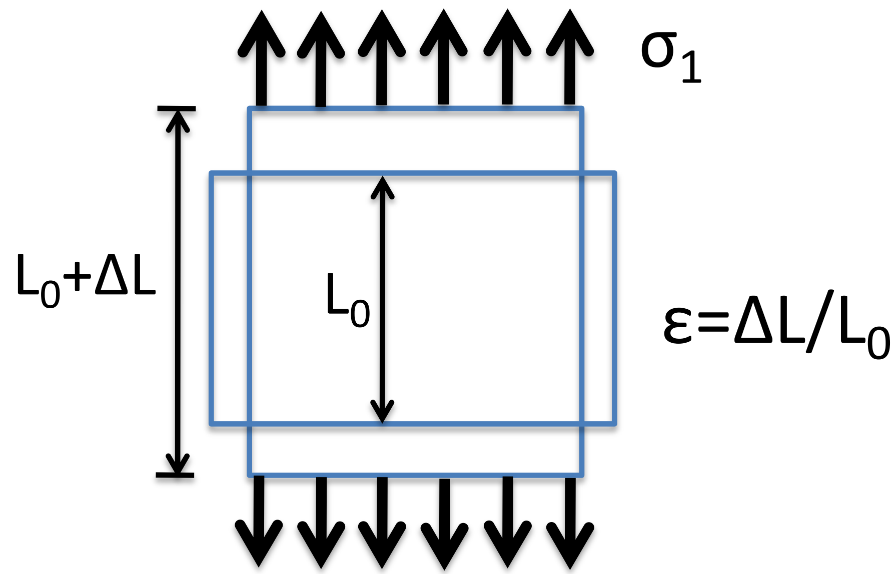
- B Hooke's Law for uniaxial stress (cont.): $\varepsilon_1 = \sigma_1/E$
  - 1 $\sigma_2 = \sigma_3 = 0$
  - 2 $\varepsilon_2 = \varepsilon_3 = -\nu\varepsilon_1$
    - a ν = Poisson's ratio
    - b ν is dimensionless
    - c Strain in one direction tends to induce strain in another direction



20. Rheology & Linear Elasticity

III Linear elasticity (cont.)

C Linear elasticity in 3D for homogeneous isotropic materials



By superposition:

1 $\varepsilon_{xx} = \sigma_{xx}/E - (\sigma_{yy}+\sigma_{zz})(\nu/E)$

2 $\varepsilon_{yy} = \sigma_{yy}/E - (\sigma_{zz}+\sigma_{xx})(\nu/E)$

3 $\varepsilon_{zz} = \sigma_{zz}/E - (\sigma_{xx}+\sigma_{yy})(\nu/E)$

20. Rheology & Linear Elasticity

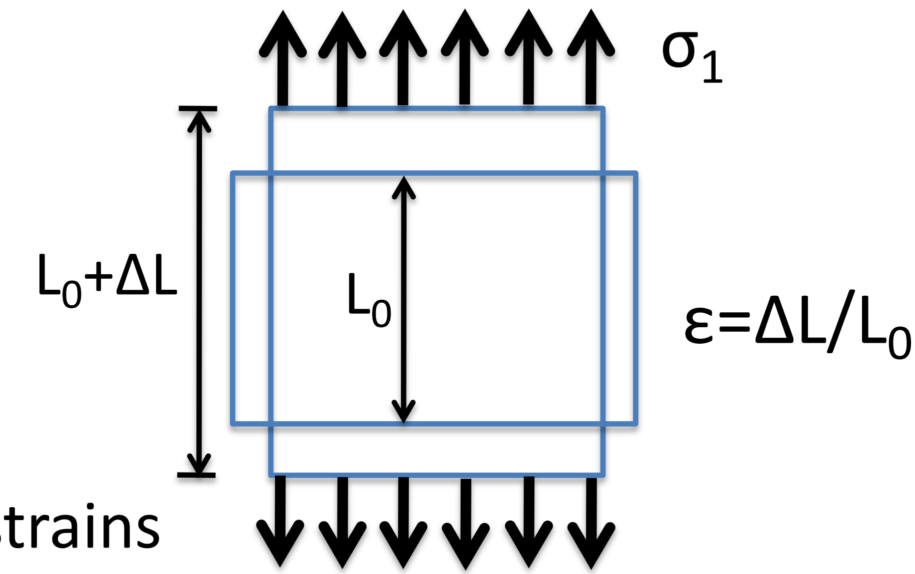
III Linear elasticity (cont.)

C Linear elasticity in 3D for homogeneous isotropic materials (cont.)

4 Directions of principal stresses and principal strains coincide

5 Extension in one direction can occur without tension

6 Compression in one direction can occur without shortening

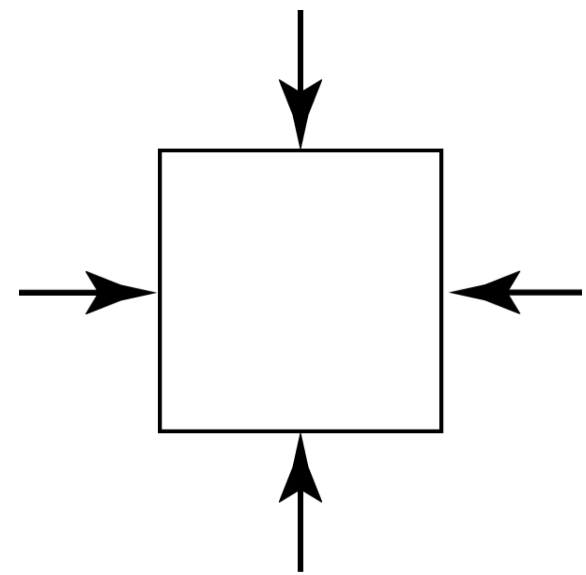


20. Rheology & Linear Elasticity

III Linear elasticity

E Special cases

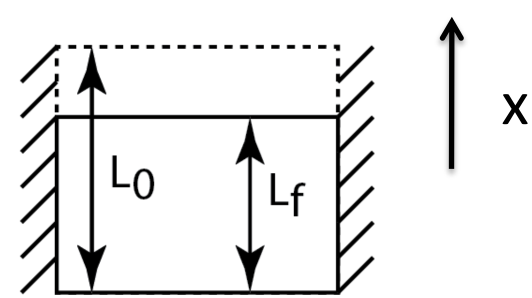
1 Isotropic (hydrostatic) stress



a $\sigma_1 = \sigma_2 = \sigma_3$

b No shear stress

2 Uniaxial strain

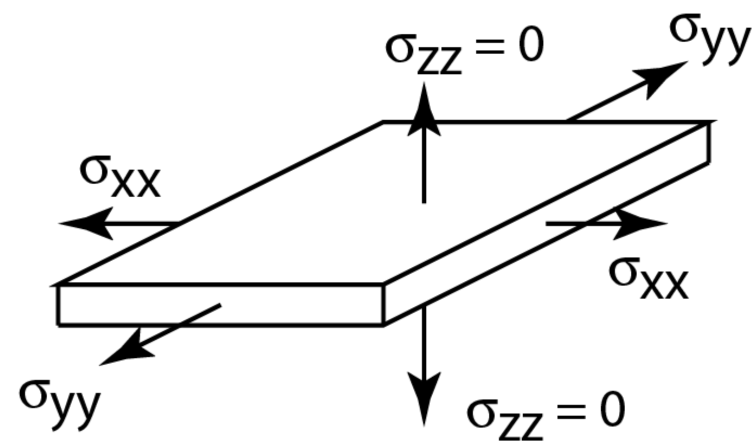


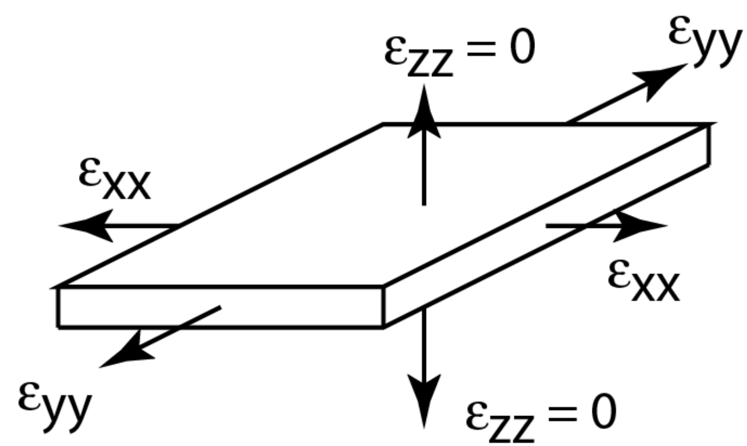
a $\varepsilon_{xx} = \varepsilon_1 \neq 0$

b $\varepsilon_{yy} = \varepsilon_{zz} = 0$

20. Rheology & Linear Elasticity

III Linear elasticity
- E Special cases
  - 3 Plane stress (2D)
    - $\sigma_z = 0$
    - "Thin plate" case
  - 4 Plane strain (2D)
    - $\varepsilon_z = 0$
    - a Displacement in z-direction is constant (e.g., zero)
    - b Plate is confined between rigid walls
    - c "Thick plate" case





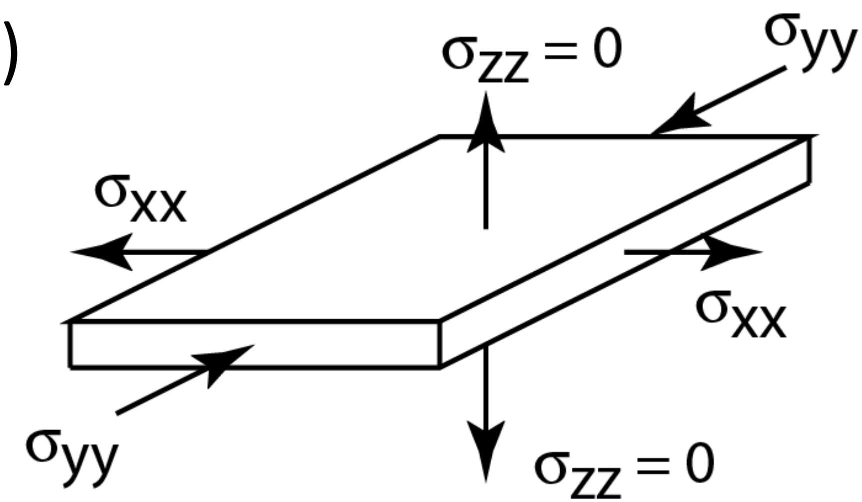
20. Rheology & Linear Elasticity

III Linear elasticity

E Special cases

5 Pure shear stress (2D)

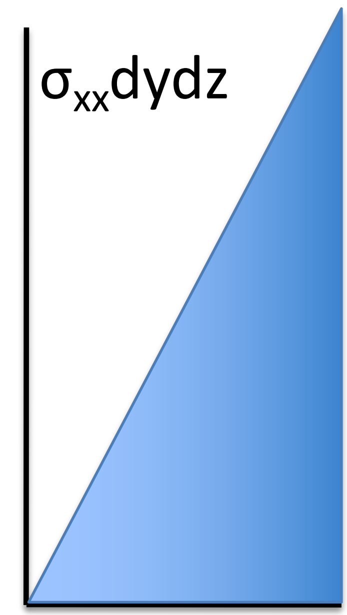
$\sigma_{xx} = -\sigma_{yy}$; $\sigma_{zz} = 0$



20. Rheology & Linear Elasticity

III Linear elasticity

F Strain energy (W_0) for uniaxial stress



1 $$W = \int_0^u F\,du = \int_0^{du} (\sigma_{xx} dydz)\left(\frac{du}{dx}\right) dx$$

2 $W = (1/2)(\sigma_{xx}dydz)\ (\varepsilon_{xx}dx)$

3 $W = (1/2)\ (\sigma_{xx}\varepsilon_{xx})\ (dxdydz)$

4 $W_0 = W/(dxdydz)$

W_0 = strain energy density

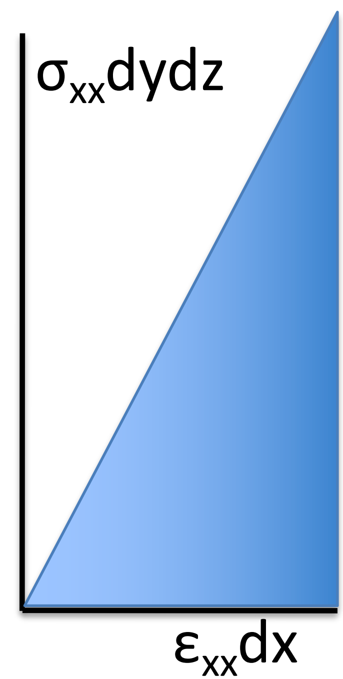
5 $W_0 = (1/2)(\sigma_{xx}\varepsilon_{xx})$

20. Rheology & Linear Elasticity

III Linear elasticity

G Strain energy (W_0) in 3D

$$W_0 = \quad (1/2)(\sigma_1\varepsilon_1+\sigma_2\varepsilon_2+\sigma_3\varepsilon_3)$$



20. Rheology & Linear Elasticity

III Linear elasticity

D Relationships among different elastic moduli

1 $G = \mu$ = shear modulus

$G = E/(2[1+\nu])$

$\varepsilon_{xy} = \sigma_{xy}/2G$

2 λ = Lame' constant

$\lambda = E\nu/([1 + \nu][1 - 2\nu])$

3 K = bulk modulus

$K = E/(3[1 - 2\nu])$

4 β = compressibility

$\beta = 1/K$

$\Delta = \varepsilon_{xx}+\varepsilon_{yy}+ \varepsilon_{zz}= -p/K$

p = pressure

5 P-wave speed: V_p

$$V_p = \sqrt{\left(K + \frac{4}{3}\mu\right)\Big/\rho}$$

6 S-wave speed: V_s

$$V_s = \sqrt{\mu/\rho}$$