17. Mohr Circle for Tractions

I Main Topics

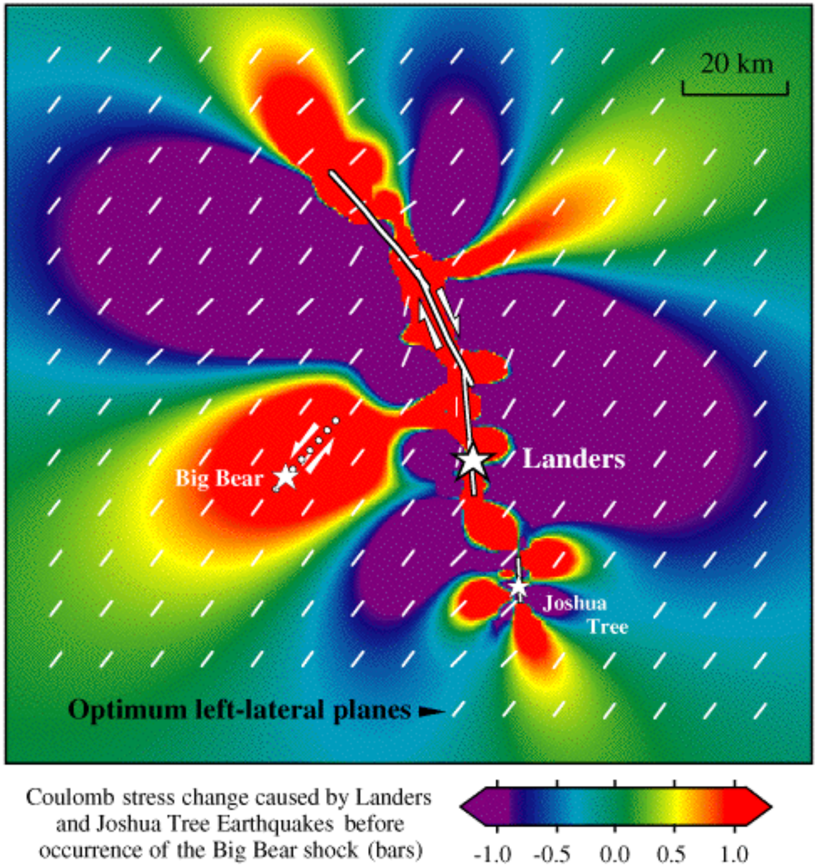
A Stresses vs. tractions

B Mohr circle for tractions

C Example

17. Mohr Circle for Tractions

- From King et al., 1994 (Fig. 11)
- Coulomb stress change caused by the Landers rupture. The left-lateral ML=6.5 Big Bear rupture occurred along dotted line 3 hr 26 min after the Landers main shock. The Coulomb stress increase at the future Big Bear epicenter is 2.2-2.9 bars.



http://earthquake.usgs.gov/research/modeling/papers/landers.php

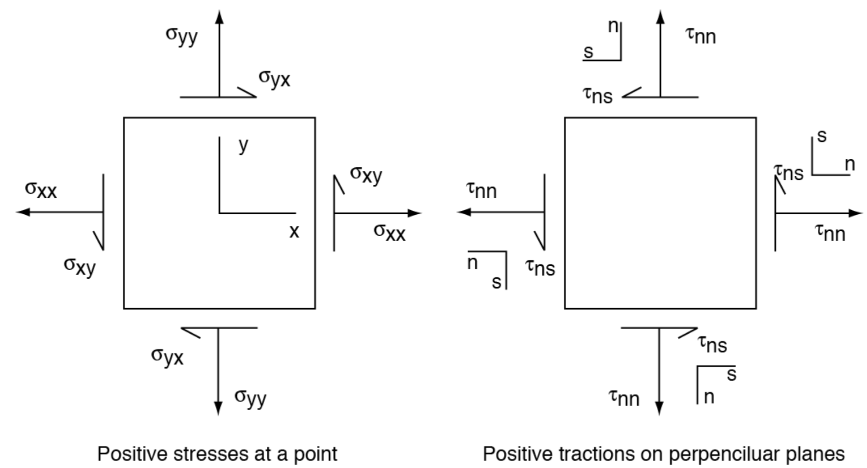
17. Mohr Circle for Tractions

II Stresses vs. tractions

A Similarities between stresses and tractions

1 Same dimensions (force per unit area)

2 The normal stress acting on a plane matches the normal traction



Note the use of double subscripts here on the tractions;
This unconventional

17. Mohr Circle for Tractions

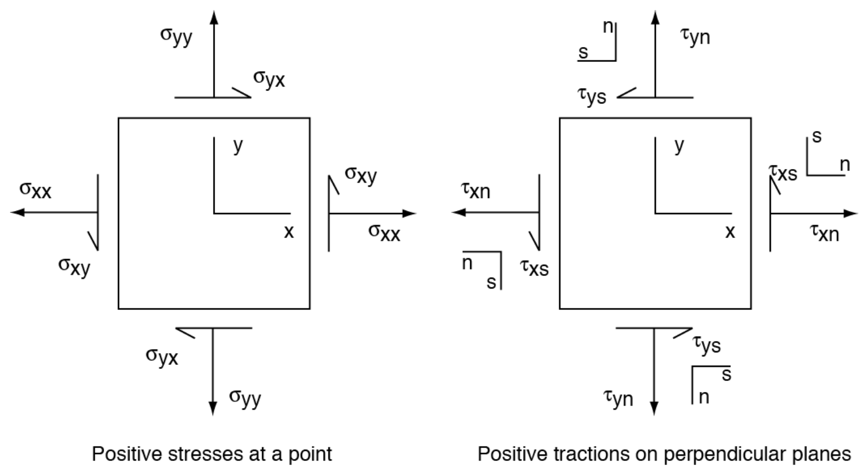
II Stresses vs. tractions (cont.)

B Differences between stresses and tractions

1 Stresses are tensor quantities and tractions are vectors.

2 The stress state is defined at a point using a fixed reference frame, whereas a traction is defined on a plane with a reference frame that floats with the plane.

3 Shear stress components on perpendicular planes have the same sign, whereas shear tractions on perpendicular planes have opposite signs.



Note that the first subscript (*n*) on the tractions has been replaced by "x" and "y" here

17. Mohr Circle for Tractions

III Mohr circle for tractions

A $\tau_n = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta$

B $\tau_s = (\sigma_2 - \sigma_1)\sin\theta\cos\theta$

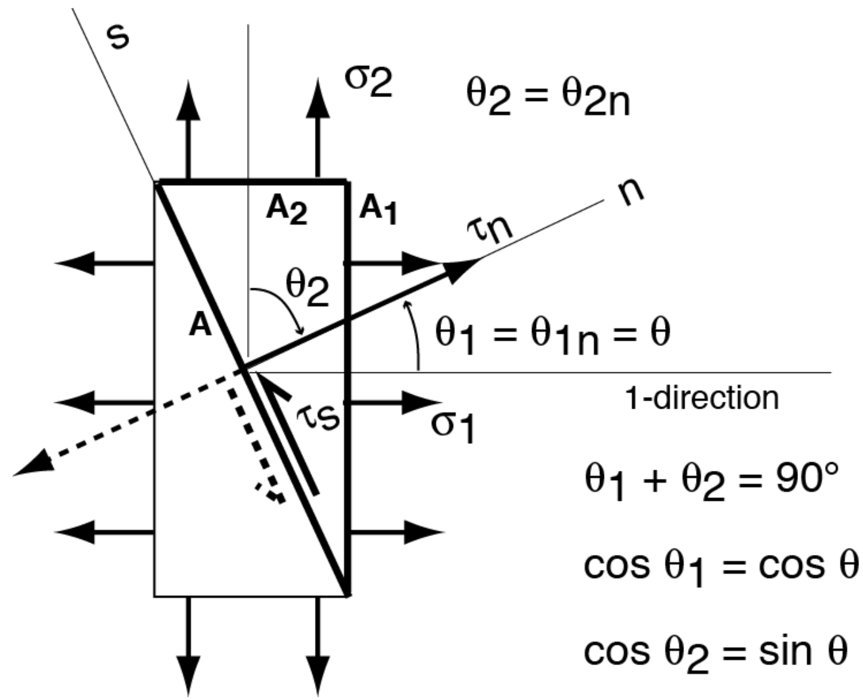
Now $\cos^2\theta = (1/2)(1+\cos 2\theta)$

$\sin^2\theta = (1/2)(1-\cos 2\theta)$

$\sin\theta\cos\theta = (1/2)(\sin 2\theta)$

C $\tau_n = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta$

D $\tau_s = \frac{-(\sigma_1-\sigma_2)}{2}\sin 2\theta$



17. Mohr Circle for Tractions

III Mohr circle for tractions

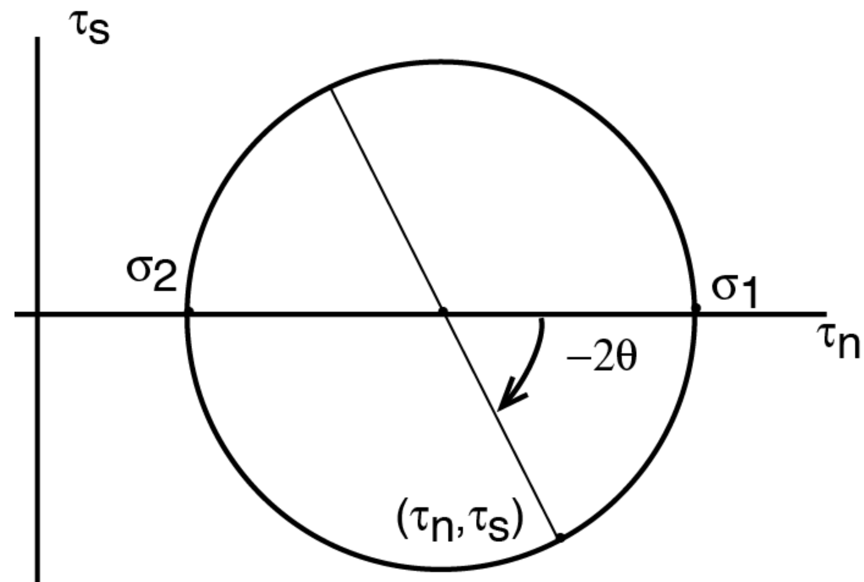
C $$\tau_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

D $$\tau_s = \frac{-(\sigma_1 - \sigma_2)}{2}\sin 2\theta$$

Now $$c = \frac{\sigma_1 + \sigma_2}{2} \qquad r = \frac{\sigma_1 - \sigma_2}{2}$$

E $$\tau_n = c + r\cos(-2\theta)$$

F $$\tau_s = r\sin(-2\theta)$$



$\tau_n = ((\sigma_1 + \sigma_2)/2) + ((\sigma_1 - \sigma_2)/2) \cos(-2\theta)$

$\tau_s = ((\sigma_1 - \sigma_2)/2) \sin(-2\theta)$

Equations of a Mohr circle for tractions
Relate tractions on planes of different orientation
c is mean normal stress (traction)
r is maximum shear traction (the circle radius)
σ_1 is the most tensile stress
σ_2 is the least tensile stress

17. Mohr Circle for Tractions

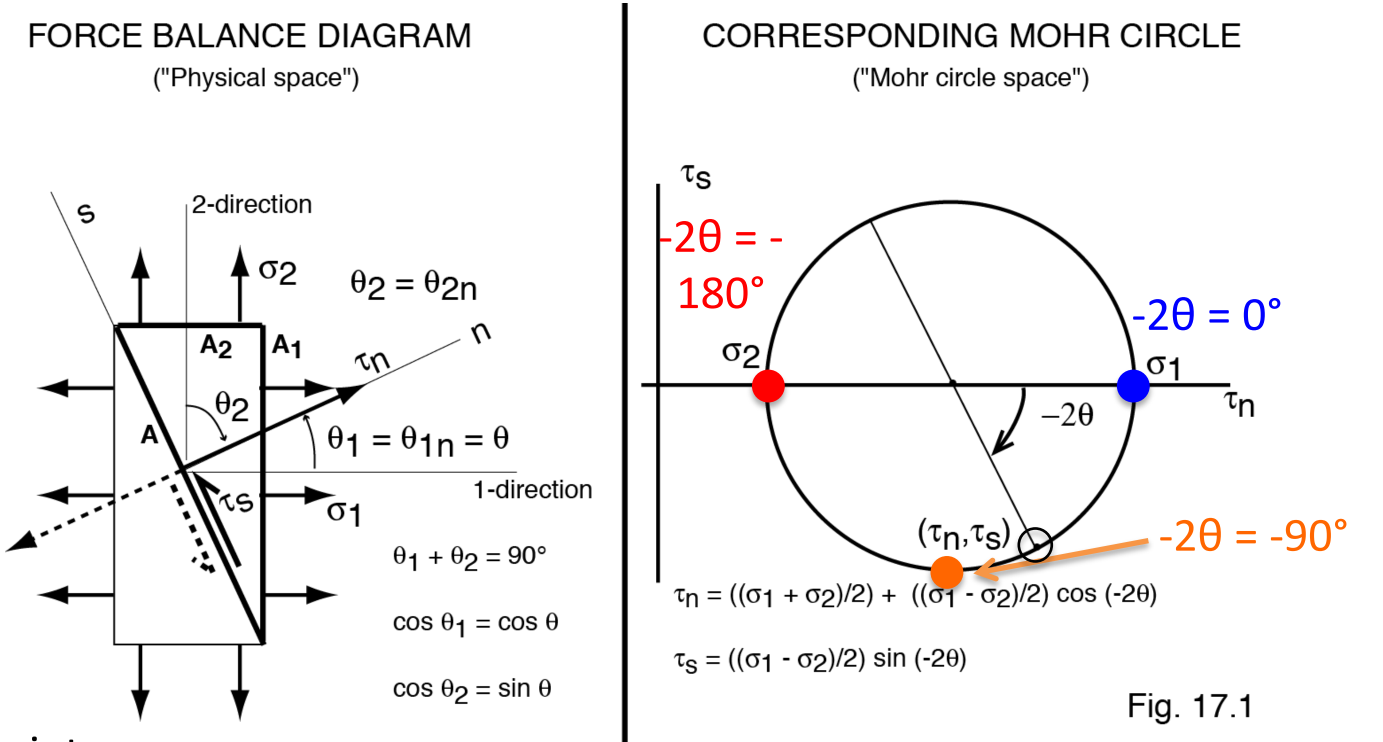


Fig. 17.1

G Key points

1. $\theta = \theta_{1n}$ is the angle between <u>the normal</u> to the plane σ_1 acts on and <u>the normal</u> to the plane of interest
2. If positive θ is counterclockwise in "physical space", -2θ is clockwise in "Mohr circle space"

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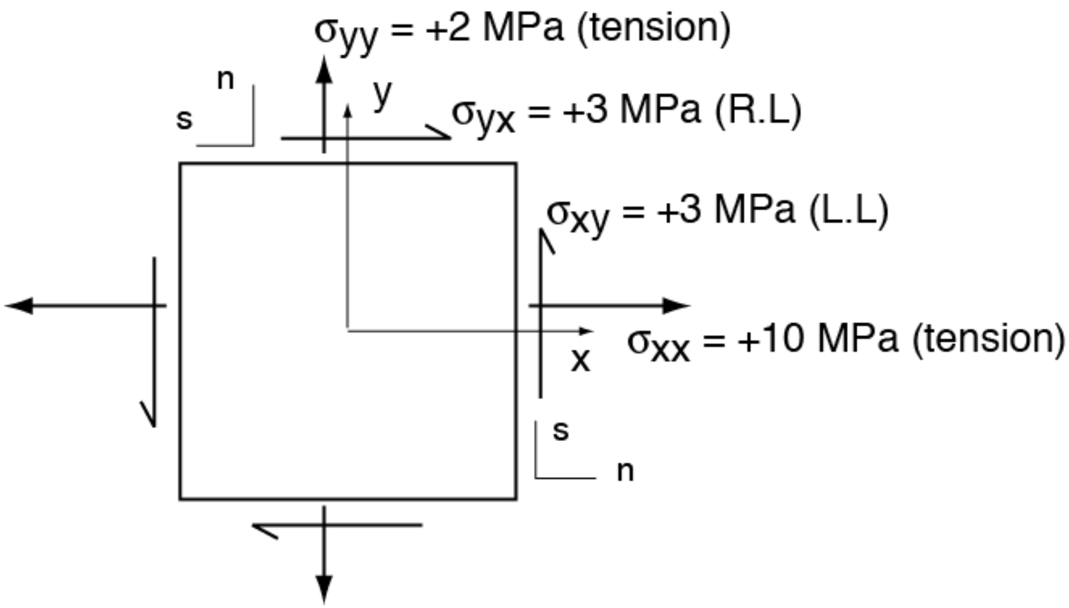


Stresses		Tractions	
σ_{xx}	+10 MPa	τ_{xn}	+10 MPa
σ_{xy}	+3 MPa	τ_{xs}	+3 MPa
σ_{yx}	+3 MPa	τ_{ys}	-3 MPa
σ_{yy}	+2 MPa	τ_{yn}	+2 MPa

Note that the magnitude of the normal stresses and normal tractions are equal. So $\tau_1 = \sigma_1$ below.

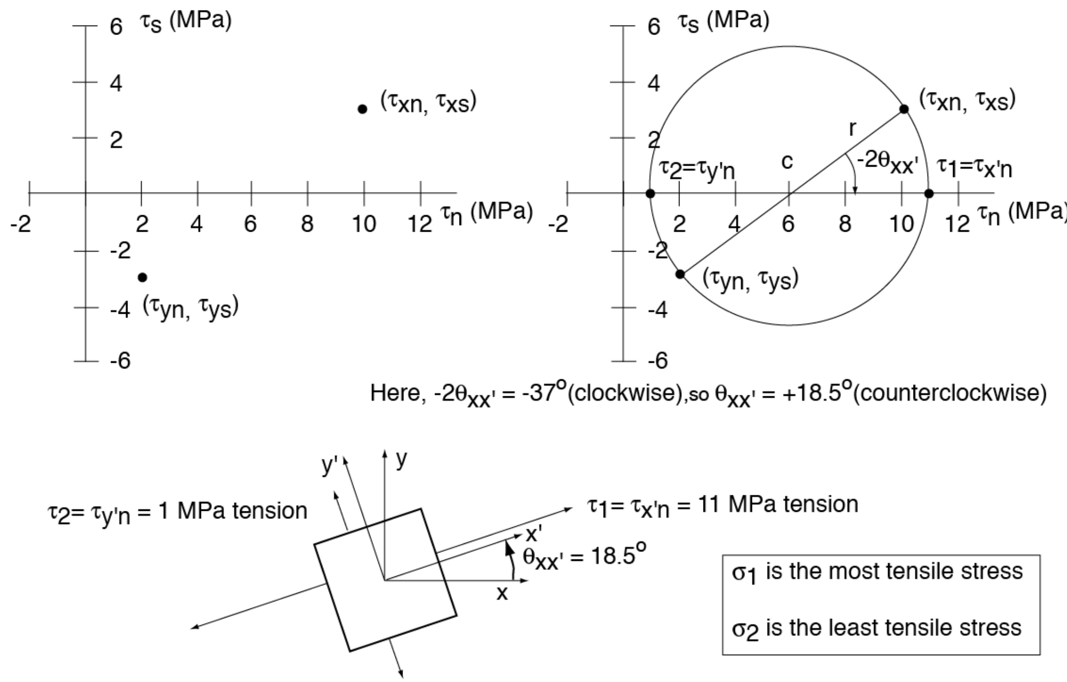
Example 1 using Mohr circle to find principal stresses

- *Suppose σ_{xx} = +10 MPa (tension), σ_{xy} = +3 MPa (left lateral shear), σ_{yy} = +2 MPa (tension), and σ_{yx} = +3 MPa (right lateral shear).*

A) Draw a box in a reference frame and clearly label the stresses on its sides; *this is a critically important step*.

B) Determine the stresses and tractions on the faces of the box. Here, we use the tensor "on-in" convention.

17. Mohr Circle for Tractions



C Plot and label the points on a set of labelled τ_n, τ_s axes.

D Draw the Mohr circle through the points by finding the center (c) and radius (r) of the circle.

E Label the principal magnitudes τ_1 and τ_2 ($\tau_1 > \tau_2$); they come from the intersection of the circle with the normal stress (τ_n) axis.

F Assign reference axes to the principal directions; I chose x' for the τ_1-direction.

G Label the negative double angle between the traction pair that act on a plane with a known normal direction (here, x or y) and the traction pair that act on a plane with an unknown direction (e.g., x').

H Draw and label a new reference frame and box showing the principal stresses, making sure to use the double angle relationship correctly.