16. STRESS AT A POINT

I Main Topics

A Stress vector (traction) on a plane

B Stress at a point

C Principal stresses

D Transformation of principal stresses to tractions in 2D

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http://hvo.wr.usgs.gov/kilauea/update/images.html

16. STRESS AT A POINT

I Stress vector (traction) on a plane

A $\vec{\tau} = \lim_{A \to 0} \vec{F} / A$

B Traction vectors can be added as vectors

C A traction vector can be resolved into normal (τ_n) and shear (τ_s) components

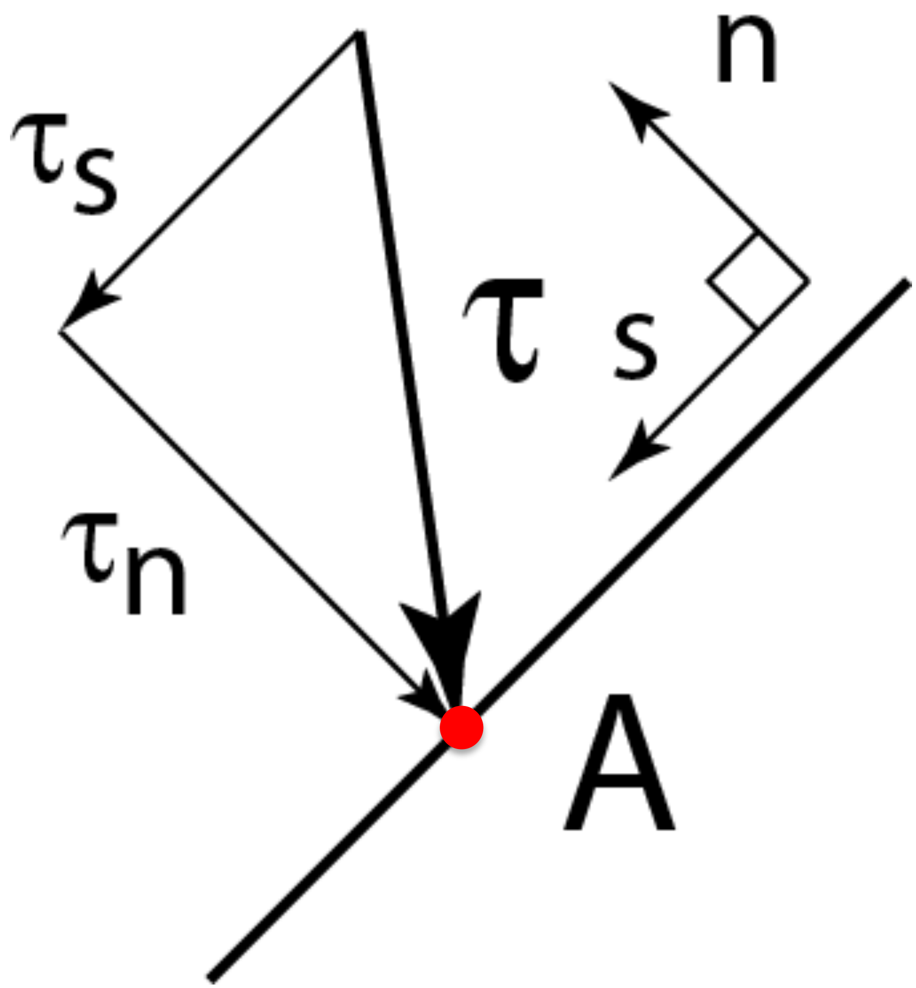
1 A normal traction (τ_n) acts perpendicular to a plane

2 A shear traction (τ_s) acts parallel to a plane

D Local reference frame

1 The n-axis is normal to the plane

2 The s-axis is parallel to the plane



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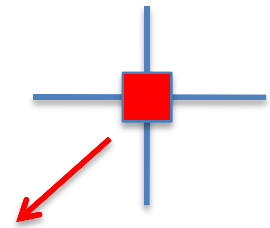


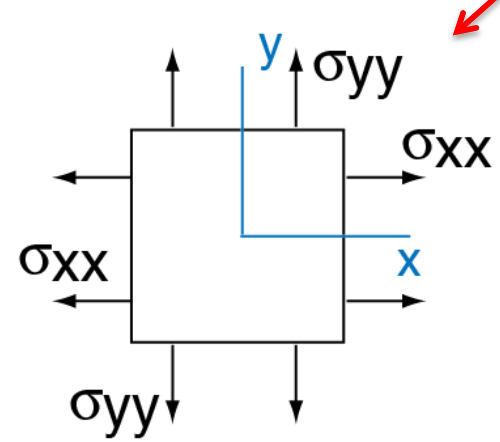
III Stress at a point (cont.)

A Stresses refer to balanced internal "forces (per unit area)". They differ from force vectors, which, if unbalanced, cause accelerations

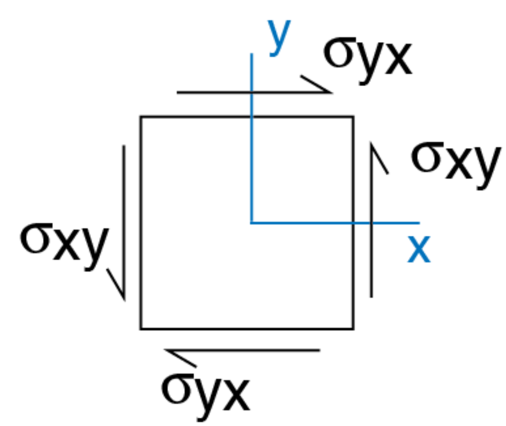
B "On -in convention": The stress component σ_{ij} acts on the plane normal to the i-direction and acts in the j-direction

1 Normal stresses: i=j

2 Shear stresses: i≠j



Normal stresses



Shear stresses

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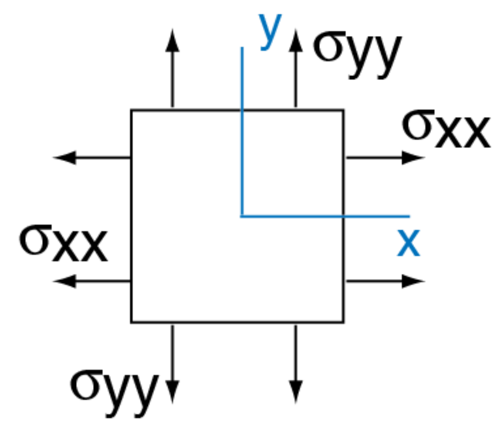
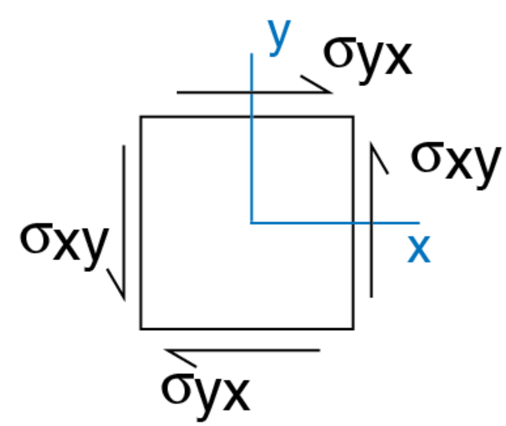
III Stress at a point

- C Dimensions of stress: force/unit area
- D Convention for stresses
  1. Tension is positive
  2. Compression is negative
  3. Follows from on-in convention
  4. Consistent with most mechanics books
  5. Counter to most geology books



Normal stresses



Shear stresses

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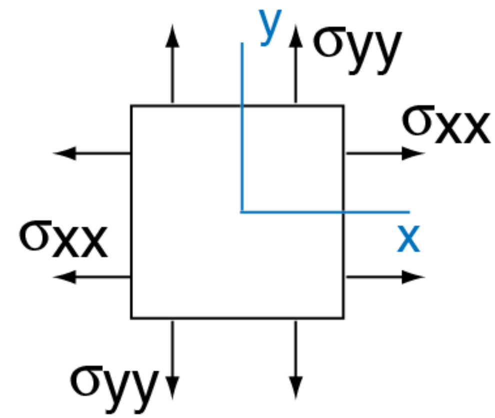
III Stress at a point

C $\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$ 2-D
4 components

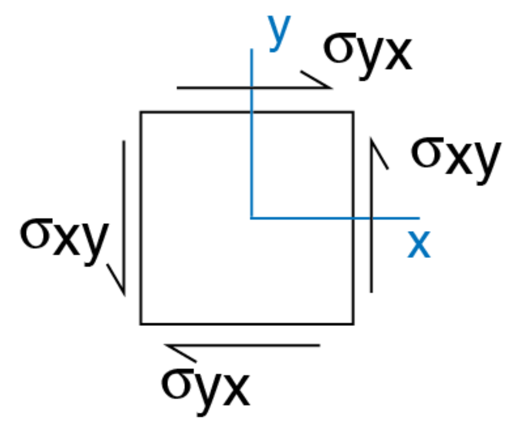
D $\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$ 3-D
9 components

E In nature, the state of stress can (and usually does) vary from point to point

F For rotational equilibrium, $\sigma_{xy} = \sigma_{yx}$, $\sigma_{xz} = \sigma_{zx}$, $\sigma_{yz} = \sigma_{zy}$



Normal stresses



Shear stresses

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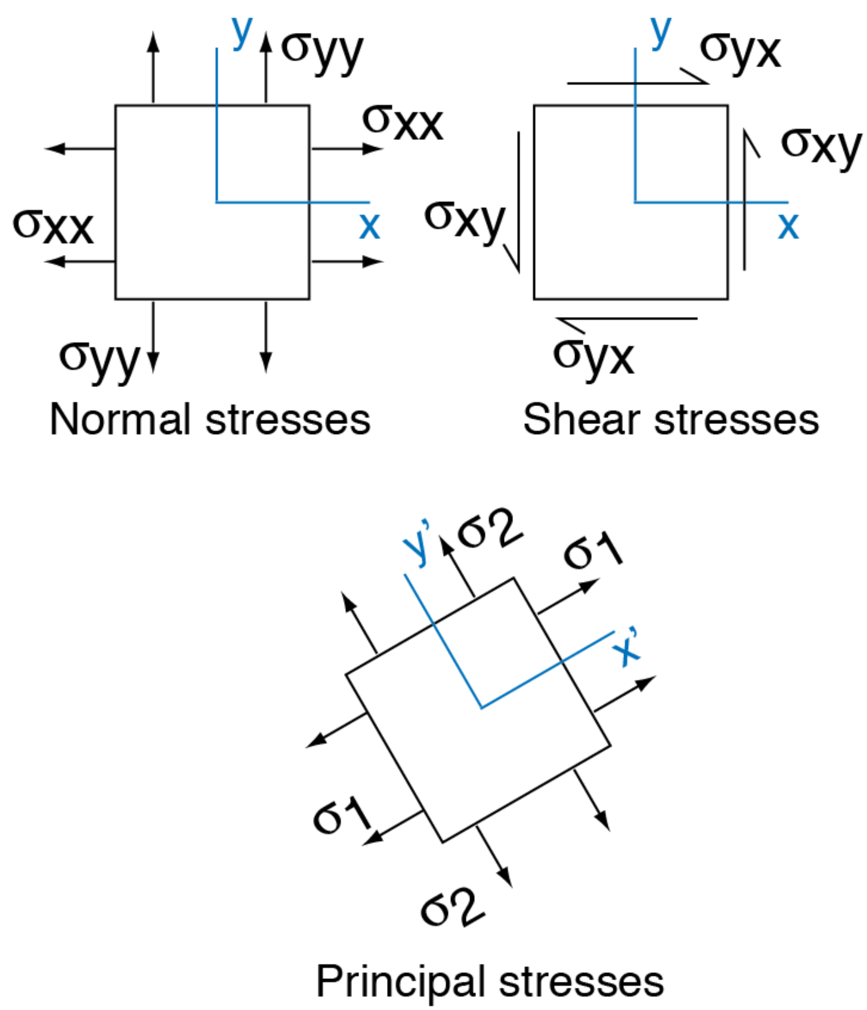
IV Principal Stresses (these have magnitudes and orientations)

- A Principal stresses act on planes which feel no shear stress
- B The principal stresses are normal stresses.
- C Principal stresses act on perpendicular planes
- D The maximum, intermediate, and minimum principal stresses are usually designated σ_1, σ_2, and σ_3, respectively.
- E Principal stresses have a single subscript.



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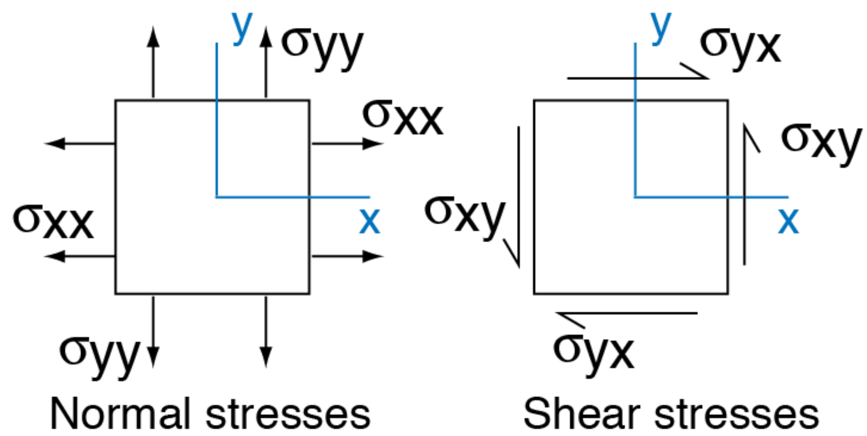
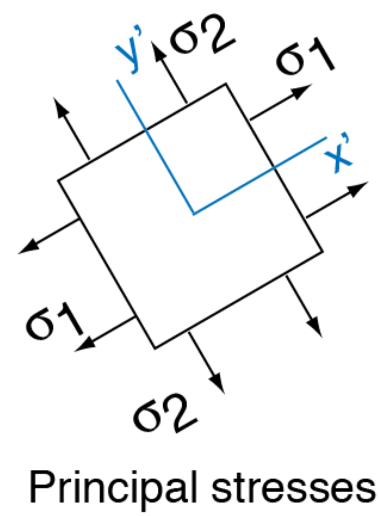
IV Principal Stresses (cont.)

F <u>Principal stresses represent the stress state most simply</u>



G $$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$ 2-D
2 components

H $$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$ 3-D
3 components



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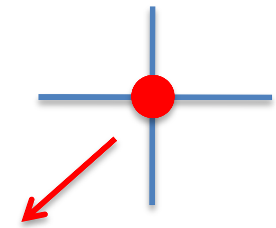


V Transformation of principal stresses to tractions in 2D

A Description of terms

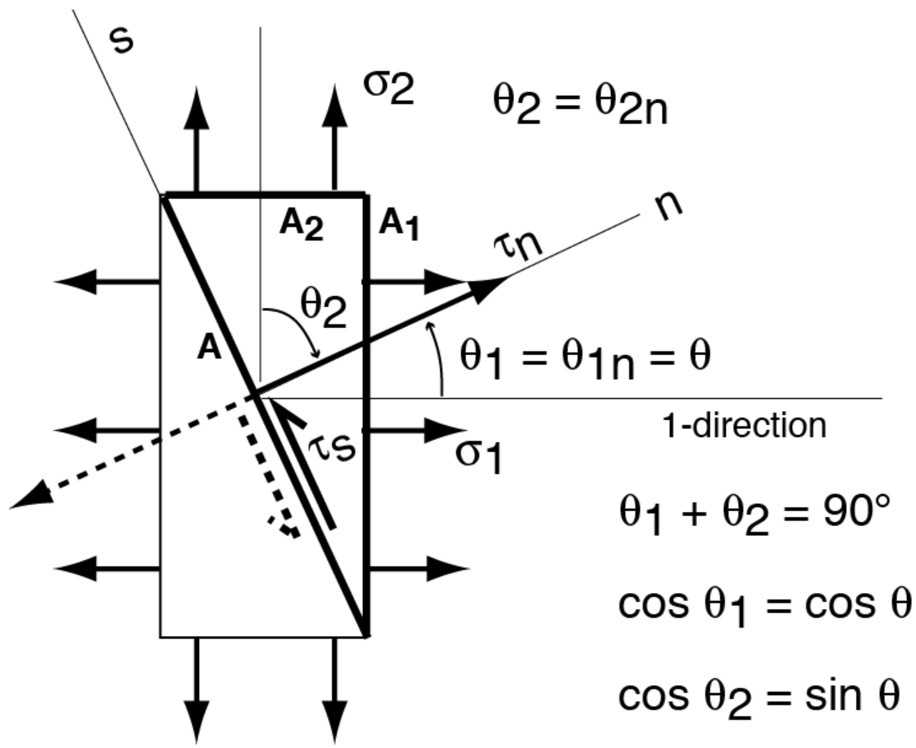
1 Three planes A, A_1, and A_2 form the sides of a triangular prism; these have normals in the n-, 1-, and -2-directions, respectively.

2 Plane A_1 is acted on by known normal stress σ_1.

3 Plane A_2 is acted on by known normal stress σ_2.

4 The n-direction is at angle θ_1 (=θ) with respect to the 1-direction, and at angle θ_2 with respect to the 2-direction.

5 The s-direction is at a counter-clockwise 90° angle relative to the n-direction (like y and x).



16. STRESS AT A POINT

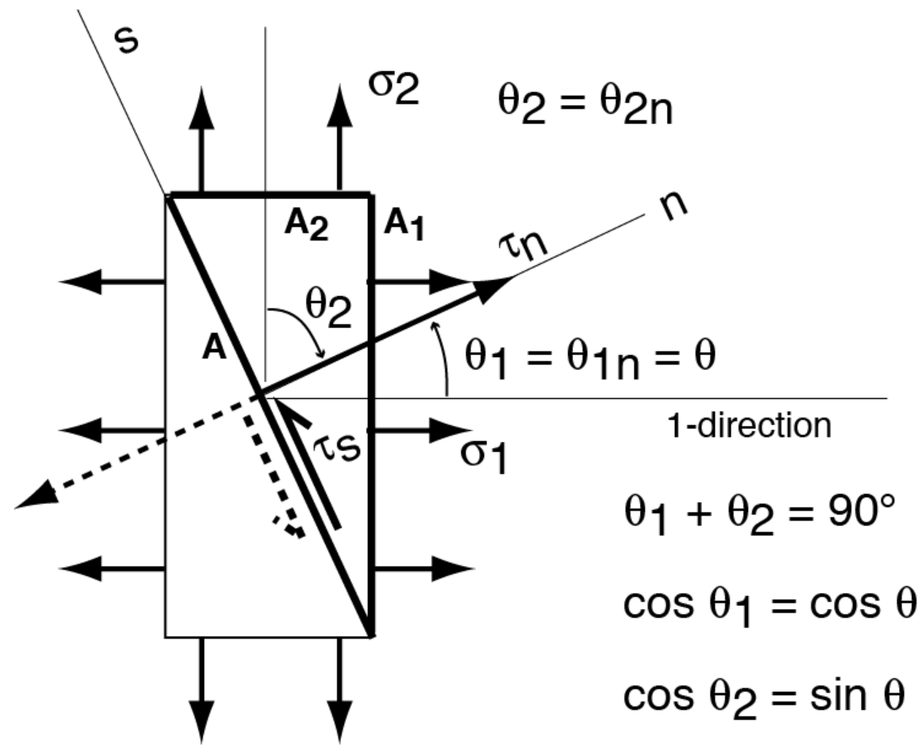
V Transformation of principal stresses to tractions in 2D

B Approach

Find weighting factors that determine contributions of known stresses to desired tractions and sum contributions

1 $\tau_n = w_{n1}\,\sigma_1 + w_{n2}\,\sigma_2$

2 $\tau_s = w_{s1}\,\sigma_1 + w_{s2}\,\sigma_2$



16. STRESS AT A POINT

C Contribution of σ_1 (on face A_1 of area A_1) to τ_n (on face A of area A)

Start with the definition of traction:

1 $\tau_n^{(1)} = F_n^{(1)} / A$

Find unknowns $F_n^{(1)}$ and A from knowns σ_1 and θ.

First find the force F_1 associated with σ_1

2 $F_1 = \sigma_1 A_1$ Force = (stress)(area)

Find $F_n^{(1)}$, the component of F_1 in the n-direction

3 $F_n^{(1)} = F_1 \cos\theta_1$

Find A in terms of A_1

$A_1 = A \cos\theta_1$ (see diagram at right)

4 $A = A_1 / \cos\theta_1$

Contribution of σ_1 to τ_n:

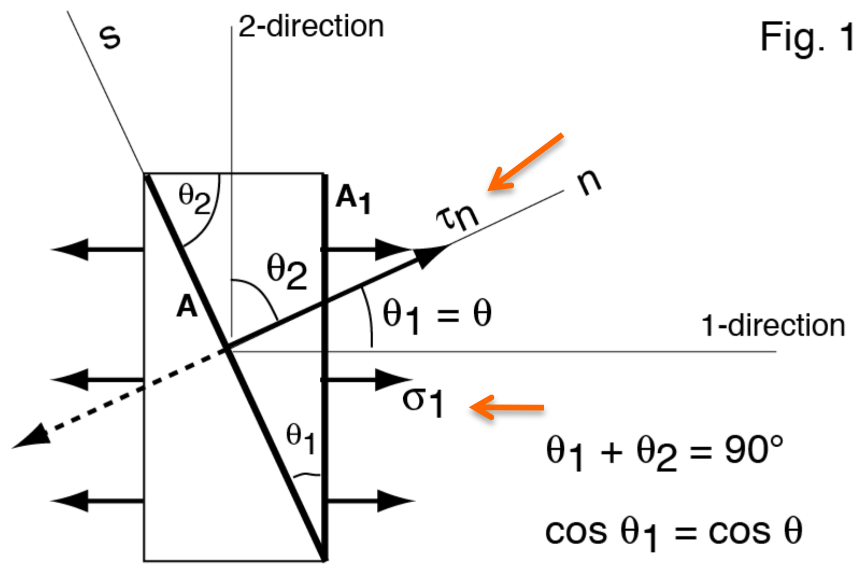
5a $\tau_n^{(1)} = F_n^{(1)} / A = F_1 \cos\theta_1 / (A_1 / \cos\theta_1)$

5b $\boxed{\tau_n^{(1)} = (F_1 / A_1) \cos\theta_1 \cos\theta_1 = \sigma_1 \cos\theta_1 \cos\theta_1}$

Weighting factor w_{n1}

6 $w_{n1} = \cos\theta_1 \cos\theta_1 = \cos\theta \cos\theta$

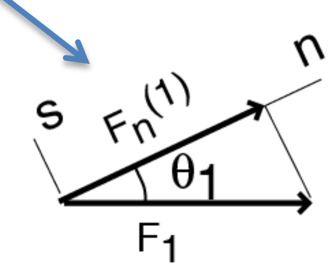
Fig. 16.1



$\theta_1 + \theta_2 = 90°$

$\cos\theta_1 = \cos\theta$

$\cos\theta_2 = \sin\theta$



16. STRESS AT A POINT

D Contribution of σ_2 (on face A_2 of area A_2) to τ_n (on face A of area A)

Start with the definition of traction:

1 $\tau_n^{(2)} = F_n^{(2)}) / A$

Find unknowns $F_n^{(2)}$ and A from knowns σ_2 and θ.

First find the force F_2 associated with σ_2

2 $F_2 = \sigma_2 A_2$ Force = (stress)(area)

Find $F_n^{(2)}$, the component of F_2 in the n-direction

3 $F_n^{(2)} = F_2 \cos\theta_2$

Find A in terms of A_2

$A_2 = A \cos\theta_2$ (see diagram at right)

4 $A = A_2 / \cos\theta_2$

Contribution of σ_2 to τ_n:

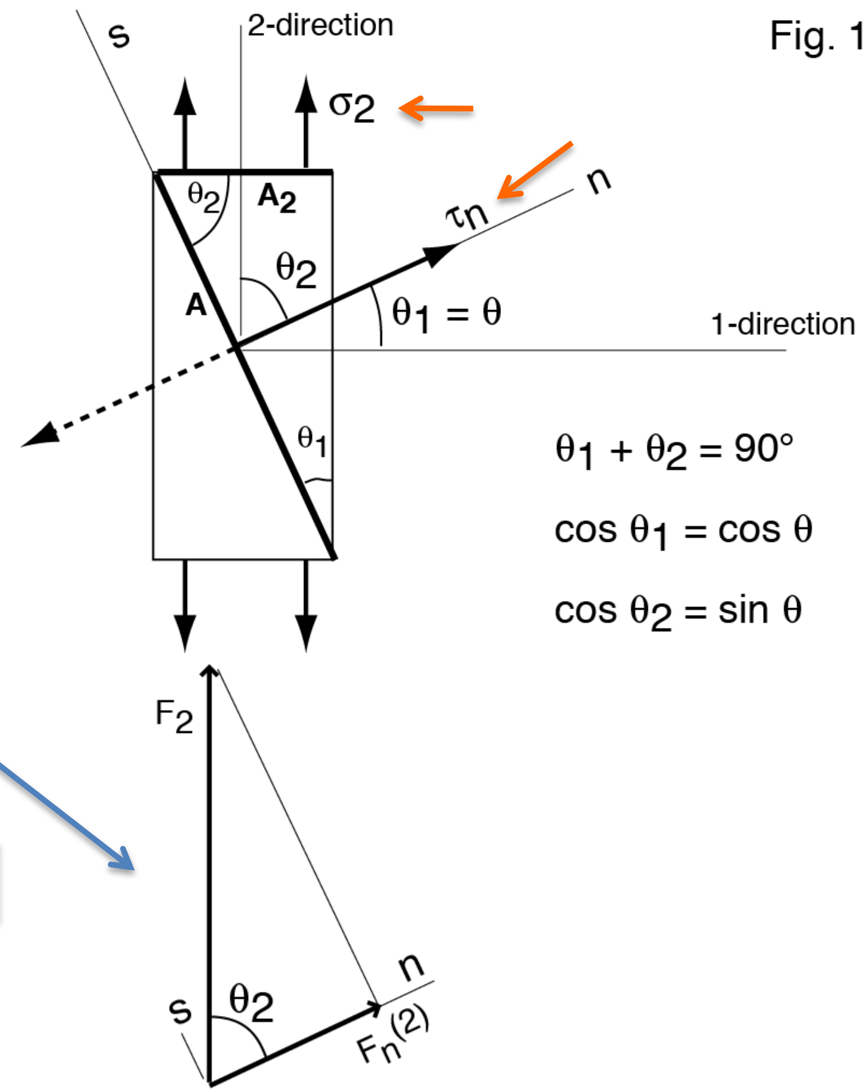
5a $\tau_n^{(2)} = F_n^{(2)} / A = F_2 \cos\theta_2 / (A_2/\cos\theta_2)$

5b $\boxed{\tau_n^{(2)} = (F_2 / A_2) \cos\theta_2 \cos\theta_2 = \sigma_2 \cos\theta_2 \cos\theta_2}$

Weighting factor w_{n2}

6 $w_{n2} = \cos\theta_2 \cos\theta_2 = \sin\theta \sin\theta$

Fig. 16.2



16. STRESS AT A POINT

E Contribution of σ_1 (on face A_1 of area A_1) to τ_s (on face A of area A)

Start with the definition of traction:

1 $\tau_s^{(1)} = F_s^{(1)} / A$

Find unknowns $F_s^{(1)}$ and A from knowns σ_1 and θ.

First find the force F_1 associated with σ_1

2 $F_1 = \sigma_1 A_1$ Force = (stress)(area)

Find $F_s^{(1)}$, the component of F_1 in the s-direction

3 $F_s^{(1)} = -F_1 \cos\theta_2$

Find A in terms of A_2

$A_1 = A \cos\theta_1$ (see diagram at right)

4 $A = A_1/\cos\theta_1$

Contribution of σ_1 to τ_n of σ_1:

5a $\tau_s^{(1)} = F_s^{(1)} / A = -F_1 \cos\theta_2 / (A_1/\cos\theta_1)$

5b $\boxed{\tau_s^{(1)} = -(F_1 / A_1) \cos\theta_2 \cos\theta_1 = -\sigma_1 \cos\theta_2 \cos\theta_1}$

Weighting factor w_{s1}

6 $w_{s1} = -\cos\theta_2 \cos\theta_1 = -\sin\theta \cos\theta$

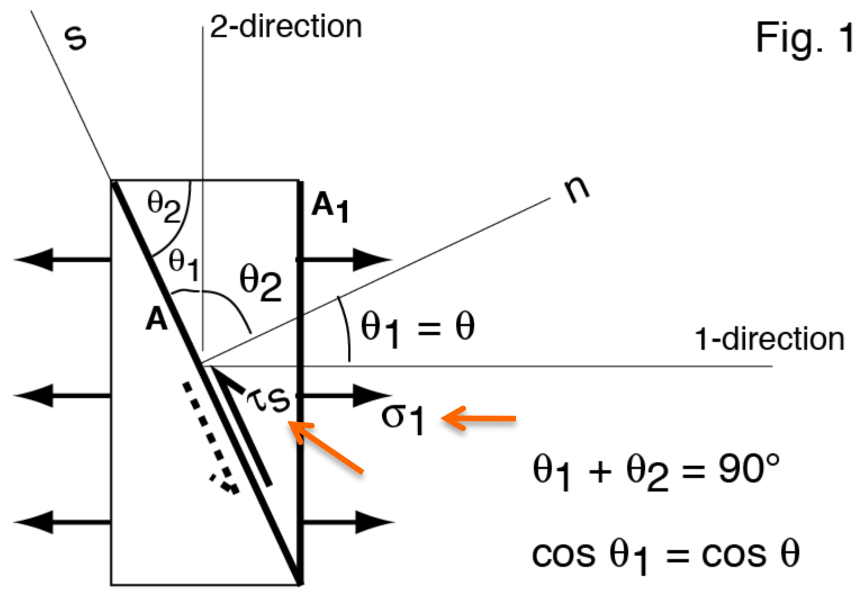
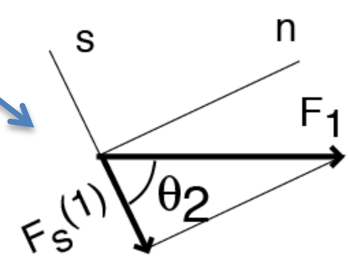


Fig. 16.3

$\theta_1 + \theta_2 = 90°$

$\cos\theta_1 = \cos\theta$

$\cos\theta_2 = \sin\theta$



16. STRESS AT A POINT

F Contribution of σ_2 (on face A_1 of area A_1) to τ_s (on face A of area A)

Start with the definition of traction:

1 $\tau_s^{(2)} = F_s^{(2)} / A$

Find unknowns $F_s^{2)}$ and A from knowns σ_2 and θ.

First find the force F_2 associated with σ_2

2 $F_2 = \sigma_2 A_2$ Force = (stress)(area)

Find $F_s^{(2)}$, the component of F_2 in the s-direction

3 $F_s^{(2)} = F_2 \cos\theta_1$

Find A in terms of A_1

$A_1 = A \cos\theta_1$ (see diagram at right)

4 $A = A_2/\cos\theta_2$

Contribution of σ_1 to τ_n of σ_1:

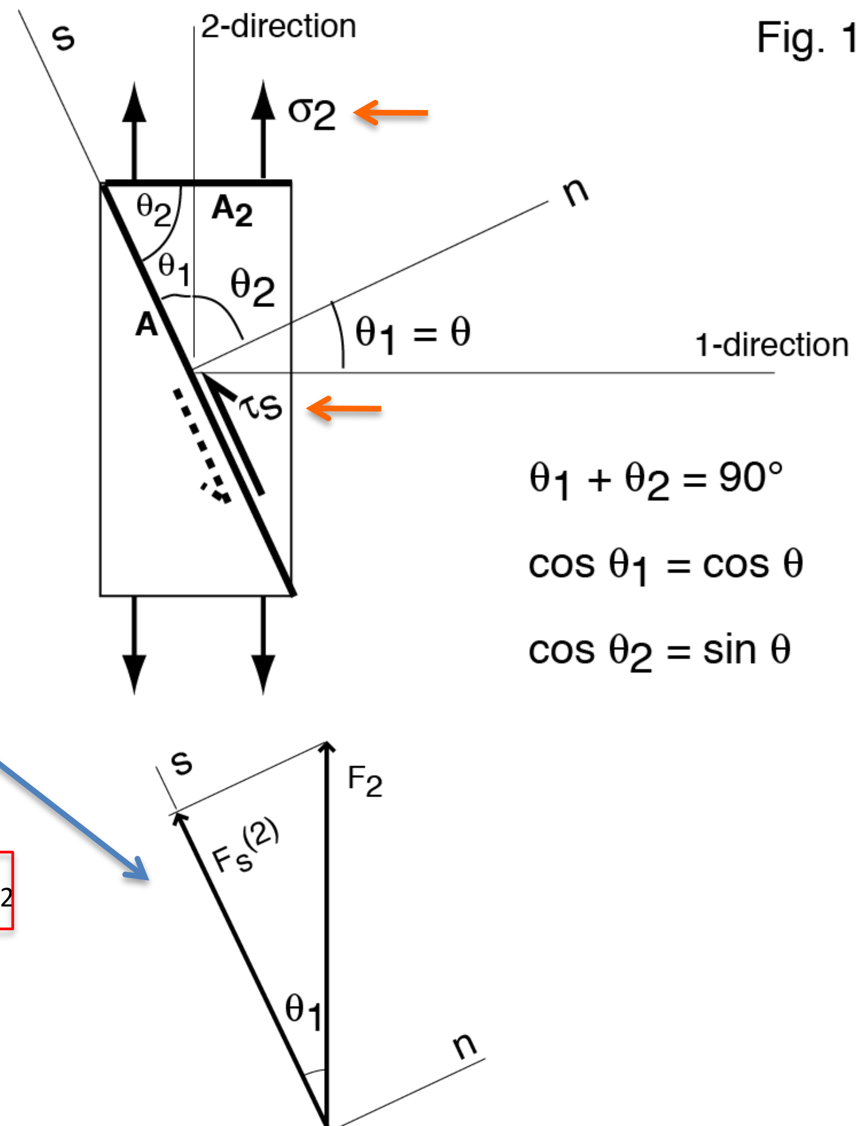
5a $\tau_s^{(2)} = F_s^{(2)} / A = F_2 \cos\theta_1 / (A_2/\cos\theta_2)$

5b $\boxed{\tau_s^{(2)} = (F_2 / A_2) \cos\theta_1 \cos\theta_2 = \sigma_2 \cos\theta_1 \cos\theta_2}$

Weighting factor w_{s2}

6 $w_{s2} = \cos\theta_1 \cos\theta_2 = \cos\theta \sin\theta$

Fig. 16.4



16. STRESS AT A POINT

V Transformation of principal stresses to tractions in 2D

G Original equations

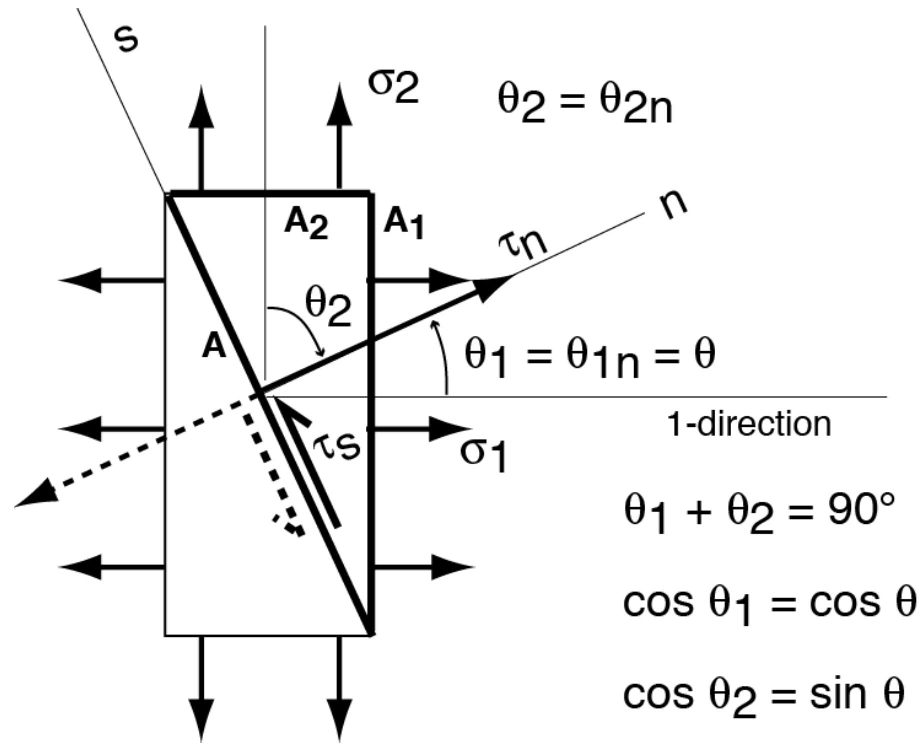
1 $\tau_n = w_{n1}\,\sigma_1 + w_{n1}\,\sigma_2$

2 $\tau_s = w_{s1}\,\sigma_1 + w_{s1}\,\sigma_2$

H Original equations

1 $\tau_n = \cos\theta\cos\theta\,\sigma_1 + \sin\theta\sin\theta\,\sigma_2$

2 $\tau_s = -\sin\theta\cos\theta\,\sigma_1 + \sin\theta\cos\theta\,\sigma_2$



Weighting factors are products of two direction cosines