15. FINITE STRAIN & INFINITESIMAL STRAIN (AT A POINT)

I Main Topics

A The finite strain tensor *E*

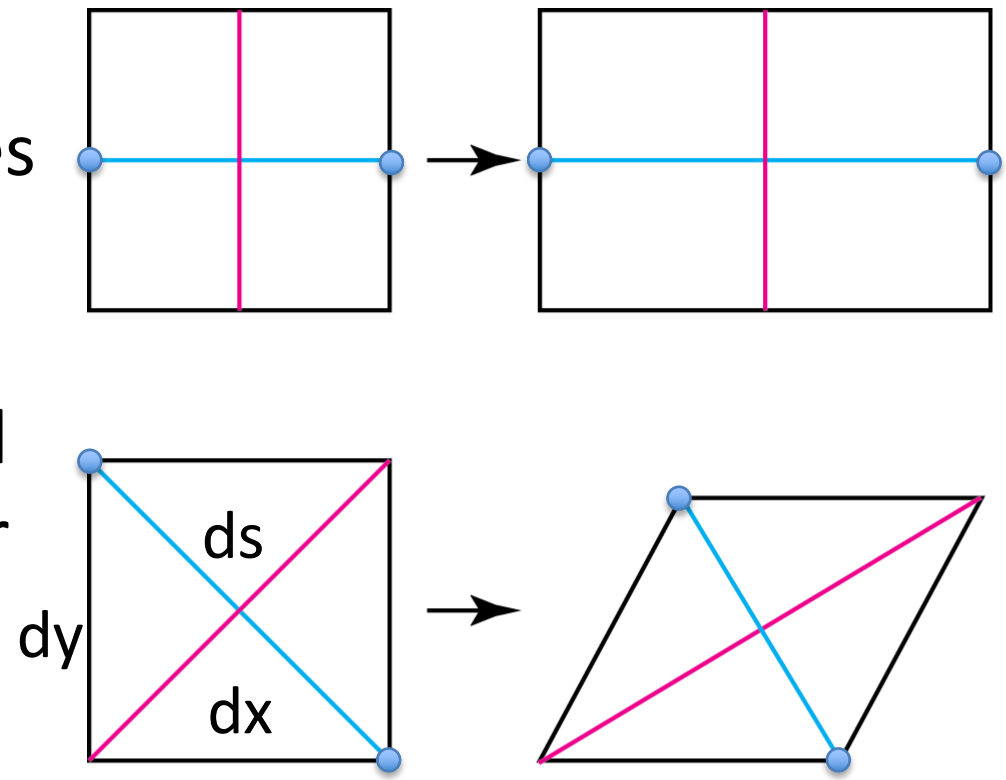
B Infinitesimal strain and its tensor ε

C Deformation paths for finite strain

15. FINITE STRAIN & INFINITESIMAL STRAIN

II The finite strain tensor *E*

A Used to find the changes in the squares of distances $(ds)^2$ between points in a deformed body based on differences in their initial positions



Sides of lower boxes maintain their length, but the diagonals change length

15. FINITE STRAIN & INFINITESIMAL STRAIN*

II The finite strain tensor *E*

B Derivation of [E]

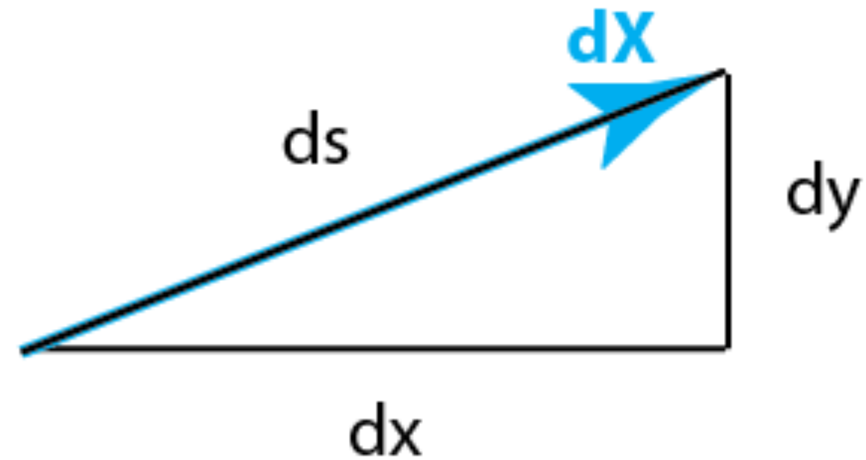
1 $(ds)^2 = (dx)^2 + (dy)^2$

2 $(ds)^2 = \begin{bmatrix} dx & dy \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix}$

3 $\begin{bmatrix} dx \\ dy \end{bmatrix} = [dX]$

4 $\begin{bmatrix} dx & dy \end{bmatrix} = [dX]^T$

Consider vector d**X** and its length



$(dx)^2 + (dy)^2 = (ds)^2 = d\mathbf{X} \cdot d\mathbf{X}$

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II The finite strain tensor *E*

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1 $(ds)^2 = (dx)^2 + (dy)^2$

2 $(ds)^2 = \begin{bmatrix} dx & dy \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix}$

3 $\begin{bmatrix} dx \\ dy \end{bmatrix} = [dX]$

4 $\begin{bmatrix} dx & dy \end{bmatrix} = [dX]^T$

5 $(ds)^2 = [dX]^T[dX] = [dX]^T[I][dX],$

$where\ [I] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now consider vector d**X'**

6 $[dX'] = [F][dX]$ From lecture 14:

7 $(ds')^2 = \begin{bmatrix} dx' & dy' \end{bmatrix} \begin{bmatrix} dx' \\ dy' \end{bmatrix}$

$= [dX']^T[dX']$

8 $(ds')^2 = [[F][dX]]^T[[F][dX]]$

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B Derivation of [E] (cont.)

Now find the difference between the two different dot products, noting that $[[F][dX]]^T = [dX]^T[F]^T$

8 $$(ds')^2 = \left[[F][dX]\right]^T \left[[F][dX]\right]$$

9 $$(ds')^2 - (ds)^2 = [dX]^T [F]^T [F][dX] - [dX]^T [I]^T [dX]$$

10 $$(ds')^2 - (ds)^2 = [dX]^T \left[[F]^T [F] - [I]\right][dX]$$

11 $$\frac{\left\{(ds')^2 - (ds)^2\right\}}{2} = \frac{[dX]^T \left[[F]^T [F] - [I]\right][dX]}{2}$$

12 $$\frac{1}{2}\left\{(ds')^2 - (ds)^2\right\} = [dX]^T [E][dX]$$

$$[E] \equiv \frac{1}{2}\left[[F]^T [F] - [I]\right]$$

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C Meaning of [E]

1 $\frac{1}{2}\left\{(ds')^2-(ds)^2\right\}=[dX]^T[E][dX]$

Given E and dX (the difference in initial positions) of points), then one can find (half) the difference in the squares of the lengths of lines connecting the points before and after deformation

2 $[E]\equiv\frac{1}{2}\Big[[F]^T[F]-[I]\Big]$

3 $[E]\equiv\frac{1}{2}\Big[[J_u+I]^T[J_u+I]-[I]\Big]$

[*E*] gives half the change of squares of line segment lengths, but what do the <u>*terms*</u> of [*E*] mean?

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D Expansion of [E]

$$[E] \equiv \frac{1}{2}\Big[\big[J_u + I\big]^T \big[J_u + I\big] - \big[I\big]\Big]$$

1 $$J_u = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}$$

2 $$E = \frac{1}{2}\left[\left[\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]^T \left[\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]$$

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D Expansion of [E]

$$[E] \equiv \frac{1}{2}\left[[J_u + I]^T [J_u + I] - [I]\right]$$

3 $$[E] = \frac{1}{2}\left[\begin{bmatrix} \frac{\partial u}{\partial x}+1 & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}+1 \end{bmatrix}\begin{bmatrix} \frac{\partial u}{\partial x}+1 & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y}+1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]$$

4 $$[E] = \frac{1}{2}\begin{bmatrix} \left(\frac{\partial u}{\partial x}+1\right)\left(\frac{\partial u}{\partial x}+1\right)+\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial u}{\partial y}\right)-1 & \left(\frac{\partial u}{\partial x}+1\right)\left(\frac{\partial v}{\partial x}\right)+\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial v}{\partial y}+1\right) \\ \left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial u}{\partial x}+1\right)+\left(\frac{\partial v}{\partial y}+1\right)\left(\frac{\partial u}{\partial y}\right) & \left(\frac{\partial v}{\partial d}\right)\left(\frac{\partial v}{\partial x}\right)+\left(\frac{\partial v}{\partial y}+1\right)\left(\frac{\partial v}{\partial y}+1\right)-1 \end{bmatrix}$$

The meanings of the terms in [*E*] still are not intuitive

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III Infinitesimal strain

A Derivation of infinitesimal strain tensor [ε]

1 $$[E]=\frac{1}{2}\begin{bmatrix} \left(\frac{\partial u}{\partial x}+1\right)\left(\frac{\partial u}{\partial x}+1\right)+\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial u}{\partial y}\right)-1 & \left(\frac{\partial u}{\partial x}+1\right)\left(\frac{\partial v}{\partial x}\right)+\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial v}{\partial y}+1\right) \\ \left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial u}{\partial x}+1\right)+\left(\frac{\partial v}{\partial y}+1\right)\left(\frac{\partial u}{\partial y}\right) & \left(\frac{\partial v}{\partial d}\right)\left(\frac{\partial v}{\partial x}\right)+\left(\frac{\partial v}{\partial y}+1\right)\left(\frac{\partial v}{\partial y}+1\right)-1 \end{bmatrix}$$

If the displacement derivatives are <<1, their *products* *with each other* can be neglected

2 $$[\varepsilon]=\frac{1}{2}\begin{bmatrix} \frac{\partial u}{\partial x}+\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial x}+\frac{\partial u}{\partial y} & \frac{\partial v}{\partial y}+\frac{\partial v}{\partial y} \end{bmatrix}==\frac{1}{2}\left[[J_u]+[J_u]^T\right]$$

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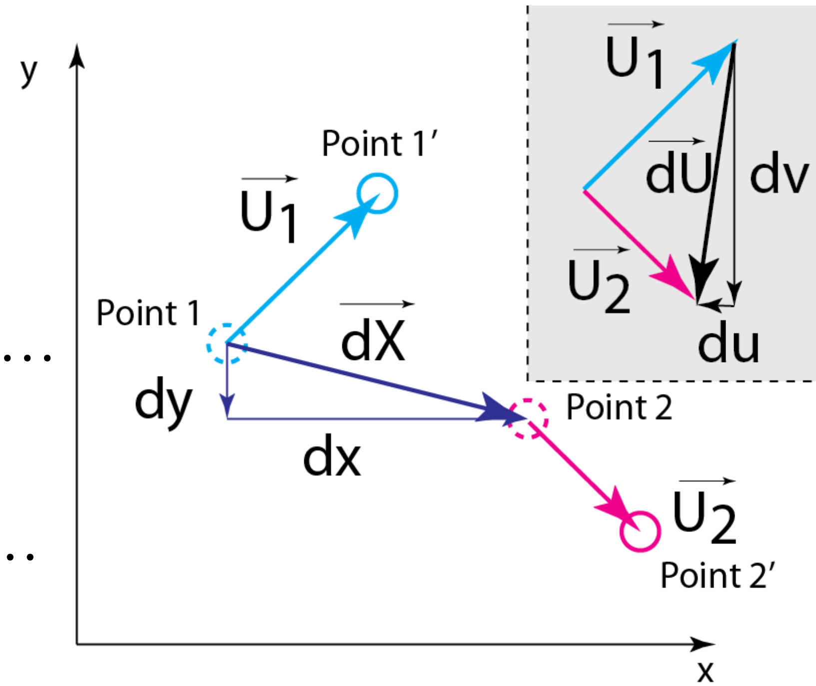
IV Infinitesimal strain

B Taylor series expansion

We seek $\mathbf{U_2}$ given $\mathbf{U_1}$ and $\mathbf{dX}$



1 $$u_2 = u_1 + du = u_1 + \left(\frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy \right) + \ldots$$

2 $$v_2 = v_1 + dv = v_1 + \left(\frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy \right) + \ldots$$

3 $$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} + \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} + \ldots \Rightarrow [U_2] = [U_1] + [dU] \approx [U_1] + [J_u][dX]$$

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B Taylor series expansion (cont.)

Now split $[J_u]$ into two matrices: the infinitesimal strain matrix $[\varepsilon]$ and the anti-symmetric rotation matrix $[\omega]$

$$[J_u]=\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} \qquad [J_u]^T=\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{bmatrix}\frac{\partial v}{\partial x}$$

$$[\varepsilon]=\frac{[J_u]+[J_u]^T}{2}=\frac{1}{2}\begin{bmatrix} \frac{\partial u}{\partial x}+\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial x}+\frac{\partial u}{\partial y} & \frac{\partial v}{\partial y}+\frac{\partial v}{\partial y} \end{bmatrix}$$

Symmetric

$$[\omega]=\frac{[J_u]-[J_u]^T}{2}=\frac{1}{2}\begin{bmatrix} 0 & \frac{\partial u}{\partial y}-\frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} & 0 \end{bmatrix}$$

Anti-symmetric

$$[\varepsilon]+[\omega]=[J_u]$$

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B Taylor series expansion (cont.)

$$[\varepsilon]=\frac{1}{2}\begin{bmatrix}\frac{\partial u}{\partial x}+\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\\ \frac{\partial v}{\partial x}+\frac{\partial u}{\partial y} & \frac{\partial v}{\partial y}+\frac{\partial v}{\partial y}\end{bmatrix}=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy}\\ \varepsilon_{yx} & \varepsilon_{yy}\end{bmatrix};\ \varepsilon_{xy}=\varepsilon_{yx}$$

Symmetric

$$[\omega]=\frac{1}{2}\begin{bmatrix}0 & \frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\\ \frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} & 0\end{bmatrix}=\begin{bmatrix}0 & \omega_{xy}\\ \omega_{yx} & 0\end{bmatrix};\ \omega_{xy}=-\omega_{yx}$$

Anti-symmetric

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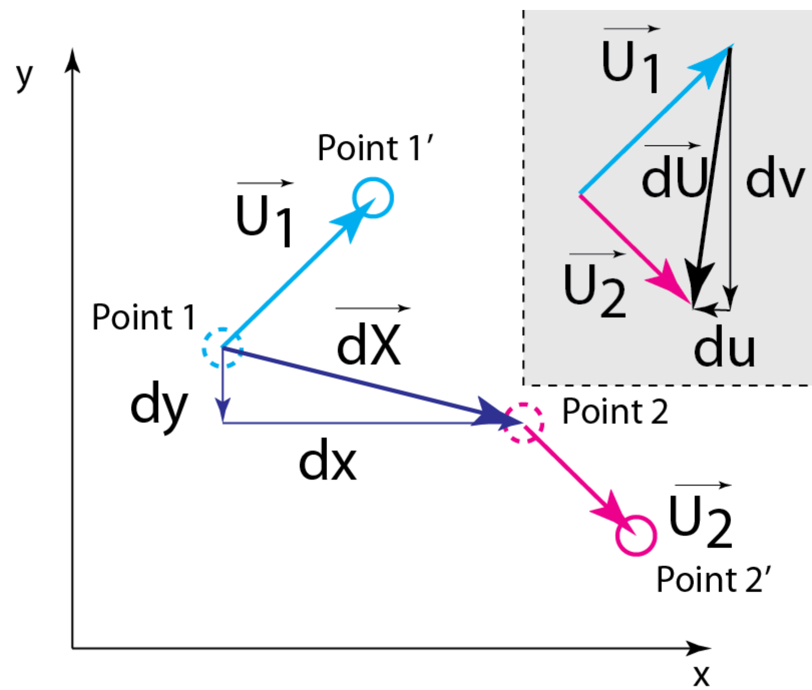
IV Infinitesimal strain

C Taylor series expansion

$$[U_2] \approx [U_1] + [J_u][dX] = [U_1] + [[\varepsilon] + [\omega]][dX]$$



Infinitesimal deformation can be decomposed into:

a translation (given by $[U_1]$),

a strain (given by [ε]),

and

a rotation (given by [ω])

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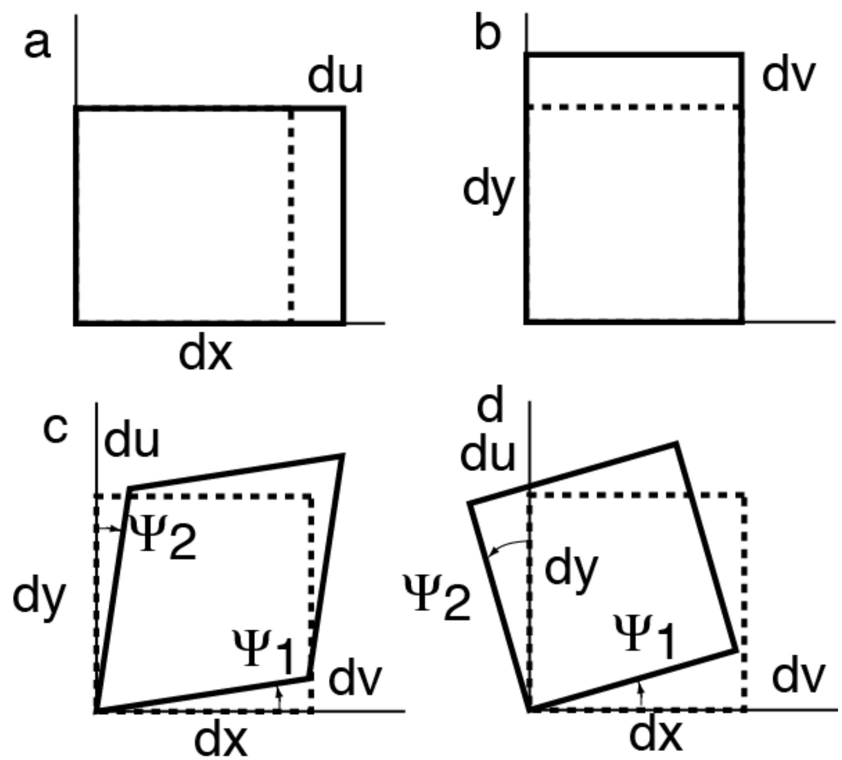
$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2}\left(\Psi_1 - \Psi_2\right) = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\omega_{xy} = -\omega_{xy} = \frac{1}{2}\left(\Psi_1 + \Psi_2\right) = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$



For small angles, Ψ = tanΨ

Positive angles are measured about the z-axis using a right hand rule. In (c) the angle Ψ_2 is clockwise (negative), but du is positive. In (d) Ψ_2 is counter-clockwise, and du< 0.

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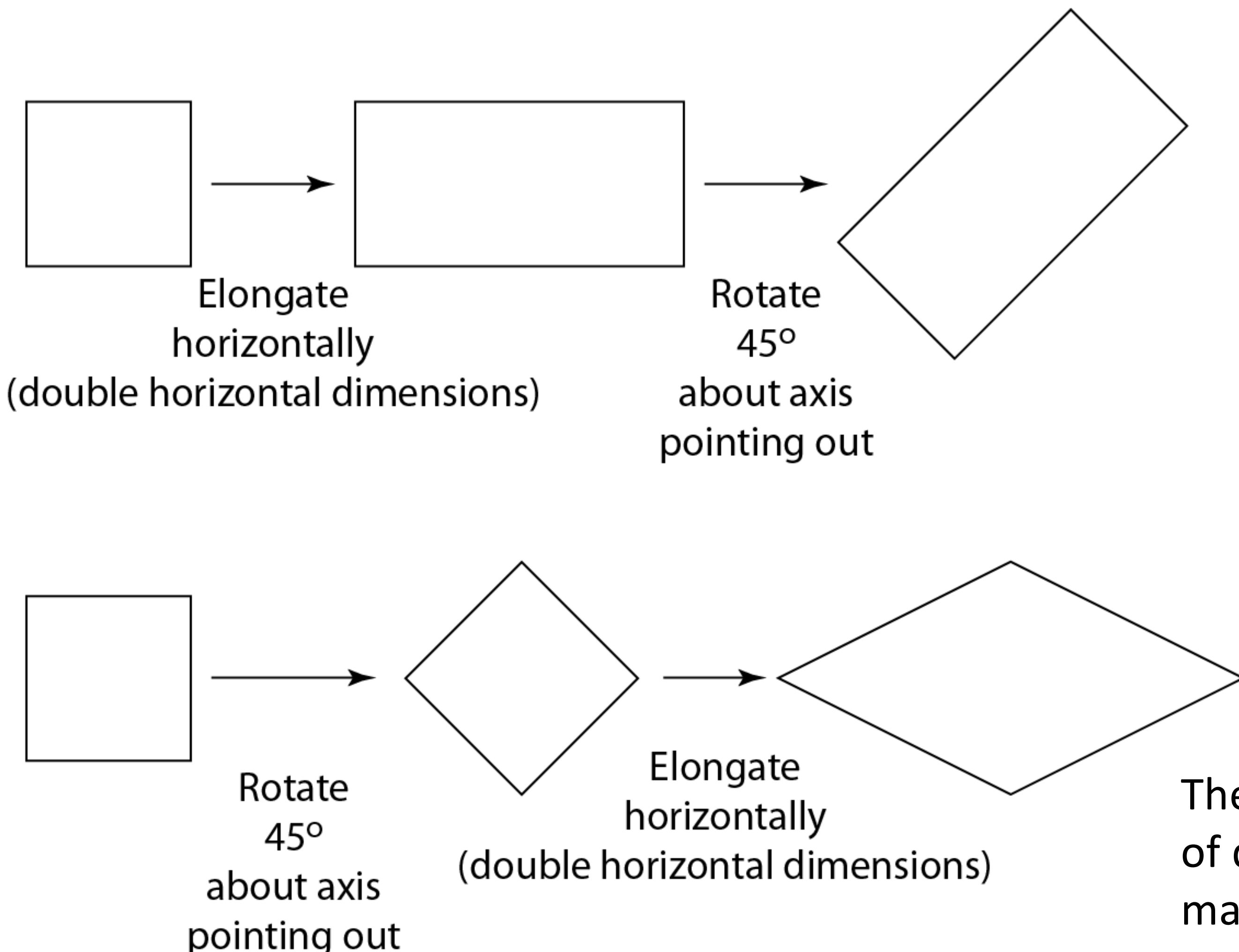

The sequence of deformation matters for large strains

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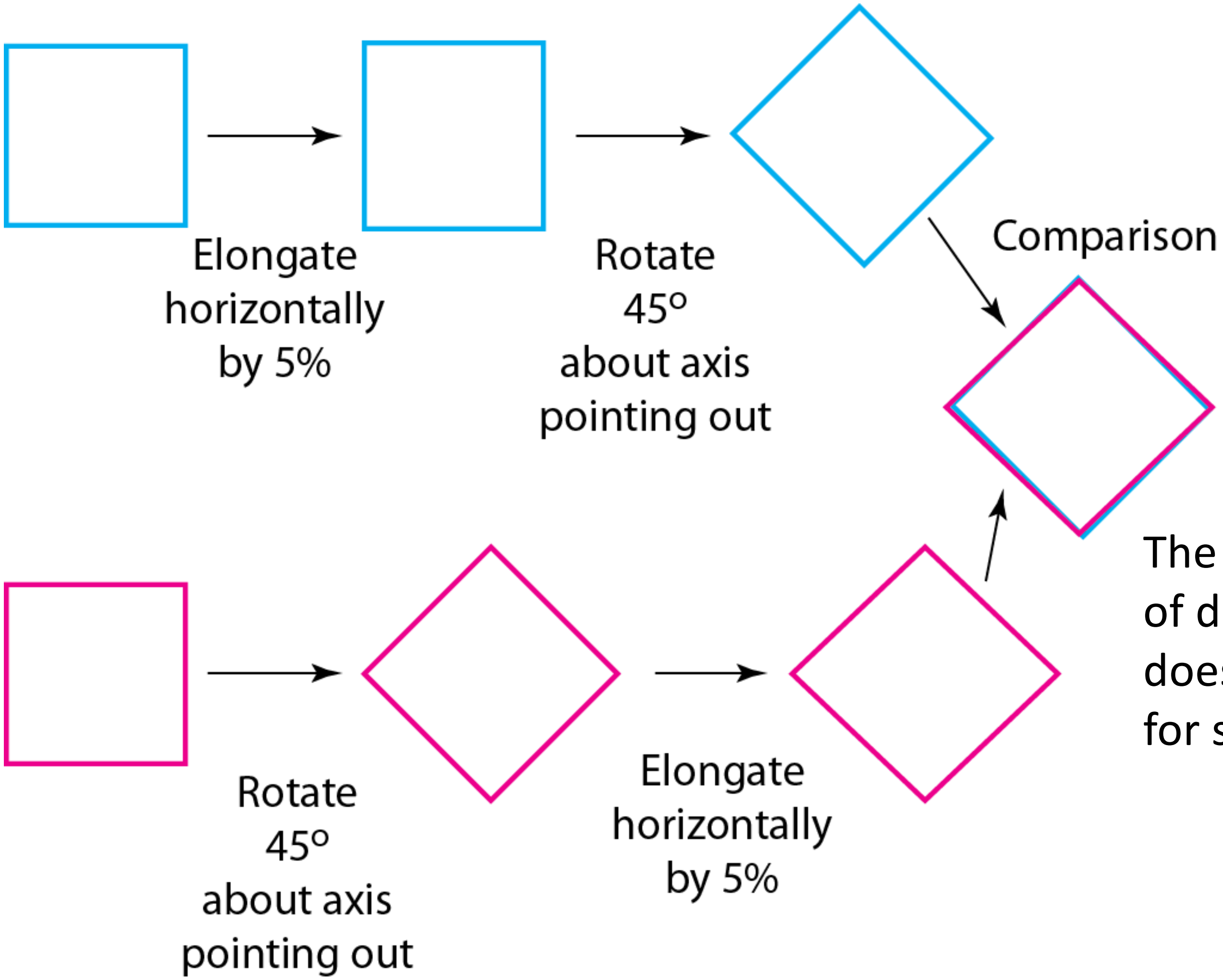

The sequence of deformation does not matter for small strains

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III Deformation paths for finite strain

Consider two different deformations

A Deformation 1

$$[F_1] = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

B Deformation 2

$$[F_2] = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$$

15. FINITE STRAIN & INFINITESIMAL STRAIN

III Deformation paths for finite strain

Consider two different deformations

A Deformation 1

$$[F_1] = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

B Deformation 2

$$[F_2] = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$$

C F2 acts on F1

$$[F_2][F_1] = \begin{bmatrix} a_2 a_1 + b_2 c_1 & a_2 b_1 + b_2 d_1 \\ c_2 a_1 + d_2 c_1 & c_2 b_1 + d_2 d_1 \end{bmatrix}$$

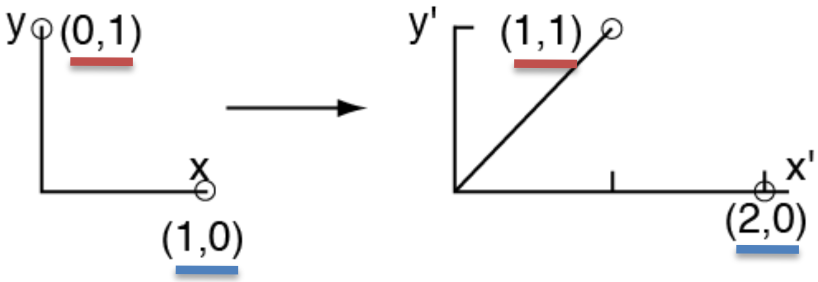
D F1 acts on F2

$$[F_1][F_2] = \begin{bmatrix} a_1 a_2 + b_1 c_2 & a_1 b_2 + b_1 d_2 \\ c_1 a_2 + d_1 c_2 & c_1 b_2 + d_1 d_2 \end{bmatrix}$$

E The sequence of finite deformations matters – unless off-diagonal terms in $[F_1]$ and $[F_2]$ are small

15. FINITE STRAIN & INFINITESIMAL STRAIN*

[F1]



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

[F1]

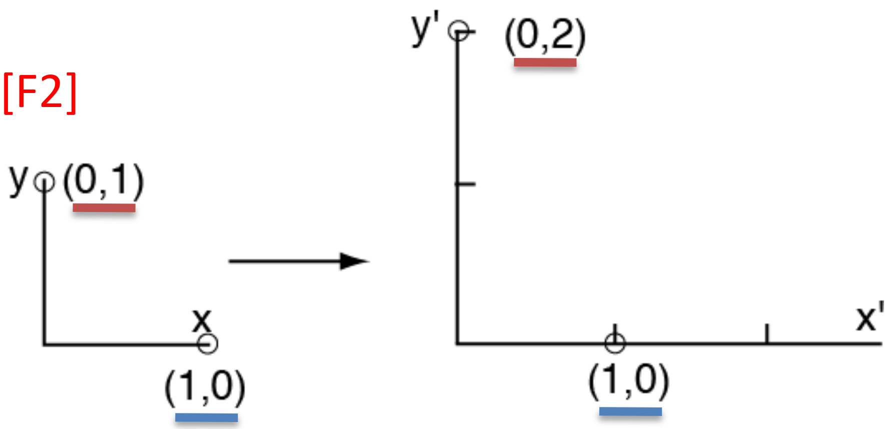
$$\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

[F1]

This shows the effect of [F1] all by itself

Two vectors, <1,0> and <0.1>, can be handled at the same time

[F2]



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

[F2]

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

[F2]

This shows the effect of [F2] all by itself

15. FINITE STRAIN & INFINITESIMAL STRAIN*

Now consider [F2] acting after [F1] (i.e., [F2] acts on [F1])

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} = [F_2][F_1]$$

[F2] [F1]

Now consider [F1] acting after [F2]] (i.e., [F1] acts on [F2])

$$\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = [F_1][F_2]$$

[F1] [F2]

$$[F_2][F_1] \neq [F_1][F_2]$$

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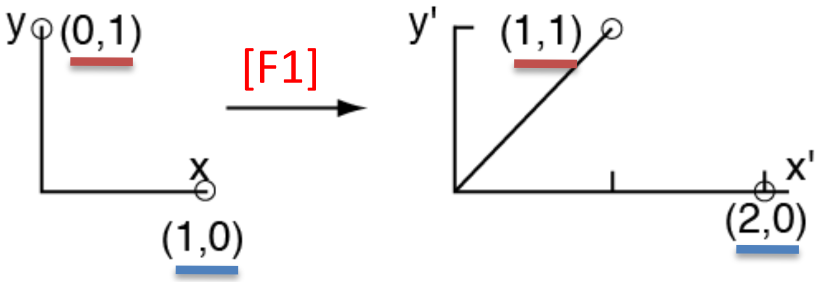
Individual effects

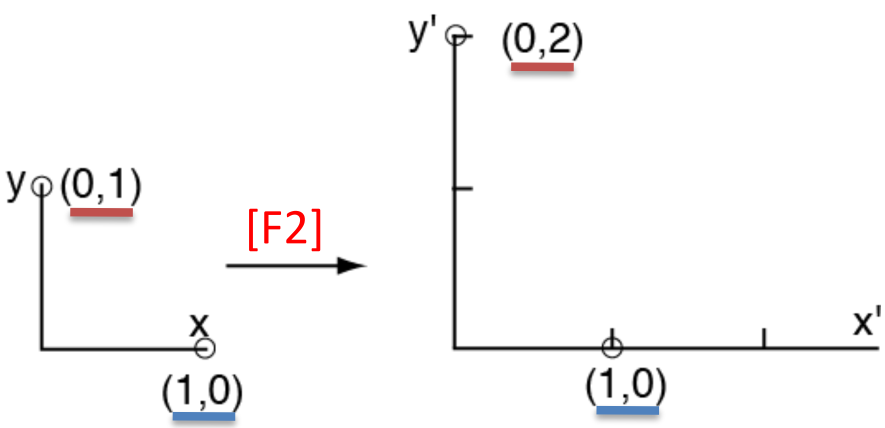


$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

[F1]

$$\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

[F1]



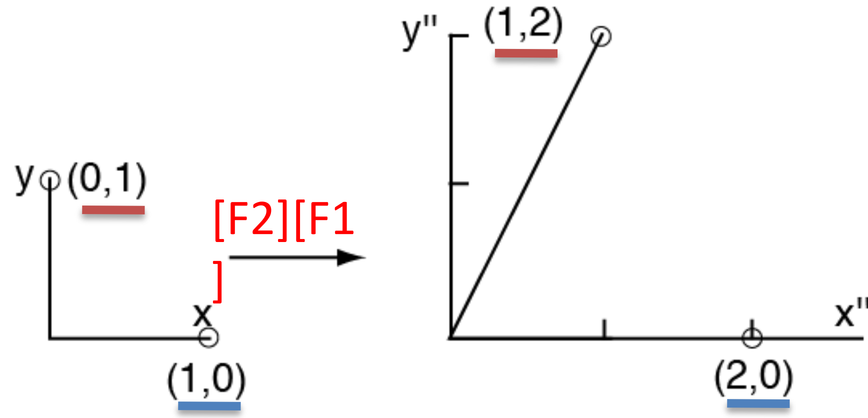
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

[F2]

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

[F2]

Combined effects

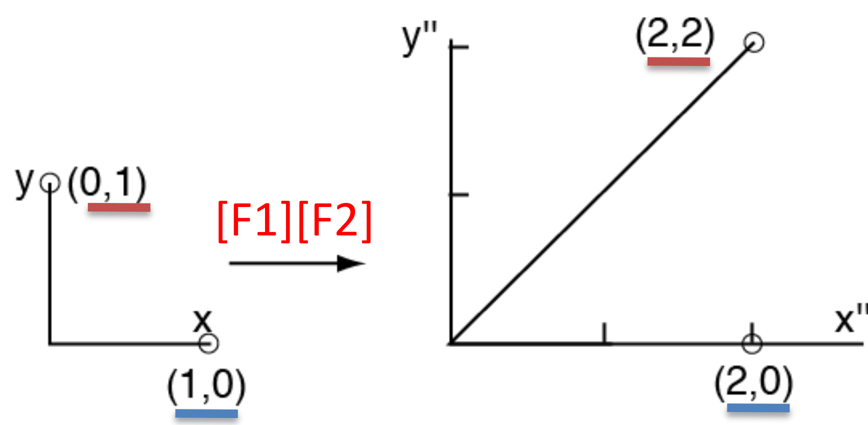


$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

[F2][F1]

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

[F2][F1]



$$\begin{bmatrix} x''' \\ y''' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

[F1][F2]

$$\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[F_2][F_1] \neq [F_1][F_2]$$

15. FINITE STRAIN & INFINITESIMAL STRAIN

IV Infinitesimal strain and the infinitesimal strain tensor [ε]

A Infinitesimal strain

Deformation where the displacement derivatives in [J_u] are small relative to one so that the products of the derivatives are very small and can be ignored.

B An approximation to finite strain

$$[\varepsilon] \equiv \frac{1}{2}\begin{bmatrix} \left(\frac{\partial u}{\partial x}\right)+\left(\frac{\partial u}{\partial x}\right) & \left(\frac{\partial u}{\partial y}\right)+\left(\frac{\partial v}{\partial x}\right) \\ \left(\frac{\partial u}{\partial y}\right)+\left(\frac{\partial v}{\partial x}\right) & \left(\frac{\partial v}{\partial y}\right)+\left(\frac{\partial v}{\partial y}\right) \end{bmatrix}$$

$$[\varepsilon] = \frac{1}{2}\left[[J_u]+[J_u]^T\right]$$

$$[J_u] = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} \qquad [J_u]^T = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{bmatrix}$$

15. FINITE STRAIN & INFINITESIMAL STRAIN

IV Infinitesimal strain and the infinitesimal strain tensor [ε] (cont.)

- C Why consider [ε] if it is an approximation?
  - 1 Relevant to important geologic deformations
    - A Fracture
    - B Earthquake deformation
    - C Volcano deformation

Gelatin Volcano Experiment

http://www.spacegrant.hawaii.edu/class_acts/WebImg/gelatinVolcano.gif

15. FINITE STRAIN & INFINITESIMAL STRAIN

C Why consider [ε] if it is an approximation? (cont.)

2 Terms of the infinitesimal strain tensor [ε] have clear geometric meaning

3 Can apply principal of superposition (addition)

4 Infinitesimal deformation is essentially independent of the deformation sequence

5 Amenable to sophisticated mathematical treatment (e.g., elasticity theory)

6 Quantitative predictive ability

Gelatin Volcano Experiment

http://www.spacegrant.hawaii.edu/class_acts/WebImg/gelatinVolcano.gif

15. FINITE STRAIN & INFINITESIMAL STRAIN*

C Why consider [ε] if it is an approximation? (cont.)

7 Infinitesimal strain example

$$F_3 = \begin{bmatrix} 1.02 & 0.01 \\ 0 & 1.01 \end{bmatrix} \longrightarrow J_{u(3)} = \begin{bmatrix} 0.02 & 0.01 \\ 0 & 0.01 \end{bmatrix}$$

$$F_4 = \begin{bmatrix} 1.01 & 0 \\ 0 & 1.02 \end{bmatrix} \longrightarrow J_{u(4)} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}$$

$$[F_4][F_3] = \begin{bmatrix} 1.01 & 0 \\ 0 & 1.02 \end{bmatrix}\begin{bmatrix} 1.02 & 0.01 \\ 0 & 1.01 \end{bmatrix} = \begin{bmatrix} 1.0302 & 0.0100 \\ 0.0000 & 1.0302 \end{bmatrix}$$

$$[F_3][F_4] = \begin{bmatrix} 1.02 & 0.01 \\ 0 & 1.01 \end{bmatrix}\begin{bmatrix} 1.01 & 0 \\ 0 & 1.02 \end{bmatrix} = \begin{bmatrix} 1.0302 & 0.0101 \\ 0.0000 & 1.0302 \end{bmatrix}$$

$$[I] + \left[\left[J_{u(3)}\right] + \left[J_{u(4)}\right]\right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.02 & 0.01 \\ 0 & 0.01 \end{bmatrix} + \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix} = \begin{bmatrix} 1.0300 & 0.0100 \\ 0.0000 & 1.0300 \end{bmatrix}$$

Sequence results can be obtained regardless of the order of events, but also by superposition (bottom row)

Results nearly indistinguishable