13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

I Main Topics (see chapters 14 and 18 of Means, 1976)

A Fundamental principles of continuum mechanics

B Position vectors and coordinate transformation equations

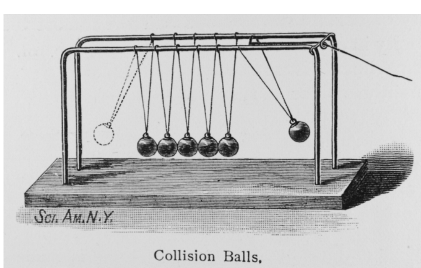
C Displacement vectors and displacement equations

D Deformation

E Homogeneous and inhomogeneous strain

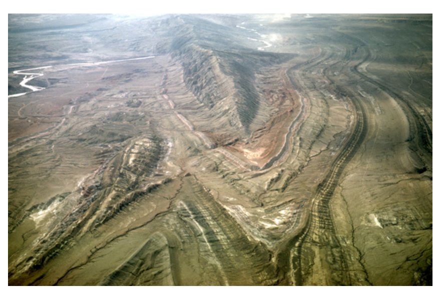
13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

Transition From Particle Mechanics to Continuum Mechanics



Newton's Pendulum

http://www.lhup.edu/~dsimanek/scenario/collision-r.jpg



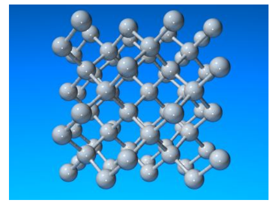
Sheep Mountain Anticline, Wyoming

http://www.geology.wisc.edu/~maher/air/air07.htm

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

II Fundamental principles of continuum mechanics

A Number of particles is sufficiently large that the concept of bulk material behavior is meaningful



http://www.webelements.com/_media/elements/allotropes/C/C-diamond.jpg

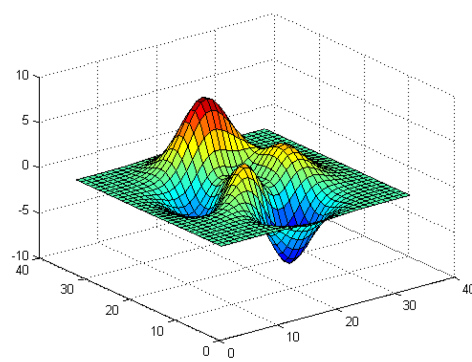
http://www.gemstonetilsoldout.com/images/diamond-gemstone.jpg

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

II Fundamental principles of continuum mechanics

B Relates natural world to the realm of mathematics

C Densities of mass, momentum, and energy exist (no "holes")

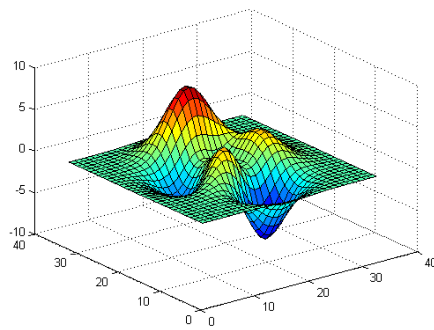


13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

II Fundamental principles of continuum mechanics

D Examples of continuous properties

1 Density $\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V}$



So certain derivatives have to exist

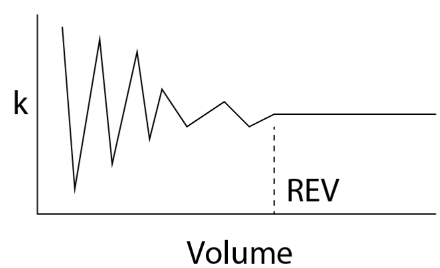
13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

II Fundamental principles of continuum mechanics

D Examples of continuous properties (cont.)

2 Hydraulic conductivity ("permeability")

E Scale matters



Note that the concept of derivatives becomes difficult at certain scales

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

II Fundamental principles of continuum mechanics

- F Variability
  - 1 Heterogeneity: material property depends on position

beg.utexas.edu

Cathedral Peak, CA

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

II Fundamental principles of continuum mechanics

- F Variability
  - 2 Anisotropy: material property depends on orientation

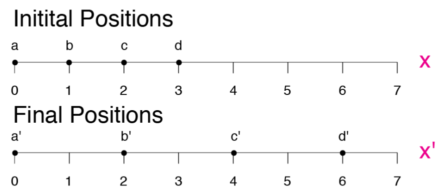
Hand sample of gneiss

http://en.wikipedia.org/wiki/File:Gneiss.jpg

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

III Position vectors and coordinate transformation equations

- A **X** = initial (undeformed) position vector
- B **X'** = final (current, or deformed) position vector (at time Δt)
- C Coordinate transformation equations
  - **1** **X'** = f(**X**)
    Lagrangian: final position a function of initial position
  - **2** **X** = g(**X'**)
    Eulerian: initial position a function of final position



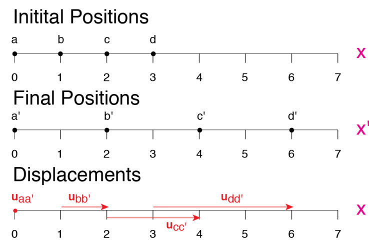
x' = 2x (Lagrangian)
x = x'/2 (Eulerian)

10/3/12 GG303 9

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

IV Displacement vector (**U**)

- A **U** = **X'** - **X**
  - 1 x-component: u_x or u
  - 2 y-component: u_y or v
  - 3 z-component: u_z or w
- B Lagrangian **U**(**X**): displacement in terms of initial position
- C Eulerian **U**(**X'**): displacement in terms of final position



x' = 2x (Lagrangian)
x = x'/2 (Eulerian)

Lagrangian displacement equation
u = u(x) = x' - x = 2x - x = x

Eulerian displacement equation
u = u(x') = x' - x = x' - x'/2 = x' /2

10/3/12 GG303 10

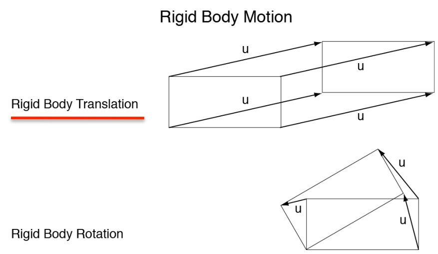
13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

V Deformation: rigid body motion + change in size and/or shape

A Rigid body translation

1. No change in the length of line connecting any points
2. All points displaced by an equal vector (equal amount and direction); no displacement of points relative to one another
3. [X'] = [U] +[X]

   matrix addition (U is a constant)



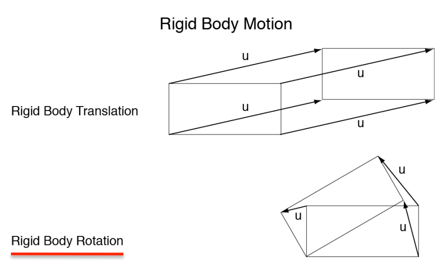
13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

V Deformation: rigid body motion + change in size and/or shape

B Rigid body rotation

1. No change in the length of line connecting any points
2. All points rotated by an equal amount about a common axis; no angular displacement of points relative to one another
3. [X'] = [a][X]

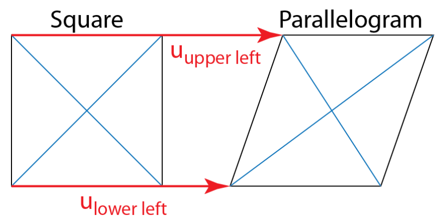
   matrix multiplication;
   rows in [a] are dir. cosines!



13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

V Deformation: rigid body motion + change in size and/or shape

- C Change in size and /or shape (distortional strain)
  - 1 At least some line segments connecting points in a body change lengths (i.e., the relative positions of points changes)
  - 2 $\vec{u}$ is not a constant throughout the body (i.e., $\vec{u}$ varies)



13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

C Change in size and /or shape (distortional strain) – cont.

- 3 Change in linear dimension
  - A Extension (or elongation): ε

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{L_1 - L_0}{L_0}$$

  - B Stretch: S

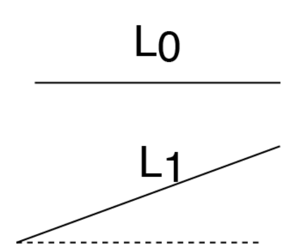
$$S = \frac{L_1}{L_0} = \frac{L_0}{L_0} + \frac{L_1 - L_0}{L_0} = 1 + \varepsilon$$

  - C Quadratic elongation: λ

$$\lambda = \left(\frac{L_1}{L_0}\right)^2 = S^2$$

  - D <u>All are dimensionless</u>

Elongation



$\varepsilon = (L_1 - L_0) / L_0$

$S = L_1 / L_0$

$\lambda = (L_1 / L_0)^2$

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

C Change in size and /or shape (distortional strain) – cont.

3 Shear strain: γ

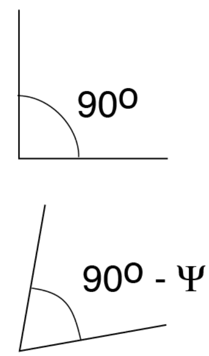
a Describes change in right in angle between originally perpendicular lines

b $\gamma = \tan\psi$

For small Ψ, tanΨ➔Ψ

c Dimensionless

Shear Strain



$\gamma = \tan \Psi$

10/3/12 GG303 15

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

V Deformation: rigid body motion + change in size and/or shape (cont.)

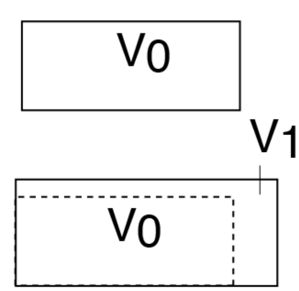
D Change in volume (dilational strain)

1 Dilation (Δ)

$$\Delta = \frac{\Delta V}{V_0} = \frac{V_1 - V_0}{V_0}$$

2 Dimensionless

Dilation



$\Delta = (V_1 - V_0) / V_0$

10/3/12 GG303 16

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

D Change in volume (dilational strain) – cont.

3 Example

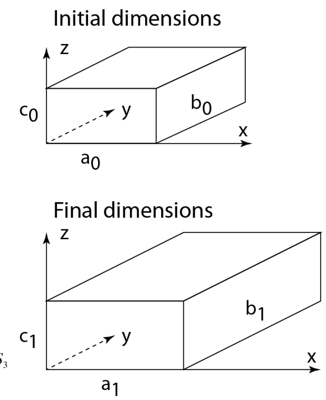
$$\Delta = \frac{\Delta V}{V_o} = \frac{V_1 - V_o}{V_o}$$

$$V_0 = \begin{vmatrix} a_o & 0 & 0 \\ 0 & b_o & 0 \\ 0 & 0 & c_o \end{vmatrix} = a_o b_o c_o$$

$$V_1 = \begin{vmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{vmatrix} = \begin{vmatrix} a_0(1+\varepsilon_1) & 0 & 0 \\ 0 & b_0(1+\varepsilon_2) & 0 \\ 0 & 0 & c_0(1+\varepsilon_3) \end{vmatrix} = a_1 b_1 c_1$$

$$V_1 = a_0 b_0 c_0 \begin{vmatrix} (1+\varepsilon_1) & 0 & 0 \\ 0 & (1+\varepsilon_2) & 0 \\ 0 & 0 & (1+\varepsilon_3) \end{vmatrix} = a_0 b_0 c_0 \begin{vmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{vmatrix} = a_0 b_0 c_0 S_1 S_2 S_3$$

$$\Delta = \frac{V_1 - V_0}{V_0} = \frac{a_0 b_0 c_0 S_1 S_2 S_3 - a_0 b_0 c_0}{a_0 b_0 c_0} = S_1 S_2 S_3 - 1 \approx \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

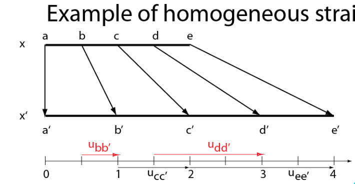


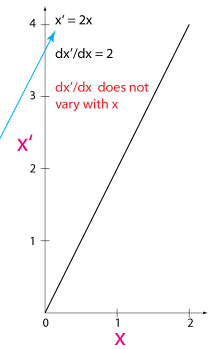
For small strains (ε<<1)

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

VI Homogeneous and inhomogenous strain

Example of homogeneous strain in one dimension





x' = 2x Lagrangian

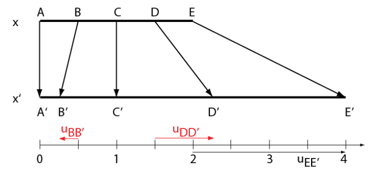
u = u(x) = x' - x = 2x - x = x Lagrangian

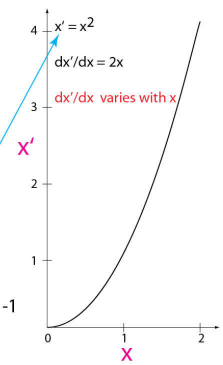
$\varepsilon = \Delta L/L_0$
$= \{ (x_2' - x_1') - (x_2 - x_1) \} / (x_2 - x_1)$
$= \{ (x_2' - x_2) - (x_1' - x_1) \} / (x_2 - x_1)$
$= \{u_2 - u_1\} / (x_2 - x_1) = \Delta u/\Delta x$

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

VI Homogeneous and inhomogenous strain

Example of inhomogeneous strain in one dimension





$x' = x^2$ Lagrangian

$u = u(x) = x' - x = x^2 - x$ Lagrangian

$$\varepsilon = \lim_{\Delta x \to 0} \Delta L / L_0 = \lim_{\Delta x \to 0} \Delta u / \Delta x = du/dx = 2x - 1$$