10. ROTATIONS (I)

I Main Topics

A Uses of rotation in geology (and engineering)

B Concepts behind rotation

C Appendix

(Geometry on a Sphere for Plate Tectonics)

10. ROTATIONS (I)

II Uses of rotation in geology (and engineering) I

A Representation of quantities in an easier to understand form

B Plate tectonics

C Evaluating stresses and strains

D Examination of features in tilted bedding

1 Pre-tilting orientation of sedimentary structures (e.g. ripples)

2 Pre-tilting orientation of beds below angular unconformities

3 Pre-tilting orientation of paleomagnetic orientations

E To determine *in-situ* orientations of features from drill cores

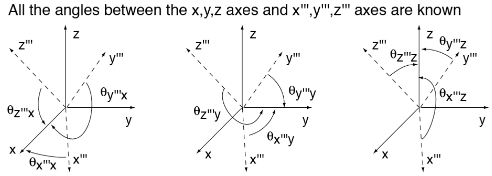
F Need to consider whether object is rotated and coordinate axes are fixed or whether the object is fixed and coordinate axes are rotated. This affects the sign(s) and sequence of the angle(s) of rotation.

10. ROTATIONS (I)

III Concepts behind rotation

A Any orthogonal coordinate system with axes x,y,z can be rotated to coincide with another orthogonal coordinate system x', y', z' by using the direction cosines relating the axes of the two systems.

All the angles between the x,y,z axes and x''',y''',z''' axes are known



$a_{x'''x} = \cos \theta_{x'''x}$	$a_{x'''y} = \cos \theta_{x'''y}$	$a_{x'''z} = \cos \theta_{x'''z}$
$a_{y'''x} = \cos \theta_{y'''x}$	$a_{y'''y} = \cos \theta_{y'''y}$	$a_{y'''z} = \cos \theta_{y'''z}$
$a_{z'''x} = \cos \theta_{z'''x}$	$a_{z'''y} = \cos \theta_{z'''y}$	$a_{z'''z} = \cos \theta_{z'''z}$

10. ROTATIONS (I)

III Concepts behind rotation (cont.)

A …direction cosines (cont.)

Dir. cosines	x' axis	y' axis	z' axis
x axis	$a_{xx'} = a_{x'x}$	$a_{xy'} = a_{y'x}$	$a_{xz'} = a_{z'x}$
y axis	$a_{yx'} = a_{x'y}$	$a_{yy'} = a_{y'y}$	$a_{yz'} = a_{z'y}$
z axis	$a_{zx'} = a_{x'z}$	$a_{zy'} = a_{y'z}$	$a_{zz'} = a_{z'z}$

Note: $a_{xz'}$ is not equal to $a_{x'z}$ because $\theta_{xz'}$ is not equal to $\theta_{x'z}$

10. ROTATIONS (I)

III Concepts behind rotation (cont.)

A Any orthogonal coordinate system with axes x,y,z can be rotated to coincide with another orthogonal coordinate system x′, y′, z′ by using the direction cosines relating the axes of the two systems.

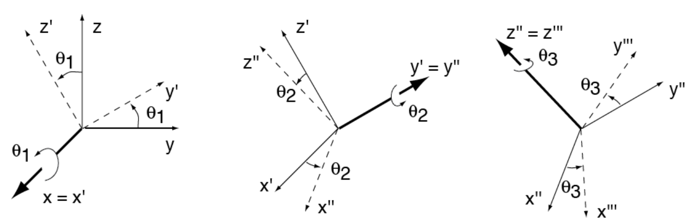
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{xx'} & a_{xy'} & a_{xz'} \\ a_{yx'} & a_{yy'} & a_{yz'} \\ a_{zx'} & a_{zy'} & a_{zz'} \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad and \quad \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} a_{x'x} & a_{x'y} & a_{x'z} \\ a_{y'x} & a_{y'y} & a_{y'z} \\ a_{z'x} & a_{z'y} & a_{z'z} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Note: $a_{xz'} \neq a_{x'z}$ because $\theta_{xz'} \neq \theta_{x'z}$; $x \neq x'$, and $z \neq z'$

10. ROTATIONS (I)

III Concepts behind rotation (cont.)

B Any orthogonal coordinate system can be rotated to coincide with another orthogonal coordinate system by three consecutive rotations **(Cayley):**



A: Rotate about x-axis B: Rotate about y'-axis C: Rotate about z"-axis

Right-hand rule: If right thumb is along rotation axis, then fingers curl in positive θ direction

10. ROTATIONS (I)

B …three consecutive rotations **(Cayley)** (cont.)

1 Rotate the x,y,z system about the x-axis by angle θ_1

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} a_{x'x} & a_{x'y} & a_{x'z} \\ a_{y'x} & a_{y'y} & a_{y'z} \\ a_{z'x} & a_{z'y} & a_{z'z} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

2 Rotate the x',y',z' system about the y'-axis by angle θ_2

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} a_{x''x'} & a_{x''y'} & a_{x''z'} \\ a_{y''x'} & a_{y''y'} & a_{y''z'} \\ a_{z''x'} & a_{z''y'} & a_{z''z'} \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

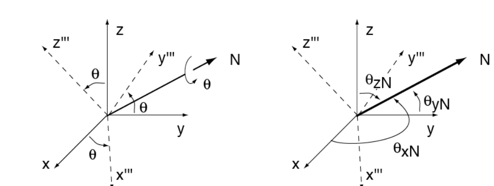
3 Rotate the x",y",z" system about the z"-axis by angle θ_3

$$\begin{bmatrix} x''' \\ y''' \\ z''' \end{bmatrix} = \begin{bmatrix} a_{x'''x''} & a_{x'''y''} & a_{x'''z''} \\ a_{y'''x''} & a_{y'''y''} & a_{y'''z''} \\ a_{z'''x''} & a_{z'''y''} & a_{z'''z''} \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix}$$

10. ROTATIONS (I)

III Concepts behind rotation (cont.)

C Any orthogonal coordinate system can be rotated to coincide with another orthogonal coordinate system by one rotation about a specially chosen axis (Euler's theorem).



This shows the angle of rotation about the rotation axis

This shows the orientation of the rotation axis relative to the three coordinate axes

$\eta_1 = \cos\theta_{xN}$ $\eta_2 = \cos\theta_{yN}$ $\eta_3 = \cos\theta_{zN}$

10. ROTATIONS (I)

III Concepts behind rotation (cont.)

C …rotation about a specially chosen axis (cont.)

$$\begin{bmatrix} x''' \\ y''' \\ z''' \end{bmatrix} = \underbrace{\begin{bmatrix} a_{x'''x''} & a_{x'''y''} & a_{x'''z''} \\ a_{y'''x''} & a_{y'''y''} & a_{y'''z''} \\ a_{z'''x''} & a_{z'''y''} & a_{z'''z''} \end{bmatrix}}_{Third\ Rotation} \left(\underbrace{\begin{bmatrix} a_{x''x'} & a_{x''y'} & a_{x''z'} \\ a_{y''x'} & a_{y''y'} & a_{y''z'} \\ a_{z''x'} & a_{z''y'} & a_{z''z'} \end{bmatrix}}_{Second\ Rotation} \left(\underbrace{\begin{bmatrix} a_{x'x} & a_{x'y} & a_{x'z} \\ a_{y'x} & a_{y'y} & a_{y'z} \\ a_{z'x} & a_{z'y} & a_{z'z} \end{bmatrix}}_{First\ Rotation} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) \right)$$

$$\begin{bmatrix} x''' \\ y''' \\ z''' \end{bmatrix} = \underbrace{\begin{bmatrix} a_{x'''x} & a_{x'''y} & a_{x'''z} \\ a_{y'''x} & a_{y'''y} & a_{y'''z} \\ a_{z'''x} & a_{z'''y} & a_{z'''z} \end{bmatrix}}_{Single\ Rotation} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

10. ROTATIONS (I)

III Concepts behind rotation (cont.)

C …rotation about a specially chosen axis (cont.)

Direction cosines in terms of three rotations (θ_1, θ_2, θ_3) about axes x, y', and z'', respectively

$a_{x'''x} = \cos\theta_3 \cos\theta_2$	$a_{x'''y} = \sin\theta_3 \cos\theta_2$	$a_{x'''z} = -\sin\theta_2$
$a_{y'''x} = -\sin\theta_3 \cos\theta_1 + \cos\theta_3 \sin\theta_2 \sin\theta_1$	$a_{y'''y} = \cos\theta_3 \cos\theta_1 + \sin\theta_3 \sin\theta_2 \sin\theta_1$	$a_{y'''z} = \cos\theta_2 \sin\theta_1$
$a_{z'''x} = \sin\theta_3 \sin\theta_1 + \cos\theta_3 \sin\theta_2 \sin\theta_1$	$a_{z'''y} = -\cos\theta_3 \sin\theta_1 + \sin\theta_3 \sin\theta_2 \cos\theta_1$	$a_{z'''z} = \cos\theta_2 \cos\theta_1$

(by multiplying matrices of slide 7)

10. ROTATIONS (I)

III Concepts behind rotation (cont.)

C …rotation about a specially chosen axis (cont.)

Direction cosines in terms of one rotation angle (θ) about axis with direction cosines η_1, η_2, η_3

$a_{x'''x} = \eta_1\eta_1(1-\cos\theta) + \cos\theta$	$a_{x'''y} = \eta_1\eta_2(1-\cos\theta) + \eta_3\sin\theta$	$a_{x'''z} = \eta_1\eta_3(1-\cos\theta) + \cos\theta$
$a_{y'''x} = \eta_2\eta_1(1-\cos\theta) - \eta_3\sin\theta$	$a_{y'''y} = \eta_2\eta_2(1-\cos\theta) + \cos\theta$	$a_{y'''z} = \eta_2\eta_3(1-\cos\theta) + \eta_1\sin\theta$
$a_{z'''x} = \eta_3\eta_1(1-\cos\theta) + \eta_2\sin\theta$	$a_{z'''y} = \eta_3\eta_2 (1-\cos\theta) - \eta_1\sin\theta$	$a_{z'''z} = \eta_3\eta_3(1-\cos\theta) + \cos\theta$

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10. ROTATIONS (I)

III Concepts behind rotation (cont.)

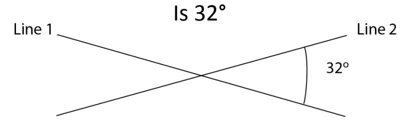
D Angles between lines

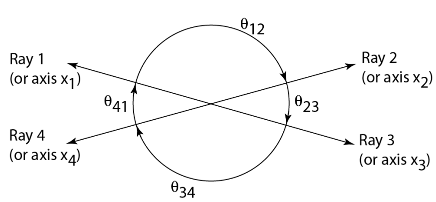
Measure the acute angle between lines

E Angles between rays, axes, or vectors ("directed half-lines")

Angles can be as great as 180°

Angle between lines 1 and 2 Is 32°





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10. ROTATIONS (I)

IV Appendix (Geometry on a Sphere for Plate Tectonics)