9. SPHERICAL PROJECTIONS (II)

I Main Topics

A Angles between lines and planes

B Fold axes of cylindrical folds

C Equal-angle and equal- area projections

D Appendices

9. SPHERICAL PROJECTIONS (II)

II Angles between lines and planes

A Angle between lines

1 The acute angle between the lines

2 Measured in the plane (great circle) containing the lines.

3 Measured along the cyclographic trace of the unique great circle representing the plane containing the two lines.

4 Can be found using the dot product or cross product of the poles

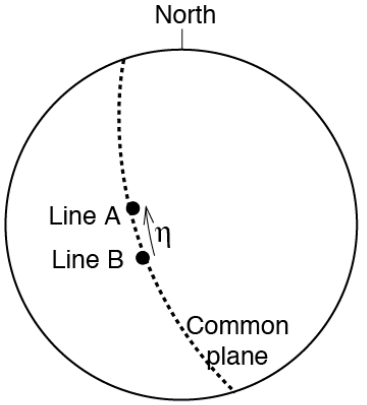
B The angle between two planes

1 The acute angle between the planes (or the poles to the planes)

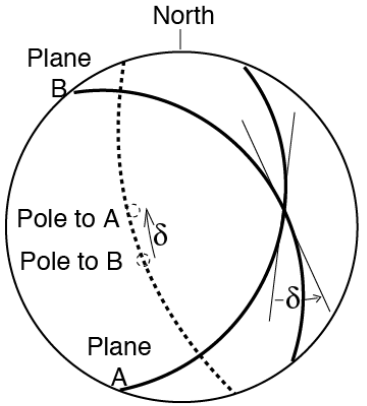
2 Measured in the plane (great circle) perpendicular to the intersection of the planes

3 Measured along the cyclographic trace of the unique great circle representing the plane containing the poles to the two planes.

4 Can be found using the dot product or cross product of the poles



Angle η between two lines



Angle δ between two planes

9. SPHERICAL PROJECTIONS (II)

III Fold axes of cylindrical folds

A Cylindrical fold

1 A folded surface

2 A surface that can be swept out by a line moving parallel to itself

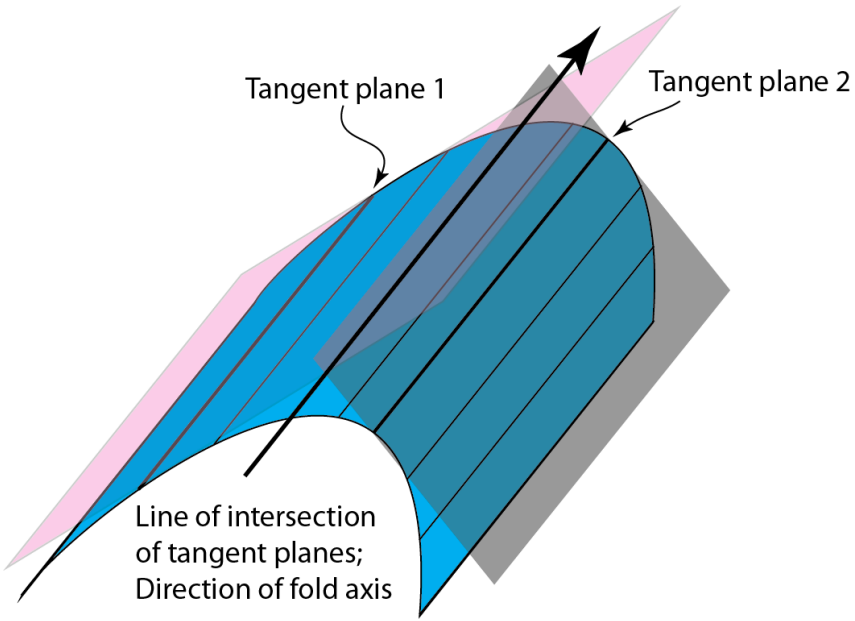
3 A "two-dimensional" fold

B Fold axis

1 The direction of a line parallel to a cylindrical fold

2 The direction of the intersection of planes tangent to a cylindrical fold

3 The direction of the cross product of normals to bedding (not shown here)



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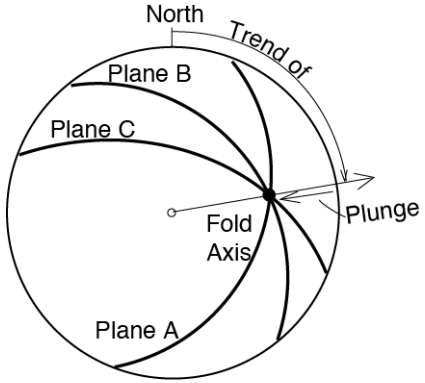
III Fold axes of cylindrical folds

B β diagram

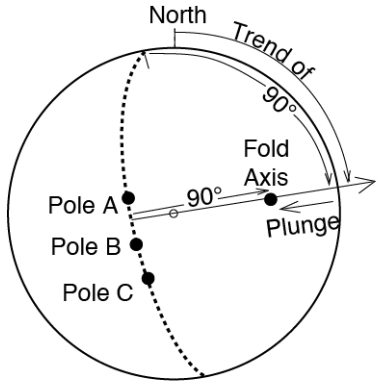
1. Fold axis is along the line of intersection of planes tangent to bedding
2. Direct method

C π diagram

1. Fold axis is perpendicular to the plane containing the poles to beds
2. Preferred over β diagram if many beds are considered
3. For two poles, the π diagram is akin to finding the cross product between the bedding plane poles.
4. Can be extended to many poles



β diagram



π diagram

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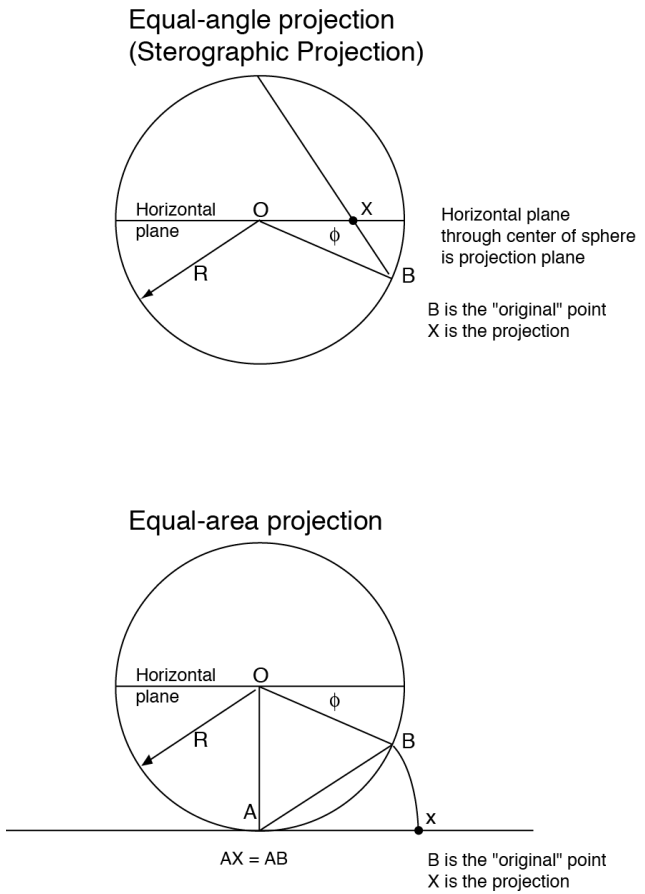
IV Types of spherical projections

- A Equal angle projection
  - 1 Shapes of plane shapes on the surface of the sphere are preserved, but their relative areas are altered.
  - 2 The angles between planes equals the angle between their cyclographic traces
  - 3 Plotted by hand with a Wulff net (see next page)
- B Equal area projection
  - 1 The relative areas of plane shapes on the surface of the sphere are preserved, but their shapes are altered.
  - 2 Good for representing the density of poles
  - 3 Plotted by hand with a Schmidt net (see next page)

Fig. 9.1



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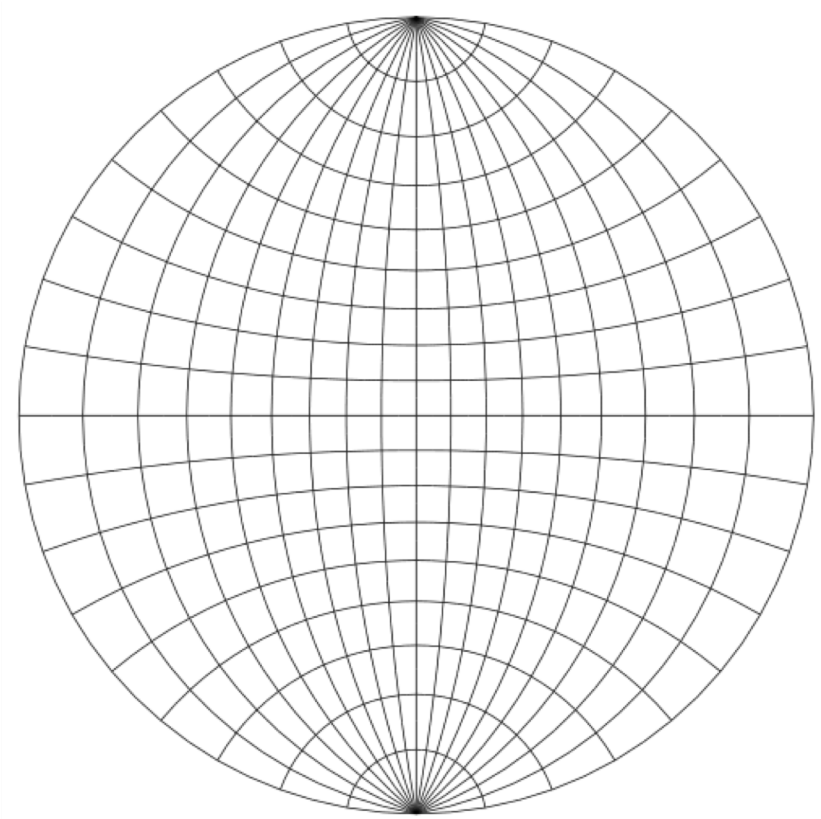
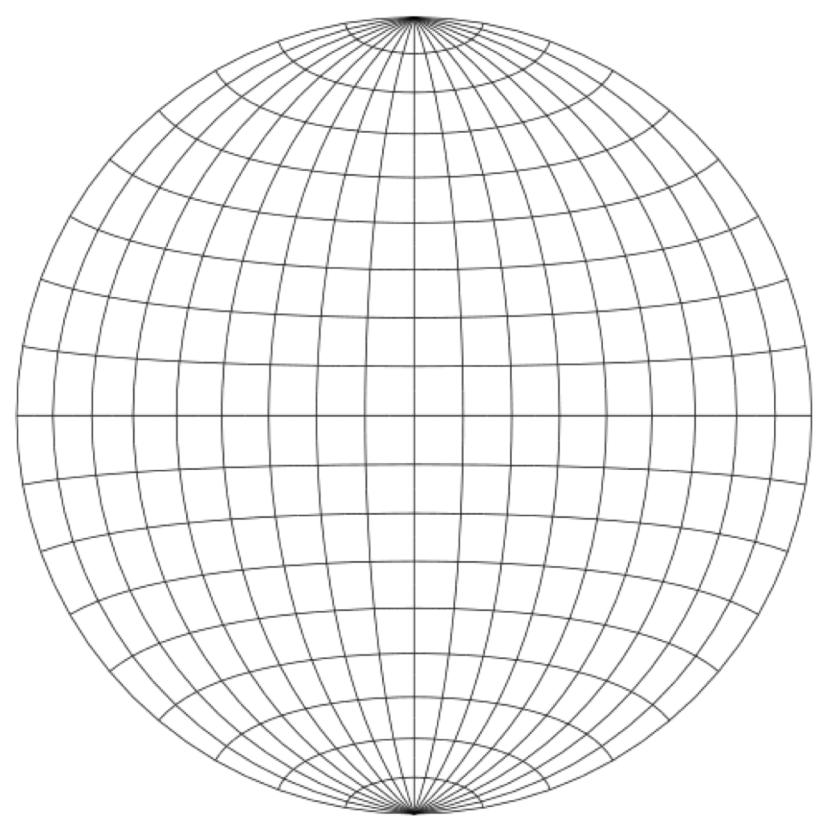
Wulff net



Schmidt net



From http://en.wikipedia.org

9. SPHERICAL PROJECTIONS (II)

(From Hobbs, Means, and Williams, 1976, An Outline of Structural Geology)

Property	Equal angle projection	Equal area projection
Net type	Wulff net	Schmidt net
Projection preserves ...	Angles	Areas
Projection does not preserve ...	Areas	Angles
A line projects as a ...	Point	Point
Great circle projection	Circle	Fourth-order quadric
Small circle projection	Circle	Fourth-order quadric
Distance from center of primitive circle to cyclographic trace measured in direction of dip	$R\tan\left(\frac{\pi}{4}-\frac{dip}{2}\right)$	$R\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{dip}{2}\right)$
Distance from center of primitive circle to pole of plane measured in the direction opposite to that of the dip	$R\tan\left(\frac{dip}{2}\right)$	$R\sqrt{2}\sin\left(\frac{dip}{2}\right)$
Distance from center of primitive circle to point that represents a plunging line	$R\tan\left(\frac{\pi}{4}-\frac{plunge}{2}\right)$	$R\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{plunge}{2}\right)$
Favored use	Measuring angular relations	Contouring orientation data

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V Appendices

A Computer programs for spherical projections (free)

B Stereographic projection of a circle

C Stereographic projection of points on the upper hemisphere

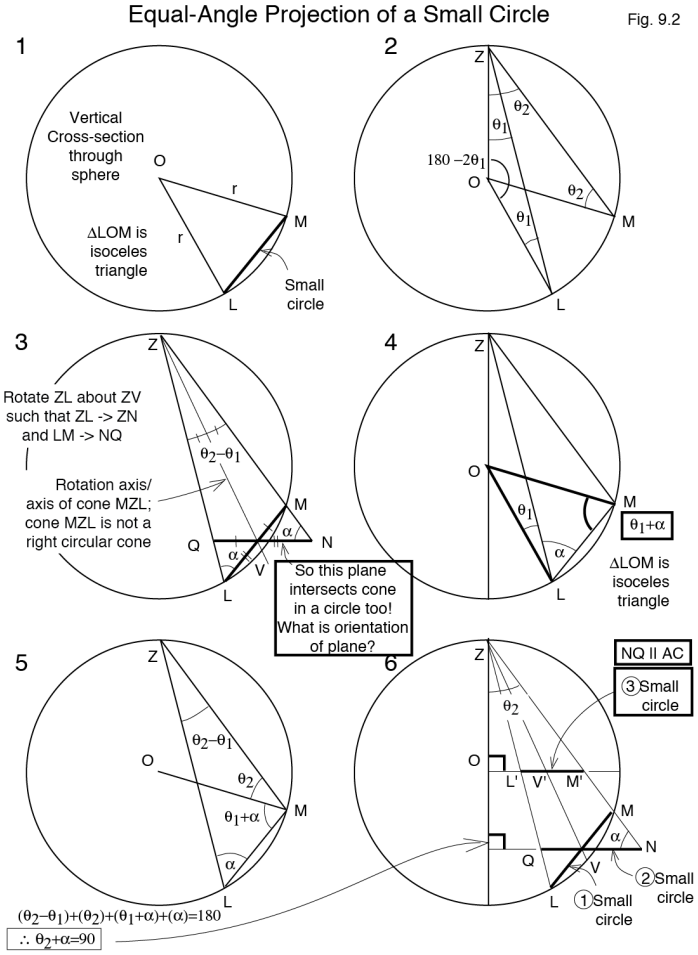
9. SPHERICAL PROJECTIONS (II)

A Computer programs for spherical projections (free)

1 S.J. Martel's Matlab code for spherical projections

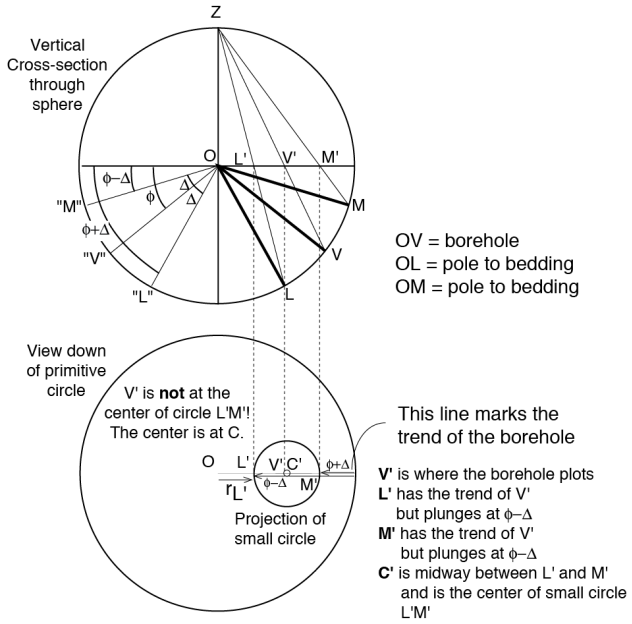
2 "Stereonet" by R. Allmendinger at Cornell University (for the Mac)

9. SPHERICAL PROJECTIONS (II)



9. SPHERICAL PROJECTIONS (II)

Equal-Angle Projection of a Small Circle (II) Fig. 9.3L



$$\frac{r_{L'}}{R} = \tan\left[\frac{90° - (\phi+\Delta)}{2}\right] \qquad r_{L'} = R \tan\left[\frac{90° - (\phi+\Delta)}{2}\right] \qquad \phi+\Delta = 90° - 2(\tan^{-1}[\, r_{L'} / R\,])$$

$$\frac{r_{M'}}{R} = \tan\left[\frac{90° - (\phi-\Delta)}{2}\right] \qquad r_{M'} = R \tan\left[\frac{90° - (\phi-\Delta)}{2}\right] \qquad \phi-\Delta = 90° - 2(\tan^{-1}[\, r_{M'} / R\,])$$

$$r_C = \frac{r_{L'} + r_{M'}}{2} \qquad \text{radius of proj. small circle} = \frac{r_{L'} - r_{M'}}{2}$$

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