# LAB 10 PHOTOELASTIC EXPERIMENT

I Main Topics
- A Photo-elastic experimental determination of stress fields
- B Review of stress contours and stress trajectories

II Photo-elastic experimental determination of stress fields
- A Theory
- B Plane stress vs. plane strain: what does the experiment simulate?

III Review of stress contours and stress trajectories
- A Stress contours: give magnitude of a particular stress component. Here the color fringes (**isochromatics)** give contours of the absolute magnitude of the difference between the principal stresses (i.e., they are contours of twice the absolute magnitude of the maximum shear stress)
- B Principal stress trajectories
  1. These give the orientation of the principal stresses
  2. At a given point, the greatest and least principal stress trajectories will be perpendicular to each other
  3. These can also be thought of as "lines of internal force", that is, as lines along which the principal stresses are transmitted. Internal forces can not be transmitted across cavities
  4. The principal stresses are in some ways analogous to streamlines in fluid flow. A cavity affects the principal stresses similar to the way an obstacle in a current affects the streamlines: the streamlines have to bend around the obstacle.

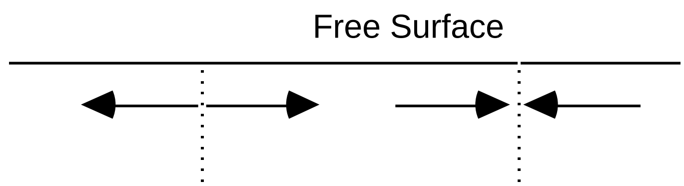
This lab will introduce you to imaging of stress fields and to stress concentrations.

In photoelastic materials the color patterns under crossed polarizing filters can be used to measure the absolute magnitude of the maximum shear stress filed throughout a material. In two-dimensional cases, the absolute magnitude of the maximum shear stress is :

(1) $$\tau_{max} = |\sigma_1 - \sigma_2|/2$$

where $\sigma_1$ is the magnitude of the greatest principal stress and $\sigma_2$ is the magnitude of the least principal stress. The color pattern does not indicate the sign of the maximum shear stress.

Now consider a free surface (i.e., a surface on which no normal or shear stresses act such). The surface of the earth is usually considered to be a free surface (the normal stress due to the weight of the atmosphere is usually ignored). A horizontal free surface is shown below:




The absence of shear traction on the free surface means that the free surface is a principal surface. The absence of a normal traction on the free surface means that the principal stress that acts on the free surface is zero. Because principal surfaces intersect at 90°, then a set of principal planes will interest the free surface at 90°; the diagram above shows a set of vertical principal planes as dotted lines. As shown above, normal stresses can act on the principal planes perpendicular to the free surface. No shear stresses will act on the principal planes perpendicular to the free surface because no shear stresses act on principal planes.

Now suppose we want to know the maximum shear stress near a free surface. One of the principal stresses is zero, so from equation (1):

(2) $$\tau_{max} = |\sigma_1|/2.$$

So the color fringes **at a free surface** also indicate the magnitude of the principal stress acting **parallel** to the free surface:

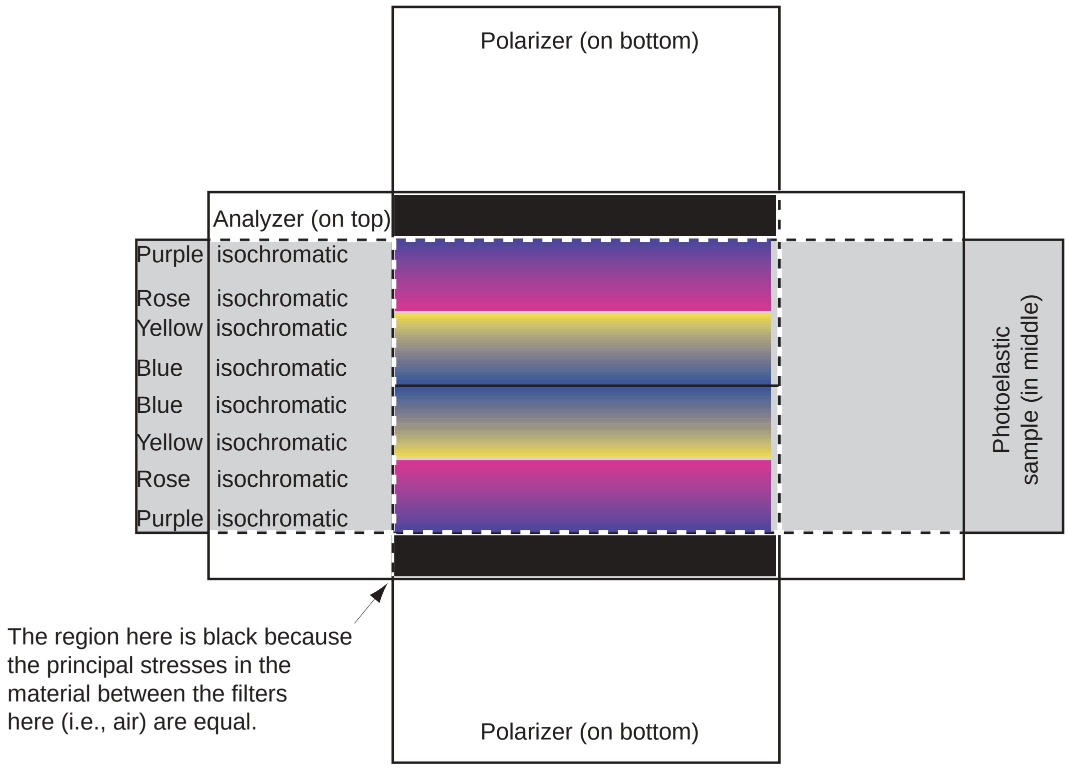
(3) $$|\sigma_1| = 2\,\tau_{max}.$$

Note that the magnitude of the maximum shear stress depends on the magnitude of the one acting principal stress at the free surface and not its sign

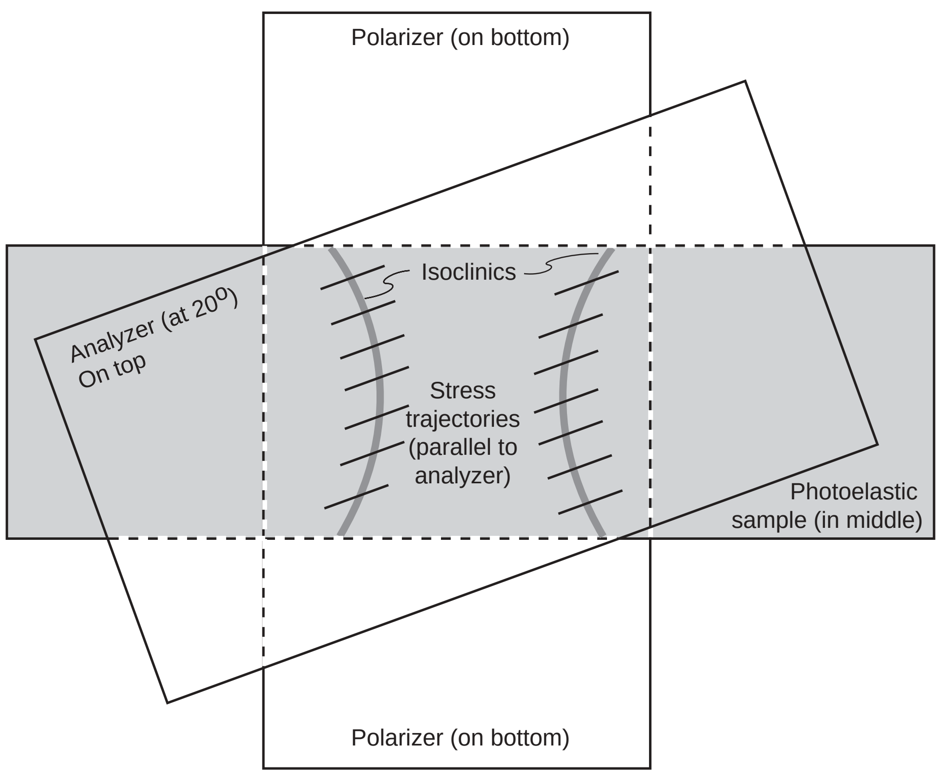
(4) $$|\sigma_1| = |-\sigma_1| = 2\,\tau_{max}.$$

On our photoelastic equipment, the digital readout is proportional to the force that is applied to a sample. The average stress across a rectangular photoelastic "blank" is this force divided by the cross sectional area of the blank.

## Isochromatics (contours of maximum shear stress) for a bent bar




## Isoclinics (black bands where a principal stress parallels the analyzer)



**Stress trajectories**

1 Trajectories in a sample with no holes or cracks **(5 pts total)**

a Turn on the projector and move it/focus it such that it illuminates an area about 2 feet on a side on the blackboard

b Tape a sheet of white paper (long axis horizontal) on the blackboard where the overhead projector is aimed.

c Place the Polaroid strip in the cardboard frame (i.e., the analyzer) on the projector, with the long axis of the analyzer perpendicular to the blackboard.

d Place the plastic strip with no holes or cuts in the loading frame and secure it with the steel pins. Make sure the sample is level.

e Place the load frame on the projector over the analyzer such that the long axis of the load frame parallels the blackboard (you will need some books or other props to raise the load frame above the analyzer). Make sure the sample projects onto the paper, and be careful with the wires connected to the meter that gives the load frame readout.

f Place one layer of Polaroid film over the center of the sample (this layer of Polaroid material is called the analyzer) and turn it so that it is parallel to the long axis of the plastic sample. The area covered by the crossed filters should appear black on the white paper (no light is transmitted).

g Refocus the projector if necessary. The projector is focused if the scratches on the plastic sample are well-focused on the piece of paper.

h Turn the knob on the load frame to apply a tension on the sample; don't compress the sample – if you do it will bow and could snap.

i Turn the upper Polaroid filter, and notice how the intensity of the transmitted light changes.

j ***Re-orient the upper Polaroid layer (the analyzer) such that cause the sample to appear black***.

k ***Where no light is transmitted (i.e., where the sample appears darkest), draw with a black felt tip pen draw tick marks parallel to the analyzer on the sheet of paper***; these are trajectories parallel to the most tensile stress. **Put opposing arrowheads on these ticks**.

l Xerox the piece of paper. At the ends of the sample, draw and label arrows showing the tension applied to the sample by the load frame. Label the ticks "Trajectories Parallel to the Most Tensile Stress." With a thin solid pencil line draw (and label) trajectories parallel to the most compressive stress; these will be perpendicular to the trajectories of the most tensile stress. **Put arrowheads that point together on these ticks**.

2 Trajectories in a sample with a circular hole

Repeat the above steps with the sample with the circular hole, but take special care with steps j and k. In this case, unlike test case 1, the sample will not be dark everywhere. In order to draw the ticks marking the trajectories parallel to the most tensile stress you will have to rotate the analyzer (the upper filter) incrementally to a variety of positions (unlike step j in part 1). Only draw the ticks where the sample is very dark, and ***remember to draw the ticks parallel to the long direction of the analyzer (this direction does not necessarily parallel any dark bands)***. This works the best if you use the following sequence:

* Start with the analyzer at ~45° to the sample (counterclockwise) and draw any stress trajectories you can see.

- Rotate the analyzer towards parallel by ~15° increments, drawing stress trajectories as you go. Draw tick marks where none where drawn previously.
- Rotate the analyzer to ~45° clockwise past parallel and draw any stress trajectories you can see. Draw tick marks where none where drawn previously.
- Rotate the analyzer towards parallel by ~15° increments, drawing stress trajectories as you go. Draw tick marks where none where drawn previously.
- Work the region from ±15° to get the stress trajectories in the areas that appear dark
- Make sure you work through all the steps, and label the stress trajectories with arrows like you did in exercise 1. **(5 pts)**
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**Stress Magnitudes**

3 Trajectories in a sample with no holes or cracks (use the real polariscope)

a Tape a new piece of paper on the blackboard and prepare a table that shows the photoelastic color in the sample center (column F) for a photoelastic blank with no hole as a function of the digitial readout number (column A). You might want to record the load at a recognizable and distinctive color (or color transition) rather than recording the color at regularly spaced load increments, but you may want to do both. **Do not exceed a load level of 240; you will damage the sample! (5 pts)**

| A Load cell reading "Units" | B Cross-section Area ($cm^2$) | C $\sigma_n$ along long axis (Units/ $cm^2$) | D $\sigma_n$ across short axis (Units/ $cm^2$) | E Maximum shear stress (Units/ $cm^2$) | (F)Color |
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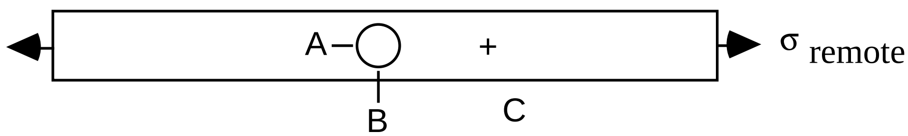
b Now insert the photoelastic strip with a hole. Load the sample up to a level of about 30 units. Based on your table above, prepare a contour map showing the maximum shear stress. Label each curve with its color (this is *data*) and maximum shear stress you think corresponds to the color (an *interpretation*) based on the table of the preceding page. For example, if indigo corresponds to a maximum shear stress of 10 units/cm2, then draw a curve labeled "10" on the piece of paper on the indigo color stripe. ***Pay particular attention to the color at the perimeter of the hole at points A and B – this point is extraordinarily important for this lab***. The colors change quickly there because of localized stress concentrations. **(5 pts)**

c Note at least two locations where the colors indicate that the maximum shear stress is zero. **(2 pts)**




Point C is 2.5 hole diameters from the hole center

d Based on your contour map, draw two profiles across the contour map of the maximum shear stress, one along the long axis of the sample, and the other across the short axis.

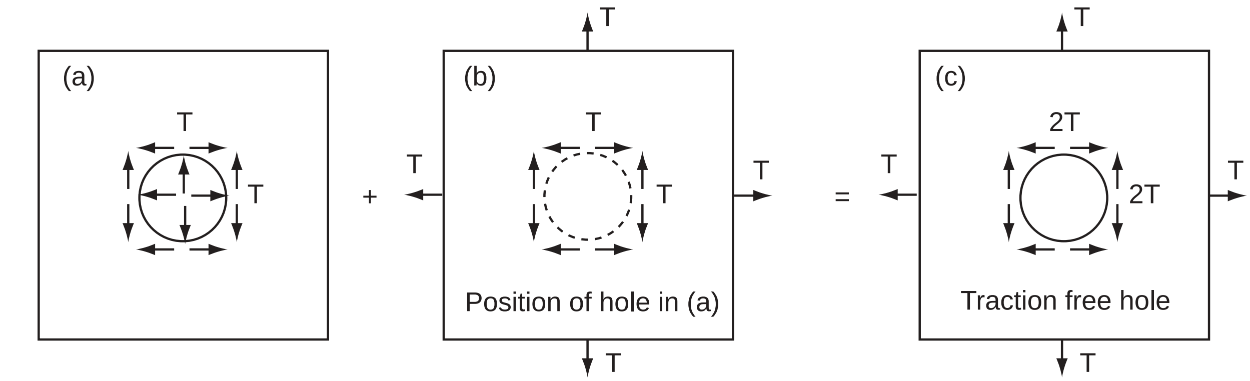
e Assuming that the normal traction acting on the free edges of the sample is zero (because the sample is unloaded there), calculate the magnitude of the ***non-zero*** normal stress at points A and B ($\sigma_n^A$ and $\sigma_n^B$). **(4 pts)** Assuming that the magnitude of the normal stress acting across the sample is zero at point C (so the only nonzero normal stress is ***parallel*** to the sample) calculate the ratios of the $\sigma_n^A/\sigma_n^C$ and $\sigma_n^B/\sigma_n^C$ The attached diagram titled "superposition" can serve as a guide to what your answer should be. **(4 pts)**

**Implications**

4) Suppose the hole represents a magma chamber. For this case where a uniaxial **tension** is applied to the sample, where might cracks (dikes) form if the applied tensile stress becomes large? Show this on a diagram and explain your reasoning. What would the orientation of the cracks be? **(5 pts)**

5) Suppose the hole represents a magma chamber. What would happen if a uniaxial **compression** were applied to the sample? Where might cracks (dikes) form if the applied compressive stress becomes large? Show this on a diagram and explain your reasoning. What would the orientation of the cracks be? The small block provided with a hole gives you a chance to check your answer and reasoning. **(5 pts)**

## SUPERPOSITION

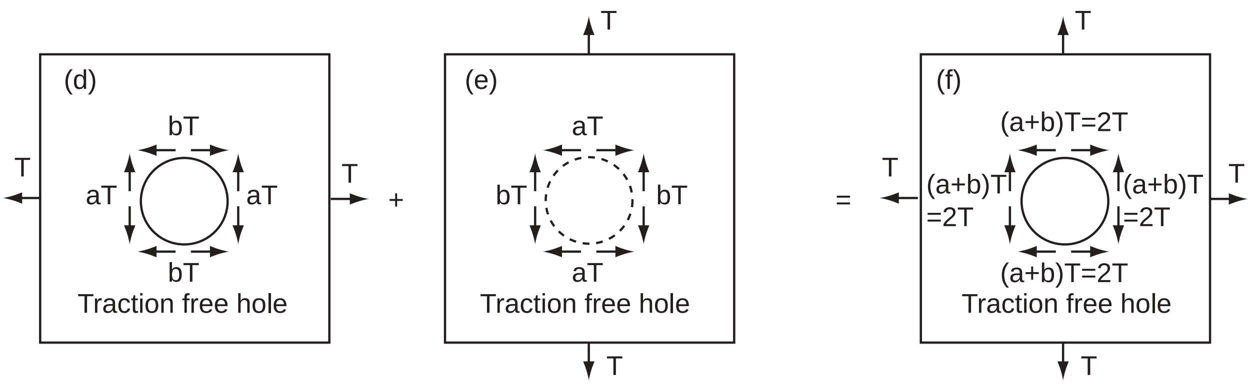



Pressure in a hole + Biaxial Tension (no hole) =

| Pressure in a hole | | Biaxial Tension (no hole) | | |
|---|---|---|---|---|
| $\sigma_{rr} = -T(a/r)^2$ | + | $\sigma_{rr} = T$ | = | $\sigma_{rr} = T - T(a/r)^2$ |
| $\sigma_{\theta\theta} = -(-T)(a/r)^2$ | | $\sigma_{\theta\theta} = T$ | | $\sigma_{\theta\theta} = T + T(a/r)^2$ |

$$\boxed{\sigma_{\theta\theta}\Big|_{r=a} = 2T}$$

Stress concentration!




Uniaxial Tension    Uniaxial Tension    Biaxial Tension