

PHOTOELASTIC STRESS EXPERIMENT: DIKES IN GELATIN

I Main Topics

A Photo-elastic experimental determination of stress fields

B Review of stress contours and stress trajectories

II Photo-elastic experimental determination of stress fields

A In isotropic materials stress is proportional to strain

B Polarized light traveling through a stressed material splits into two components polarized parallel to the direction of principal stress.

C These components travel at different speeds through the material and get out of phase.

D The total phase shift depends on the optic strain coefficient of the material, the stress state, and the thickness of the material.

E The components interfere to produce colors. The colors reflect the total phase shift, and hence the stress field.

III Review of stress contours and stress trajectories

A Stress contours: give magnitude of a stress component. Here the color fringes (**isochromatics**) give contours of the absolute magnitude of the difference between the principal stresses

B Principal stress trajectories

1 These give the orientation of the principal stresses

2 At a given point, the greatest and least principal stress trajectories will be perpendicular to each other

3 These can also be thought of as "lines of internal force", that is, as lines along which the principal stresses are transmitted. Internal forces can not be transmitted across cavities

4 The principal stresses are in some ways analogous to streamlines in fluid flow. Principal stresses deflect around a cavity the way streamlines deflect around an obstacle.

5 With monochromatic light, black lines (or black regions) appear where the analyzer (the exiting polarizing filter) is parallel to a principal stress. This black line (or region) is called an isoclinic. The isoclinic is not parallel to the principal stresses - the isoclinic shows points where the principal stresses in the material are parallel to the analyzer.

The equations relating strains (ϵ) and stresses (σ) in a homogeneous isotropic linear elastic material are:

Normal stresses and strains	Shear stresses and strains
$\epsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu[\sigma_{yy} + \sigma_{zz}])$	$\epsilon_{xy} = \frac{1+\nu}{E}(\sigma_{xy}); \epsilon_{yx} = \frac{1+\nu}{E}(\sigma_{yx})$
$\epsilon_{yy} = \frac{1}{E}(\sigma_{yy} - \nu[\sigma_{zz} + \sigma_{xx}])$	$\epsilon_{yz} = \frac{1+\nu}{E}(\sigma_{yz}); \epsilon_{zy} = \frac{1+\nu}{E}(\sigma_{zy})$
$\epsilon_{zz} = \frac{1}{E}(\sigma_{zz} - \nu[\sigma_{xx} + \sigma_{yy}])$	$\epsilon_{zx} = \frac{1+\nu}{E}(\sigma_{zx}); \epsilon_{xz} = \frac{1+\nu}{E}(\sigma_{xz})$
$\sigma_{xx} = \frac{E}{(1+\nu)} \left[\epsilon_{xx} + \frac{\nu}{(1-2\nu)} (\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}) \right]$	$\sigma_{xy} = \frac{E}{1+\nu}(\epsilon_{xy}); \sigma_{yx} = \frac{E}{1+\nu}(\epsilon_{yx})$
$\sigma_{yy} = \frac{E}{(1+\nu)} \left[\epsilon_{yy} + \frac{\nu}{(1-2\nu)} (\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}) \right]$	$\sigma_{yz} = \frac{E}{1+\nu}(\epsilon_{yz}); \sigma_{zy} = \frac{E}{1+\nu}(\epsilon_{zy})$
$\sigma_{zz} = \frac{E}{(1+\nu)} \left[\epsilon_{zz} + \frac{\nu}{(1-2\nu)} (\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}) \right]$	$\sigma_{zx} = \frac{E}{1+\nu}(\epsilon_{zx}); \sigma_{xz} = \frac{E}{1+\nu}(\epsilon_{xz})$

Here E is Young's modulus and ν is Poisson's ratio. For gelatin, the material in our experiment: $E \approx 1332 \text{ Pa}$ (i.e., 1332 Newtons per square meter).

$\nu \approx 0.5$.

For our tank experiments, we will let the z-axis point down, the x-axis parallel the short dimension of the tank, and the y-axis parallel to the long dimension of the tank. The reference frame origin is at the surface of the gelatin. We will let tensile stresses be positive; compressive stresses be negative.

In our tank, the vertical normal stress is due to the weight of the gelatin and the atmosphere:

$$\sigma_{zz} = -(\rho_{gelatin}gz + P_{air})$$

where

$\rho_{gelatin}$

$$\rho_{gelatin} = 1.02 \times 10^3 \text{ kg/m}^3, \quad g = 9.8 \text{ m/sec}^2, \quad \text{and} \quad P_{air} = 10^5 \text{ kg/m sec}^2$$

The maximum shear stress at a point is

$$|\tau_{\max}| = \frac{|\sigma_1 - \sigma_3|}{2}$$

We will be conducting a laboratory experiment using a tank of gelatin to gain insight into how dikes propagate. This lab is designed to:

- Illustrate the usefulness of experiments in geology
- Test whether dike growth can be predicted (or understood) based on (a) a knowledge of stresses, and (b) ***an operating hypothesis that dikes open in the direction of the least compressive stress and propagate in directions perpendicular to the least compressive stress.***
- Make you think about boundary conditions
- Help you "see" stresses

Read (skim) the whole lab before proceeding, and answer all 20 questions on separate pages.

Experiment A: Hypothesis development for the unperturbed state of stress (this might take about an hour) 32 PTS

Our first experiment is a thought experiment to deduce the state of stress in the tank of gelatin without any dikes present. This will give us something to test (and not just something to observe) when we conduct the actual experiment.

We start by examining the strains. If we assume (a) that the sides and bottom of the fish tank are complete rigid, and (b) that the gelatin is not sticking (too much) to the sides of the tank, then (c) the displacements in the gelatin in the x and y directions due to gravity are zero, and (d) there is no shear stress at the edges of the tank.

- 1 ***Based on the simplifying assumption that the tank walls are completely rigid and prevent horizontal displacements***, find all the strains except ϵ_{zz} for the unperturbed gelatin in the tank and write them below. The elastic strains in infinitesimal strain theory are obtained by examining how displacements change as a function of position. Use a separate page to show any work you need or to provide an explanation.

Normal strains	Shear strains
$\epsilon_{xx} = \frac{\partial u_x}{\partial x} =$ 1pt	$\epsilon_{xy} = \epsilon_{yx} = \frac{1}{2} \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) =$ 1pt
$\epsilon_{yy} = \frac{\partial u_y}{\partial y} =$ 1pt	$\epsilon_{yz} = \epsilon_{zy} = \frac{1}{2} \left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \right) =$ 1pt
$\epsilon_{zz} = \frac{\partial u_z}{\partial z}$ (Don't solve for this one)	$\epsilon_{zx} = \epsilon_{xz} = \frac{1}{2} \left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right) =$ 1pt

- 2 Using the stress-strain relationships in the last two rows of the left column of the table below and your answers to the previous question, determine the ratio between the horizontal stresses σ_{xx} and σ_{yy} . Show your work on a separate page. **5pts**

Normal stresses and strains	Shear stresses and strains
$\epsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu[\sigma_{yy} + \sigma_{zz}])$	$\epsilon_{xy} = \frac{1+\nu}{E}(\sigma_{xy}); \epsilon_{yx} = \frac{1+\nu}{E}(\sigma_{yx})$
$\epsilon_{yy} = \frac{1}{E}(\sigma_{yy} - \nu[\sigma_{zz} + \sigma_{xx}])$	$\epsilon_{yz} = \frac{1+\nu}{E}(\sigma_{yz}); \epsilon_{zy} = \frac{1+\nu}{E}(\sigma_{zy})$
$\epsilon_{zz} = \frac{1}{E}(\sigma_{zz} - \nu[\sigma_{xx} + \sigma_{yy}])$	$\epsilon_{zx} = \frac{1+\nu}{E}(\sigma_{zx}); \epsilon_{xz} = \frac{1+\nu}{E}(\sigma_{xz})$
$\sigma_{xx} = \frac{E}{1+\nu}\epsilon_{xx} + \frac{\nu E}{(1+\nu)(1-2\nu)}(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz})$	$\sigma_{xy} = \frac{E}{1+\nu}\epsilon_{xy}; \sigma_{yx} = \frac{E}{1+\nu}\epsilon_{yx}$
$\sigma_{yy} = \frac{E}{1+\nu}\epsilon_{yy} + \frac{\nu E}{(1+\nu)(1-2\nu)}(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz})$	$\sigma_{yz} = \frac{E}{1+\nu}\epsilon_{yz}; \sigma_{zy} = \frac{E}{1+\nu}\epsilon_{zy}$
$\sigma_{zz} = \frac{E}{1+\nu}\epsilon_{zz} + \frac{\nu E}{(1+\nu)(1-2\nu)}(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz})$	$\sigma_{zx} = \frac{E}{1+\nu}\epsilon_{zx}; \sigma_{xz} = \frac{E}{1+\nu}\epsilon_{xz}$

- 3 Using your answers to questions 1 and 2, the stress-strain relationships provided, and the provided value of Poisson's ratio, express all the stresses in the tank, solving for any non-zero stresses in terms of the normal stress σ_{zz} . Show your work on a separate page.

Normal stresses	Shear stresses
$\sigma_{xx} =$ 1pts	$\sigma_{xy} = \sigma_{yx} =$ 1pt
$\sigma_{yy} =$ 1pts	$\sigma_{yz} = \sigma_{zy} =$ 1pt
$\sigma_{zz} = \sigma_{zz}$	$\sigma_{zx} = \sigma_{xz} =$ 1pt

- 3 On a separate piece of paper plot three Mohr circle diagrams showing the state of stress, one each for tractions in the xy, yz, and zx planes, respectively. For the axes of the plots, show t_n/σ_{zz} and t_s/σ_{zz} instead of σ_n and τ . This is called *normalizing* the plots. **(3 pt/plot)**
- 4 What do you predict the maximum shear stress be in the tank? Write this as a function of vertical position z. **2pts**
- 5 Based on your answers so far, what do you predict regarding the orientation of the principal stresses in the tank of gelatin? Are they uniquely determined? **2pts**

Experiment B: Observations of the unperturbed state of stress in a tank of gelatin. 33 PTS

Now we test our hypotheses experimentally. Put one polarizing filter on each side of the tank of gelatin and cross the filters and shine a light through them. The filters should parallel the long axis of the tank. You can rotate the filters, but keep them perpendicular to each other.

- 6 Describe the color pattern as seen through the filters for the air and for the gelatin. **2pts**
- 7 By rotating the analyzing filter (the one away from the light), determine the principal stress trajectories in the tank. Use the isoclinics and orientations of the filters. **2pts**
- 8 Are your observations consistent with your theoretical prediction at the bottom of the previous page? Explain thoroughly. **5pts**

Read questions 9-13 and be prepared to act fast on them. Then re-cross the polarizing filters. Insert the injection needle vertically through a hole about 1/4 of the way from the center of the tank to the middle. Wiggle the needle back and forth parallel to the short dimension of the tank to make a small starter cut. Then inject a vertical dike with 3 cc of air.

- 9 Carefully describe the shape of the dike as viewed (a) across the short dimension of the tank and (b) as viewed across the long dimension of the tank. Is the dike symmetric with respect to a horizontal plane? With respect to a vertical plane? **5pts**
- 10 Describe as best you can the color field and what it implies in general terms about stresses around the air dike. Is it uniform? Pay special attention to the tips of the dike. **5pts**
- 11 What happens to the color field when the dike gets to the surface, and what does this mean in terms of the stresses in the tank? **2pts**
- 12 Describe/trace the dike path. **2pts**
- 13 Discuss whether the dike path is consistent with the operating hypothesis. **5pts**

Cross the polarizers. Insert the injection needle **into the next hole towards the tank end** but aim the needle towards the point where the surface of the gelatin intersects the tank end; this is to make an inclined dike. Wiggle the needle back and forth parallel to the short dimension of the tank to make a small starter cut. Then inject the dike with 3 cc of air.

- 14 Describe/trace the dike path. Is it consistent with the operating hypothesis? Why? **5pts**

Experiment C (Fish tank with a surface load)**35 PTS**

Now place a load on the surface that spans the short dimension of the tank and cross the polarizers, setting one horizontal and the other vertical. The load mass is 0.076 kg, and its base is 0.101 m long and 0.031 m wide. You will need to calculate the force/per unit length F_z in Newtons/meter that the load induces on the gelatin.

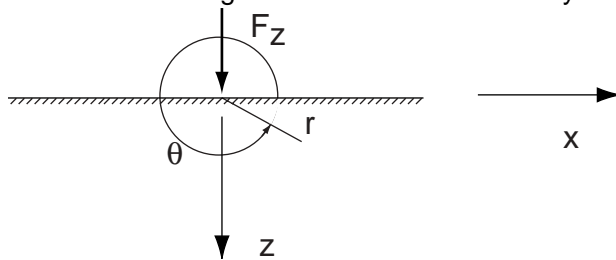
- 15 Draw the color pattern as seen through the filters for the air and for the gelatin. **5pts**
- 16 By rotating the analyzing filter (the one away from the light), determine the principal stress trajectories in the tank by finding the isoclinics and drawing trajectories along the isoclinics parallel to the analyzer. **5pts**
- 17 Inject a series of a few vertical air dikes in the tank B. After this is done, inject them with colored water so that they are visible and then trace the dike paths to scale. **5pts**

Analysis, interpretation, and conclusions

Suppose a positive force (per unit length in the y -direction) F_z is applied perpendicular to the horizontal surface of an elastic body of infinite depth in the positive z -direction (i.e., down), and no displacements occur in the y -direction. Elasticity theory predicts that, **in the absence of gravity**, the stresses in this 2-D case are (Barber, 1992, p. 139-143).

$$\sigma_{rr} = \frac{2F_z \sin^2 \theta}{\pi r} \quad \sigma_{\theta\theta} = 0 \quad \sigma_{r\theta} = 0$$

where r is the distance from the point of application of the force on the surface and positive θ is measured in a right-hand sense about the y -axis.



- 18 What is the direction of the stress trajectories that are perpendicular to the most tensile stress for a normal surface force, and what does this imply for dike paths if our hypothesis is correct and if gravity were turned off? **5pts**
- 19 What is the magnitude of the greatest shear stress as a function of radial distance r ? Express this as a function of F_z, r , and θ . **5pts**
- 20 The approximate total stress field for experiment C is given by the Matlab functions "geldike" and "traj2". The first converts the stresses due to gravity and those due to a surface force to a common reference frame and superposes them. It also finds the magnitude of the maximum shear stress and the orientation of the principal stresses for the total stress field. The traj2 function plots stress trajectories and is a modified version of matlab's "quiver" function. Using the correct value for F_z , use geldike to help test the hypothesis. What do you conclude? Where do the theoretical assumptions differ significantly from the experimental conditions? **10pts**

```

function geldike(Fz,num)
% Calculates the trajectories perpendicular to the direction
% of the least compressive stress and contours of the
% maximum shear stress for the gelatin dike experiment for
% GG303, assuming no displacements in the direction of the
% short dimension of the tank.
% Fz = the force (weight) of the surface load in Newtons (kg m / sec^2)
% If Fz = 0, one obtains the stresses in a tank with no surface load
% num = number of contours on contour plot

% Set material parameters and grid dimensions
rho = 1.02e03;
g = 9.8;
nu = 0.5;
E = 1332;
G = 444;
%X = -0.245:0.01:0.245;
%X = -0.2475:0.005:0.2475;
X = -0.0975:0.005:0.0975;
%Z = 0:0.01:0.2;
Z = 0:0.005:0.2;
Pair = 1e06;

[x,z] = meshgrid(X,Z);
r = sqrt(x.*x + z.*z);
theta = atan2(x,z);
zip = zeros(size(x));

% Find direction cosines between x,z and r,t axes
axr = x./r;
azr = z./r;
axt = azr;
azt = -axr;

% Find stresses due to gravity
szzg = -rho*g*z - Pair;
sxxg = (nu/(1-nu)) * szzg;
sxzg = zip;
szxg = zip;
syyg = (nu/(1-nu)) * szzg;

% Find stresses due to surface force Fz
%sint = sin(theta);
sint = -z./r;
srrF = (2*Fz/pi)*(sint./r);
srtF = zip;
strF = zip;
sttF = zip;

% Convert stresses due to surface forces to x,y frame
sxxF = axr.*axr.*srrF...
      + axr.*axt.*srtF...

```

```

    + axt.*axr.*strF...
    + axt.*axt.*sttF;
sxxF = axr.*azr.*srrF...
    + axr.*azt.*srtF...
    + axt.*azr.*strF...
    + axt.*azt.*sttF;
syzF = azr.*axr.*srrF...
    + azr.*axt.*srtF...
    + azt.*axr.*strF...
    + azt.*axt.*sttF;
szzF = azr.*azr.*srrF...
    + azr.*azt.*srtF...
    + azt.*azr.*strF...
    + azt.*azt.*sttF;

```

```

% Superpose the gravitational stresses and those due to
% the surface force

```

```

sxx = sxxg + sxxF;
syz = syzg + syzF;
syz = syzg + syzF;
szz = szzg + szzF;

```

```

% Contour the maximum shear stress
% The formula below can be derived from a Mohr circle
figure(1)

```

```

clf
maxshear = (1/2)*sqrt( (sxx-szz).^2 + (syz-szy).^2 );
colormap(jet)
brighten(jet,0.8)
contourf(x,-z,maxshear,num);
hold on
line([min(X),max(X)],[min(Z),min(Z)]);

```

```

xlabel('x')
ylabel('z')
axis('equal');
title('Contours of maximum shear stress')

```

```

% Contour the maximum shear stress
% The formula below can be derived from a Mohr circle
figure(2)

```

```

clf
maxshear = (1/2)*sqrt( (sxx-szz).^2 + (syz-szy).^2 );
cc = contour(x,-z,maxshear,num);
clabel(cc);
hold on
line([min(X),max(X)],[min(Z),min(Z)]);

```

```

xlabel('x')
ylabel('z')
axis('equal');
title('Contours of maximum shear stress')

```



```
% Find the trajectories perpendicular to the least
% compressive stress
figure(3)
clf
ang = 0.5*atan2(sxz,(szz-sxx)/2);
axis('equal');
index = find(x<0);
ang(index) = ang(index) + pi;
traj2(x,-z,cos(ang),sin(ang))
xlabel('x')
ylabel('z')
axis('equal');
title('Direction of most compressive stress')
```

```

function traj2(arg1,arg2,arg3,arg4,arg5,arg6)
%TRAJ2 Trajectory plot. Modified version of QUIVER version 4.1
% TRAJ2(X,Y,DX,DY) draws little headless arrows at every (X,Y) pair in
% matrices X and Y. The (DX,DY) pairs in matrices DX and DY
% determine the direction and magnitude of the arrows.
% In order for this function to work properly, DX and DY
% MUST be equal to the cosine and sine, respectively, of the
% orientation of the headless arrows.
%
% TRAJ2(x,y,DX,DY), with two vector arguments replacing the first
% two matrix arguments, must have length(x) = n and
% length(y) = m where [m,n] = size(DX) = size(DY). In this case, the
% arrows are the quads (x(j), y(i), DX(i,j), DY(i,j)).
% Note that x corresponds to the columns of DX and DY and y corresponds
% to the rows.
%
% TRAJ2(DX,DY) uses x = 1:n and y = 1:m. In this case DX and DY
% are defined over a geometrically rectangular grid.
%
% TRAJ2(X,Y,DX,DY,S) and TRAJ2(DX,DY,S) apply scalar S as a scale
% factor to the lengths of the arrow. For example, S = 2 doubles
% their relative length and S = 0.5 halves them.
%
% A final trailing string argument specifies linetype and color using
% any legal line specification as described under the plot command.
%
% For example, try
% xord = -2:.2:2;
% yord = -2:.2:2;
% [x,y] = meshgrid(xord,yord);
% z = x .* exp(-x.^2 - y.^2);
% [px,py] = gradient(z,.2,.2);
% contour(x,y,z),hold on, quiver(x,y,px,py), hold off
%
% See also GRADIENT, COMPASS, FEATHER, ROSE.
%
% Charles R. Denham, MathWorks 3-20-89
% Modified 12-19-91, LS.
% Modified by SJM 10/22/93
% Copyright (c) 1984-93 by The MathWorks, Inc.

xx = [0 1 NaN NaN NaN].';
yy = [0 0 NaN NaN NaN].';
arrow = xx + yy.*sqrt(-1);

eval(['last = arg' int2str(nargin) ';'']);
if isstr(last)
    eval(['style = arg' int2str(nargin) ';'']);
    narg = nargin - 1;
else
    style = 'r-';
    narg = nargin;
    eval(['last = arg' int2str(nargin-1) ';'']);

```

```

        if isstr(last)
            error('Only the final argument can be a string.');
```

end

```

end
if narg == 0
    error('First 2 or 4 arguments must be numeric.')
```

end

```

eval(['lastdim = min(size(arg' int2str(narg) ');');]);
if lastdim == 1
    if isstr(eval(['arg' int2str(narg)]))
        error('Scalar scale argument expected.')
```

end

```

    eval(['scale = arg' int2str(narg) ';']);
    narg = narg - 1;
else
    scale = 1;
end
if narg == 0
    error('First 2 or 4 arguments must be numeric.')
```

end

```

if min(size(arg1)) > 1
    [m,n] = size(arg1);
else
    m = max(size(arg1));
    n = max(size(arg2));
end
if narg == 2
    if isstr(arg1) | isstr(arg2)
        error('Input must be numeric.')
```

end

```

    [xx,yy] = meshgrid(1:n, 1:m);
    px = arg1;
    py = arg2;
else
    if isstr(arg1) | isstr(arg2) | isstr(arg3) | isstr(arg4)
        error('Input must be numeric.')
```

end

```

    if min(size(arg1)) == 1 & min(size(arg2)) == 1
        [xx,yy] = meshgrid(arg1,arg2);
    else
        xx = arg1;
        yy = arg2;
    end
    px = arg3;
    py = arg4;
end

% figure out delx and dely so spacing is accounted for in z
delx = xx(1,2)-xx(1,1);
dely = yy(2,1)-yy(1,1);

grid = xx + yy.*sqrt(-1); grid = grid(:);
px = px(:); py = py(:);
```

```
maxlen = max(sqrt((px/delx).^2+(py/dely).^2));
z = (px + py.*sqrt(-1)).';
% The next active line was modified 10/22/93 by SJM
% The scaling factor of 0.90 was reset to 0.40
scale = scale*0.40 ./ maxlen;
%a = scale * arrowU * z + ones(5,1) * grid.U; %mathworks suggestion
a = scale * arrow * z + ones(5,1) * grid.';
% append nan's so we get one handle
a = [a; nan*ones(1,size(a,2))];
a = a(:);
cax = newplot;
plot(real(a), imag(a), style);
next = lower(get(cax,'NextPlot'));
if ~ishold
    minx = min(min(xx));
    miny = min(min(yy));
    maxx = max(max(xx));
    maxy = max(max(yy));
    axis([minx maxx miny maxy]);
    view(0,90);
end
```

Directions for preparing ordnance-grade gelatin

Date: Tue, 14 Nov 2000 11:02:00 -0800

To: Steve Martel <martel@soest.hawaii.edu>

From: Jordan Muller <mullerj@pangea.stanford.edu>

Subject: Re: How do you cook gelatin?

This is really where the fun begins. I put a large pot with half of the amount of water for your gelatin on a lab hot plate. Let that water get really hot, although I don't think that it has to boil. Add the gelatin powder and throw in the stirring rod that should come with the hot plate or just stir it for a while yourself. After the particles look dissolved, turn off the heat. Then you can throw in the remaining half of the water. Make that water cold to help cool down the mixture (adding scalding water to the tanks is what cracked some of them). Stir a little bit, then fill the tank. We filled the tanks by siphoning the mixture. This may also help prevent cracking due to heat. The gelatin mixture we used was 1.5 wgt.% powder. This means that for one tank the mix is 6 liters of water and 90 grams of powder.

(For 5 liters of water use 75 grams of powder)