DEFORMATION AROUND A HOLE

This lab has two main objectives. The first is to develop insight into the displacement, stress, and strain fields around a hole in a sheet under an approximately uniaxial load, and to see how those fields are related. The second is to evaluate the stress concentrations at the wall of the hole for uniaxial loading and compare them to the theoretical stress concentrations at the wall of the hole for uniform biaxial loading. We do this by stretching a Lycra sheet with a hole. The solid grey original circles deform to ellipses with dashed perimeters.

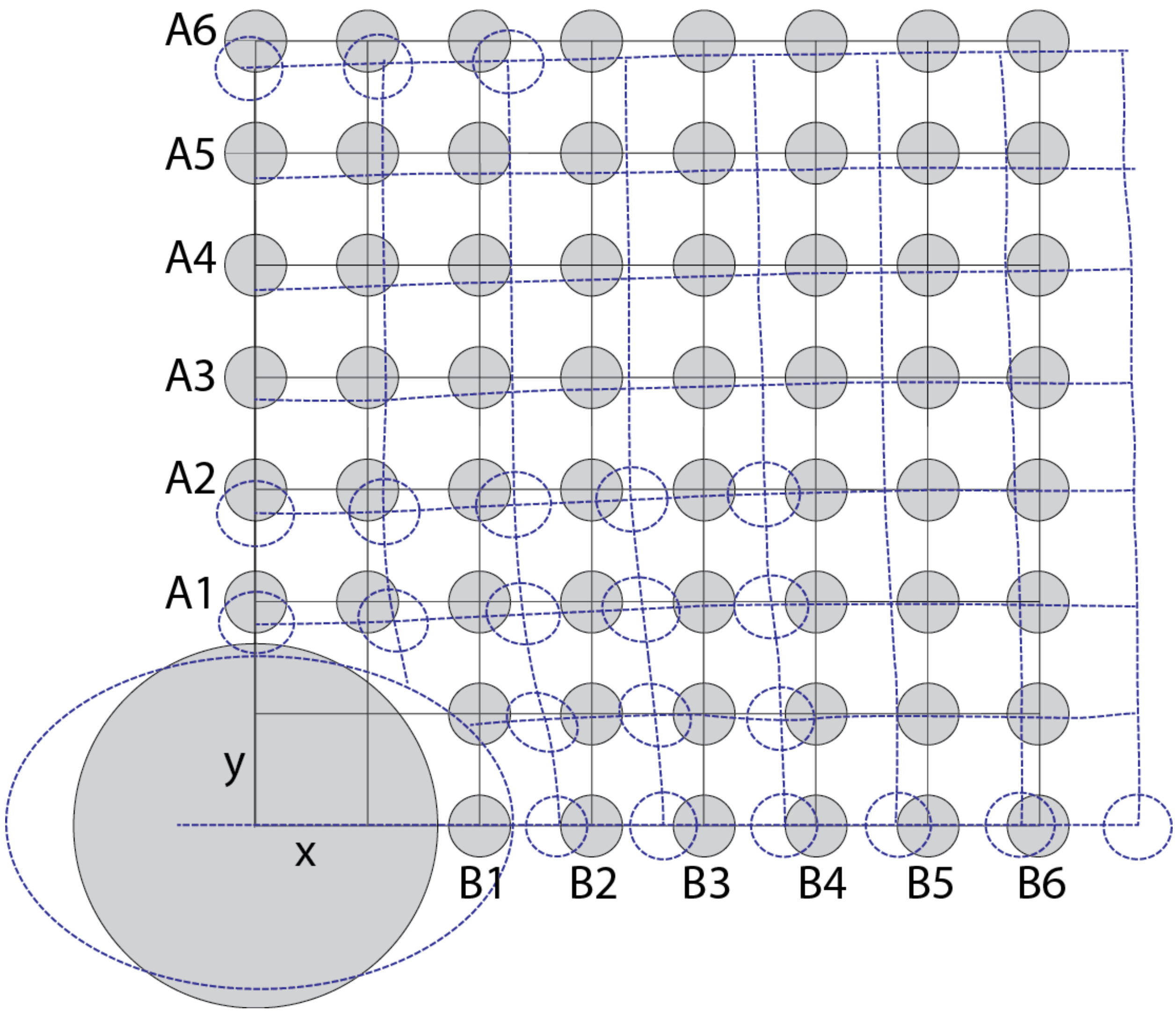
Part 1 (Displacement and strain in Lycra sheet with a circular hole

- a On page 3, plot the 22 displacement vectors connecting the centers of the undeformed circles to the centers of the deformed circles (i.e., the ellipses). (**22 pts**)
- b The Lycra puckers when the sheet with the hole is stretched. Draw on page 3 the approximate boundary of the region of puckering. (**5 pts**)
- c On page 4, draw (in red) the major axes of the 22 ellipses. (**22 pts**)
- d On page 4, draw (in blue) the minor axes of the 22 ellipses. (**22 pts**)
- e Fill out the table below. The terms x and y are the initial coordinates of the centers of the little circles; x' and y' are the final coordinates of the centers of the little circles; u_x and u_y are the displacement components of the centers of the little circles; ε_1 and ε_2 are the greatest and least elongations of the little circles; θ is the orientation of the maximum extension measured from the x-axis; and ε_1^* is the maximum extension at point B6. Give three significant figures for the positions and two significant figures for all the other quantities. (1 pt/box)

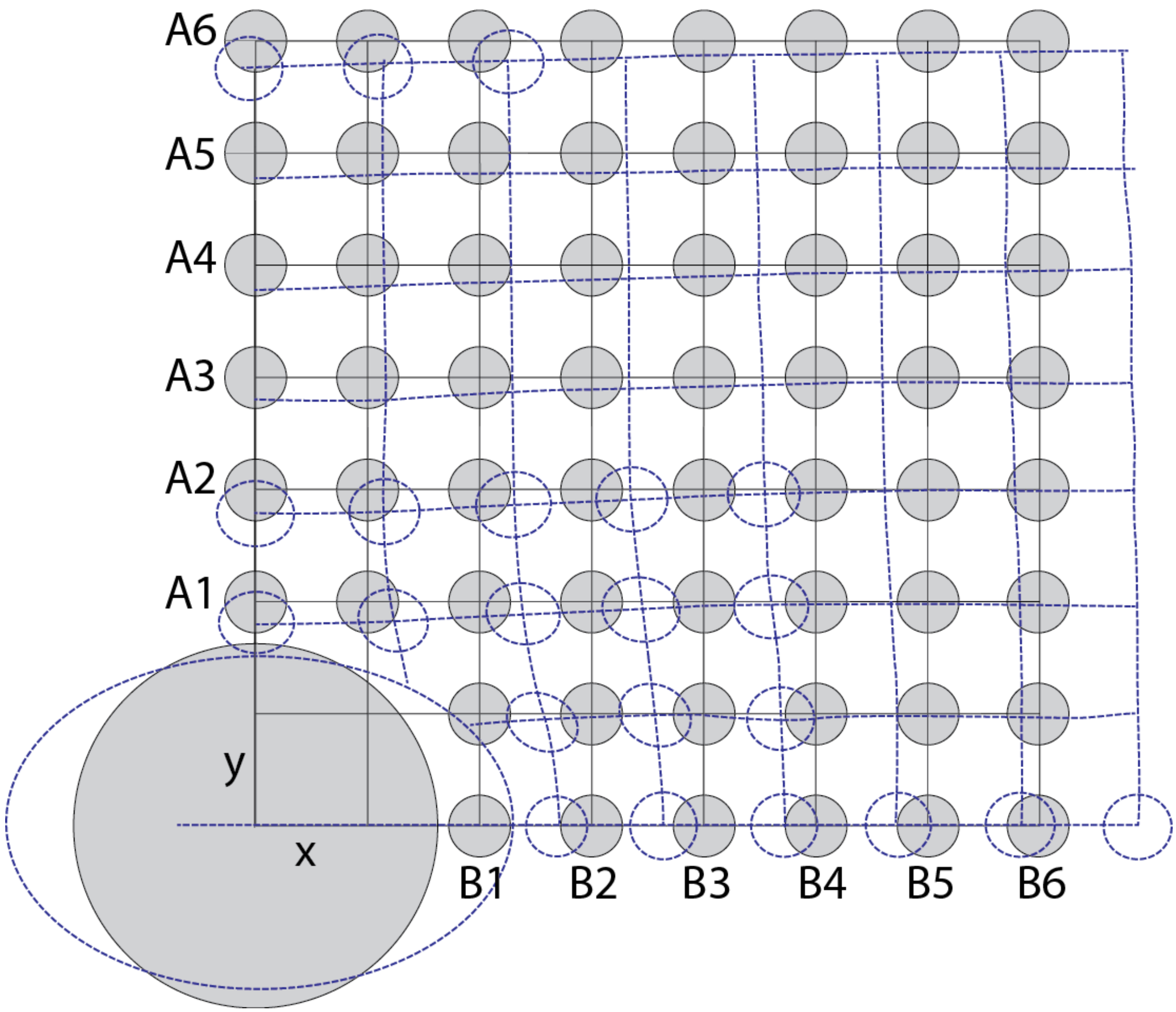
Point	x (mm)	y (mm)	x' (mm)	y' (mm)	u_x (mm)	u_y (mm)	ε_1	ε_2	θ	$\varepsilon_1/\varepsilon_1^*$	$\varepsilon_2/\varepsilon_1^*$
A1											
A2											
A3							xxxxx	xxxxx	xxxxx	xxxxx	xxxxx
A4							xxxxx	xxxxx	xxxxx	xxxxx	xxxxx
A5							xxxxx	xxxxx	xxxxx	xxxxx	xxxxx
A6											
B1											
B2											
B3											
B4											
B5											
B6											

ε_1^* is the maximum elongation at point B6. This is the point furthest from the hole, and where the strain is likely to be closest to the background level (i.e., the level least affected by the hole). The ratio $\varepsilon_1/\varepsilon_1^*$ thus gives the maximum elongation at an arbitrary point relative to the background level. That ratio will be useful for addressing question 8 below.

DISPLACEMENTS



PRINCIPAL STRAIN (STRESS) DIRECTIONS



<u>Type</u> up your answers to the following questions on a separate page

1 Is this experiment best approximated by plane strain or a plane stress? Explain your reasoning. **(2 pts)**

2 Is the strain field uniform within the plane of the fabric? Explain your reasoning. **(2 pts)**

3 If the stress field in the fabric were uniform, would the strain field in the fabric be uniform if the fabric were a perfectly linear elastic, homogeneous, isotropic material? Explain your reasoning. **(2 pts)**

4 Based on your answers to (2) and (3), do you conclude that the stress field in the fabric with a hole is uniform or not? **(1 pt)**

5 Along the y-axis, does the direction of the displacement at a point generally match the direction of the maximum stretch at the point? Explain the basis for your answer. **(2 pts)**

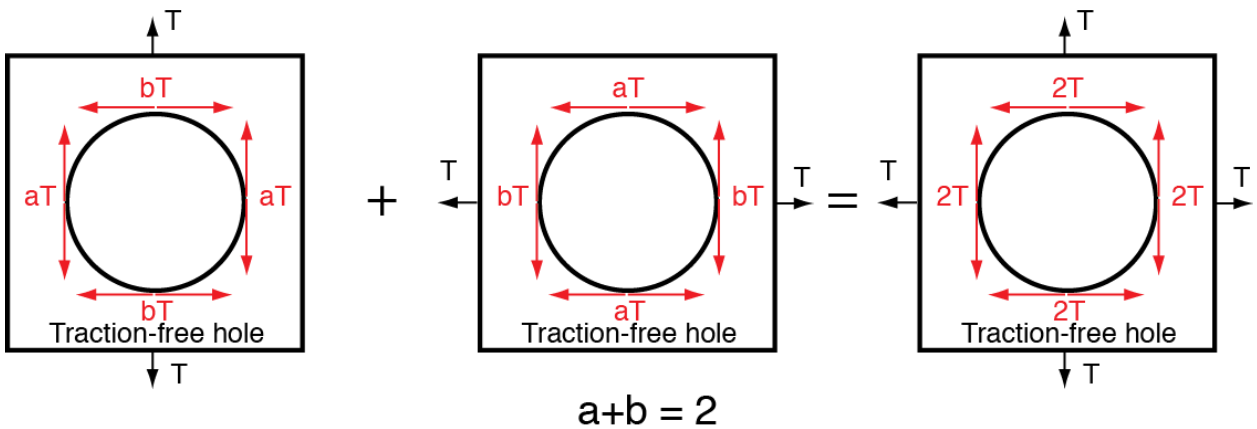
6 Ignoring air pressure, what are the tractions acting <u>on</u> the free edges of the fabric, including the wall of the hole? Are they zero? **(1 pt)**

7 Based on your answer to (6), are the stresses along the free edges essentially uniaxial (only one non-zero principal stress) or biaxial? Explain clearly and completely. **(2 pts)**

8 The superposition thought experiment in the class notes for the second lecture on the stresses around a hole implies that the sum of the stresses parallel to the wall at points A and B should equal twice the uniaxial stress far from the hole for a uniaxial test. This implies that the values of maximum normalized elongation parallel to the wall of the hole at points A and B should sum to 2. Using the value of $\varepsilon_1/\varepsilon_1^*$ for point A1 in the Table as the value of "a" (to one significant figure) calculate "b" assuming a + b = 2. State the value of "b" using **<u>one</u>** significant figure. **(2 pts)**

a =

b =



9 What is the direction of the maximum elongation at point A1 relative to the direction of the tension applied to the sheet of fabric? **(1 pt)**

10 What is the direction of the maximum tensile stress at point A1 relative to the direction of the tension applied to the sheet of fabric? See slide 23 of the rheology lecture. **(1 pt)**

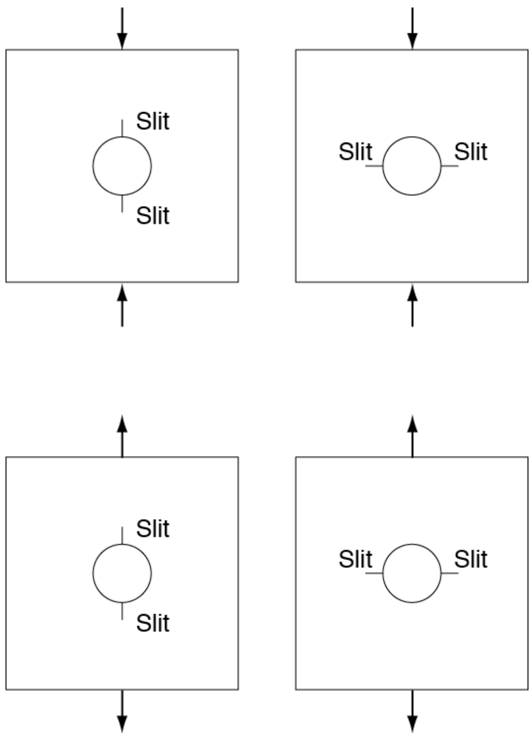
11 What is the direction of the maximum contraction at point 1B relative to the direction of the tension applied to the sheet of fabric, assuming the fabric is a linear elastic isotropic material? **(2 pts)**

12 What is the direction of the maximum compressive stress at point B1 relative to the direction of the tension applied to the sheet of fabric, assuming the fabric is a linear elastic isotropic material? **(1 pt)**

13 Describe in your own words the physical meaning of the values of “a” and “b” in question 8 in terms of the stress concentrations at points A and B. **Address both the sign and magnitude (4 pts)**

Part 2 Foam rubber experiment

14 Conduct a series of informal uniaxial stress tests on the block of foam rubber with a hole in it. Describe what happens to the labeled slits cracks in each case below. Describe whether the slits open or stay closed. **(4 pts total)**



15 Are your descriptions of the four boxes consistent with your answers from the Lycra experiment? Note that the tractions on the boundaries in the top row of figures, including on the walls of the hole, differ in sign from those in the bottom row. Explain clearly and completely. **(4 pts)**

16 Suppose the hole represents a magma chamber as seen in <u>map</u> view. For this case where a uniaxial horizontal **tension** is applied to the sample (examples in the bottom row of the diagram), where might cracks (dikes) form at the magma chamber walls, and what would the orientation of the dikes be? Show this on a diagram and explain your reasoning. **(4 pts)**

17 Suppose the hole represents a magma chamber as seen in <u>map</u> view. For this case where a uniaxial horizontal **compression** is applied to the sample (examples in the top row of the diagramD), where might cracks (dikes) form at the magma chamber walls, and what would the orientation of the dikes be? Show this on a diagram and explain your reasoning. **(4 pts)**