PHOTOELASTIC EXPERIMENT: STRESS CONCENTRATIONS

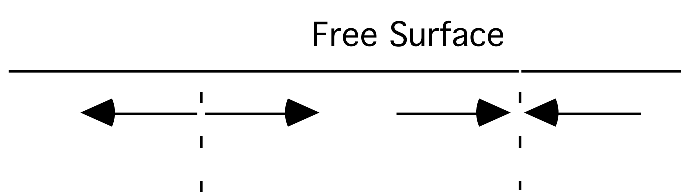
I Main Topics
- A Photo-elastic experimental determination of stress fields
- B Superposition – thought experiment

II Photo-elastic experimental determination of stress fields
- A Stress contours: give magnitude of a particular stress component.
  1. In photoelastic materials the color patterns under crossed polarizing filters can be used to measure the absolute magnitude of the difference between the principal stresses :

$$|\sigma_1 - \sigma_2|$$

where σ_1 is the magnitude of the greatest principal stress and σ_2 is the magnitude of the least principal stress.

Now consider a traction-free surface (i.e., a surface on which no normal or shear tractions act such). The surface of the earth is usually considered to be a free surface (the normal stress due to the weight of the atmosphere is usually ignored). A horizontal free surface is shown below:



The absence of shear traction on the free surface means that the free surface is a principal surface. The absence of a normal traction on the free surface means that the principal stress that acts on the free surface is zero. Because principal surfaces intersect at 90°, then a set of principal planes will interest the free surface at 90°; the diagram above shows a set of vertical principal planes as dotted lines. As shown above, normal stresses can act on the principal planes perpendicular to the free surface. No shear stresses will act on the principal planes perpendicular to the free surface because they too are principal planes.

2 As the principal stress normal to a free surface is zero, the color fringes at a free surface indicate the absolute magnitude of the principal stress acting parallel to the free surface.

B Principal stress trajectories

1 These give the orientation of the principal stresses

2 At a given point, the greatest and least principal stress trajectories will be perpendicular to each other

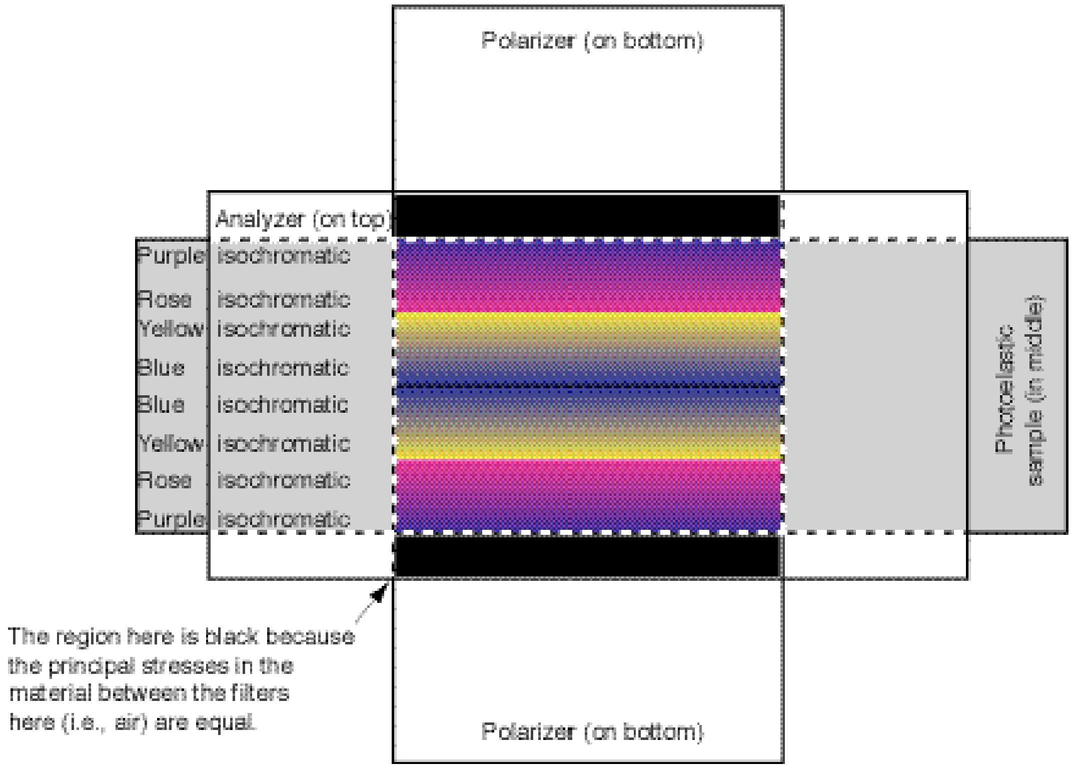
3 These can also be thought of as "lines of internal force", that is, as lines along which the principal stresses are transmitted. Internal forces can not be transmitted across cavities

4 The principal stresses are in some ways analogous to streamlines in fluid flow. A cavity affects the principal stresses similar to the way an obstacle in a current affects the streamlines: the streamlines have to bend around the obstacle.

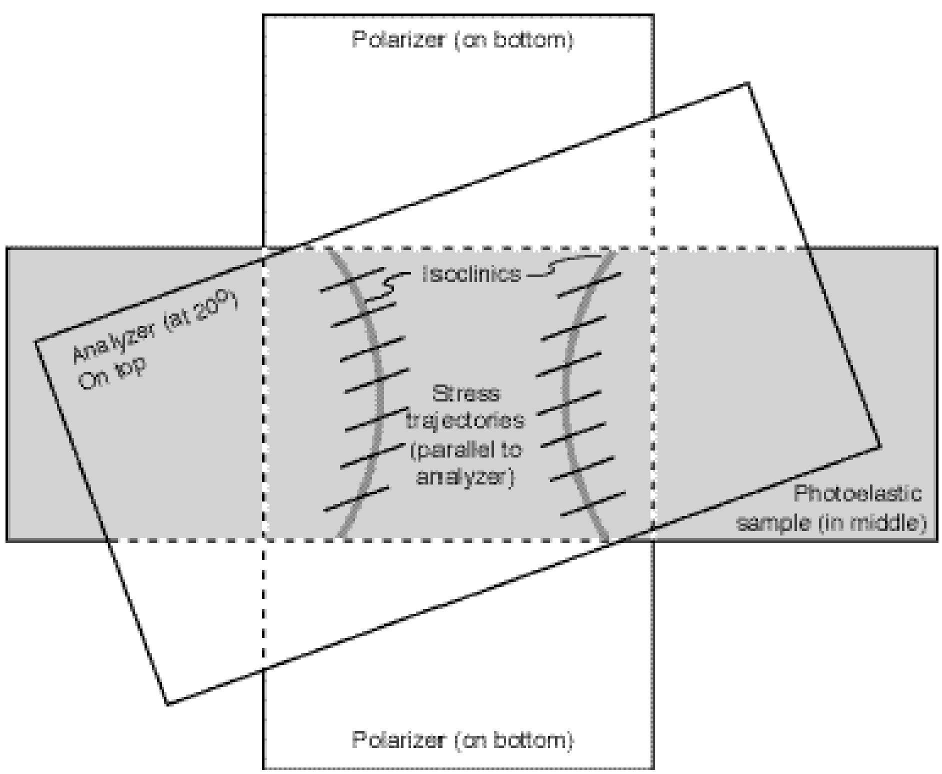
5 In an isotropic material, the directions of the principal stresses parallel the directions of the principal strains.

6 Along an **isoclinic** (a band of extinction) one principal stress direction parallels the orientation of the analyzer (one of the polarizing filters). The other principal stress direction is at 90° to this.

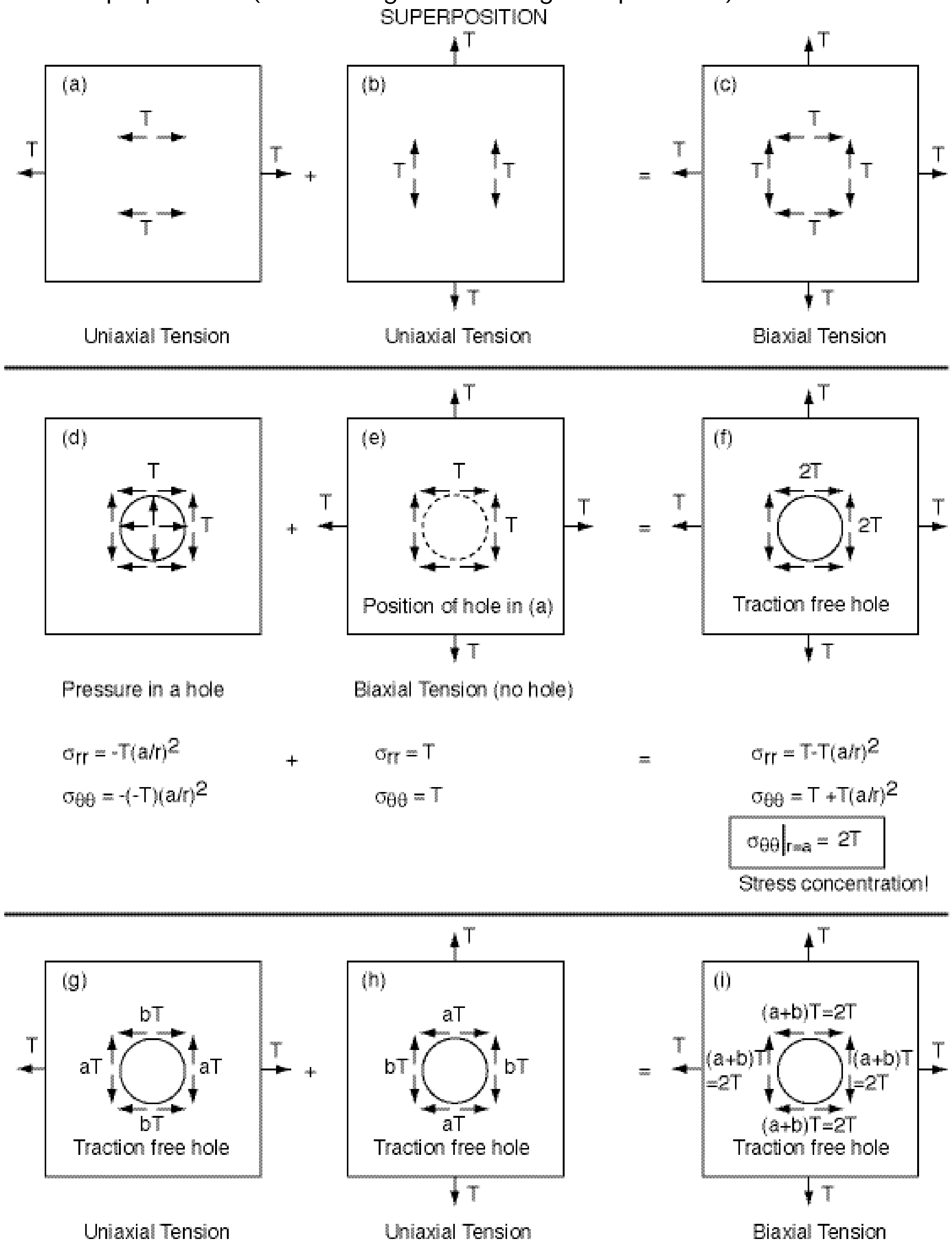
Isochromatics (contours of maximum shear stress) for a bent bar



Isoclinics (black bands where a principal stress parallels the analyzer)



III Superposition (Stress magnitude thought experiment)



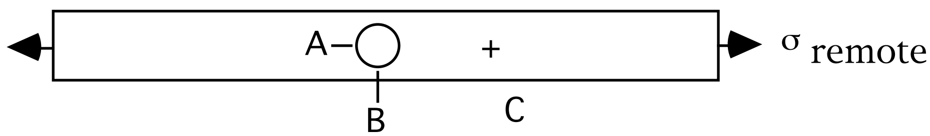
To obtain (f), the solution we derived for the stresses around a pressurized hole (d) is superposed upon the stresses in a plate under biaxial tension (e). The tractions on the *position* of the wall of the hole are compressive in (d) but tensile in (e), and they cancel each other out, yielding a traction-free hole. In other words, no normal tractions act against the wall of the hole, and no shear tractions acting along it either. Normal stresses do act *parallel* to the wall of the hole, however, so even though the hole is traction-free, the wall of the hole is not "stress-free". This distinction is important. The normal stress parallel to the wall of the hole is 2T, with one contribution of T coming from the pressurized hole (d) and the other contribution of T coming from (f).

In the experiments we place a sample with a hole under uniaxial tension T (d). The normal stress parallel to the walls of the hole (i.e., the "hoop stress") is some unknown factor different from T. In (g) the hoop stress at the left and right sides of the hole equals aT, and the hoop stress and the top and bottom of the hole equals bT. We do not know what the factors "a" and "b" are yet, but will use the results of our experiment to find out. Now imagine pulling uniaxially on the sample in a new direction (h), 90° from the direction of pull in (g). In diagram (h) the hoop stress at the top and bottom of the hole equals aT, and the hoop stress at the left and right sides of the hole equals bT. In both (g) and (h) the walls of the hole are traction-free, as are the unloaded edges of the plates. Now if conditions (g) and (h) are superposed, then we obtain the solution for the stresses around a traction-free hole in a plate under biaxial tension (h). This is an axially symmetric situation, and it must match the solution for (fg). The hoop stresses at the wall of the hole are aT + bT = 2T, so a + b = 2.

We can anticipate that the presence of the hole in (d) will cause a tensile stress concentration at the top and bottom of the hole. This is because the material removed to make the hole used to support forces within the plate, and when this material is removed, the surrounding material must pick up an extra load. We can anticipate then that "b" will be greater than 1: the horizontal tension at the top and bottom of the hole in (d) should exceed the tension at the plate edge. If "b" is greater than 1, then "a" must be less than one for the sum a + b to equal two. In this way we use the known solution for the stresses around a traction-free hole in a plate under biaxial tension to infer the stresses around a traction-free hole in a plate under uniaxial tension.

Stress Magnitude Physical Experiment

a Insert a photoelastic strip with a circular hole into real polariscope. Cross the polarizers so that the image appears dark with no load on the sample. Increase the tension on the sample until a distinctive color first appears **at point B**; this typically is a blue-violet color at a load level of ~21-22. Usually you overshoot the tension and then have to back the tension off a bit to see the first appearance of the color. Record the color and the load level. Now increase the tension if needed until that same color appears **at point C** ("far from the hole"). Record the load level. Then unload the sample. Repeat these steps for the following distinctive colors at point B: pale green (load of ~27), and yellow-pink (load level of ~32).



Color	Load level for color at B "IbTI"	Load level for color at C "ITI"	IbTI/IT I = IbI

b Repeat the procedure, but now focus on point A. Cross the polarizers so that the image appears dark with no load on the sample. Increase the tension on the sample until a distinctive color first appears **at point A**; this typically is a blue-violet color. Usually you overshoot the tension and then have to back the tension off a bit to see the first appearance of the color. Record the color and the load level. Now increase the tension if needed until that same color appears **at point C** ("far from the hole"). Record the load level. Repeat these steps for the following distinctive colors at point A: pale green, and yellow-pink. **Do not exceed a load level of 150!!!**

Color	Load level for color at A "IaTI"	Load level for color at C "ITI"	IaTI/IT I = IaI

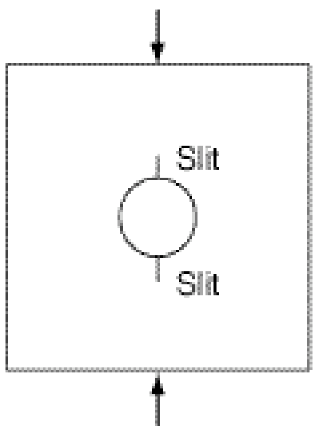
Type up your answers to the following questions:

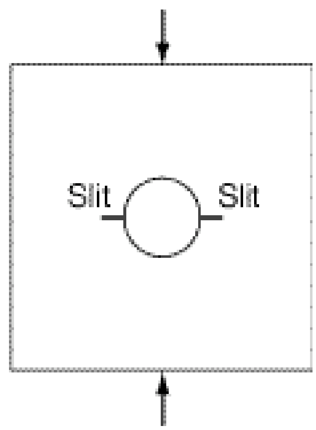
c The superposition thought experiment shows that a + b = 2 if our system could be considered as an infinitely large plate with a hole. Assuming this is correct (or at least approximately correct), use your answers to (a) and (b) to evaluate constants a and b.

a = ______________ b = ______________

d Describe in your own words what these values mean in terms of the stress concentrations at points A and B.

e Take the small block of foam rubber with the hole and squeeze the block as shown below. Describe what happens to the hole and the cracks in each case.





f Are your answers to (i) and (k) consistent? Explain clearly and completely.

g Suppose the hole represents a magma chamber as seen in map view. For this case where a uniaxial horizontal **tension** is applied to the sample, where might cracks (dikes) form at the magma chamber walls, and what would the orientation of the dikes be? Show this on a diagram and explain your reasoning.

h Suppose the hole represents a magma chamber as seen in map view. For this case where a uniaxial horizontal **compression** is applied to the sample, where might cracks (dikes) form at the magma chamber walls, and what would the orientation of the dikes be? Show this on a diagram and explain your reasoning.