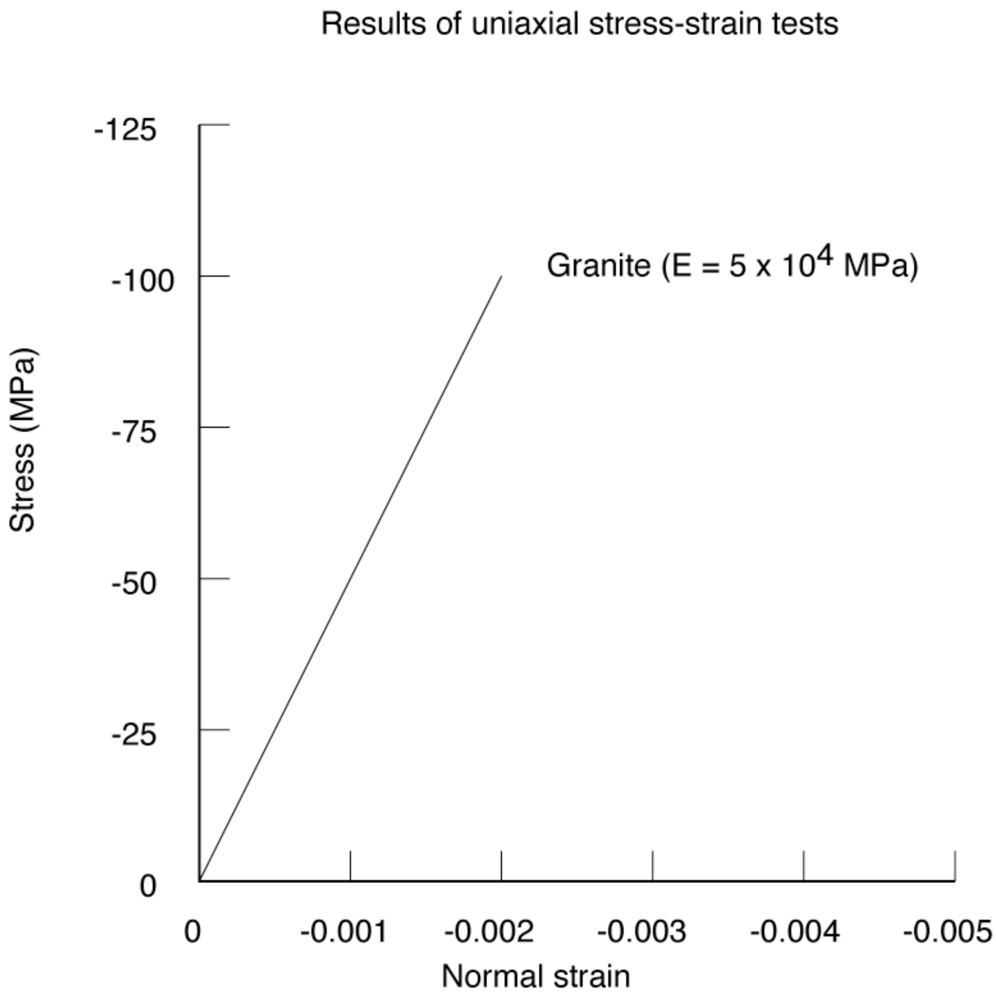
Lab 11: Introduction to Elasticity Theory (Stress-Strain relationships)

In all the questions here tension is considered positive and compression is considered negative.

1a What is the mass of a cube of water 1 meter on a side? Give the answer in metric tons; a metric ton is 1000 kg, or ~2200 pounds on Earth).
(2 pts)

1b What is the water pressure at the base of a column of water 100 meters tall? Assume gravitational acceleration = 10 m/sec^2 here. Give the answer in MPa (megaPascals). (2 pts)

2 The figure below is a stress-strain curve from a uniaxial compression test on a cylinder of Barre granite with a Young's modulus of 5 x 10^4 MPa. Draw the stress-strain curves for a sample of cemented Navajo sandstone that has a Young's modulus of 10^4 MPa and a weakly cemented Navajo sandstone that has a Young's modulus of 5 x 10^3 MPa. **(2 pts for each curve; 4 pts total)**



3 An incompressible material is one whose volume does not change during deformation (i.e., $\Delta = \Delta V/V_0 = 0$). Rubber is nearly incompressible. Using the stress-strain relationships, show that Poisson's ratio (for infinitesimal strain) is 0.5 for incompressible materials. (**3 pts**)

Describe in words why this answer makes sense in the context of a uniaxial compression test. (**1 pt**). Hints: See the Poisson ratio "definition" from lecture 20 (slide 21), and recall that the dilation ($\Delta V/V$) for infinitesimal strain is $\varepsilon_1 + \varepsilon_2 + \varepsilon_3$ (lecture 13, slide 17). (**4 pts total**)

4 To a first approximation, the non-tectonic *vertical* normal stress at shallow depths in the Earth's crust below a horizontal ground surface can be taken as

$$\sigma_{zz} = \rho g z$$

where ρ is the density of the rock, g is gravitational acceleration (9.8 m/sec^2), and z is vertical position (z= 0 corresponds to the ground surface and the positive z axis points up; see the diagram on the last page). To a first approximation, the non-tectonic *horizontal* normal stress at shallow depths in the Earth's crust commonly be taken as

$$\sigma_{xx} = \frac{\nu}{1-\nu}\rho g z$$

where the x-axis is horizontal. Derive this expression from the equations for Hooke's law (lecture 20, slide 22) if one assumes that the extension of all horizontal lines is zero at all depths (i.e., $\varepsilon_{xx} = \varepsilon_{yy}= 0$), that is, if the material is laterally confined (i.e., it acts as though it is in a container with rigid vertical walls). For a Poisson's ratio of 0.25, the typical value for many materials) this relationship is quite simple. (**4 pts total**)

5 A square plate of granite is loaded as shown on the last page, where $\sigma_{xx} = \sigma_{yy} = \sigma_{xy}$ = 200 MPa. Using the Young's modulus for granite in problem 2 and a Poisson's ratio of 0.25, determine the change in length of the diagonal ab. Assume plane stress conditions (see lecture 20, slide 25: tractions are applied at the edges of the plate but not on its face)

Solve first by using the Mohr circle for stress to find the orientation (2 pts) and magnitude (2 pts) of the principal stresses, then use Hooke's law to find the principal strains (2 pts), and then use the principal strains to find the length change of diagonal ab (2 pts).

Check the answers for the principal stresses by using the eig function in Matlab (e.g., [V,D] = eig(sigmaxy) as in the last lab). State in words what the meaning of the terms in V and D are, and support your description with any drawings you find helpful. (4 pts)

6 Suppose a horizontal cylinder of granite (see the last page) is initially unstrained and is then subjected to a constant uniform positive temperature change (i.e, it is heated). **No tractions are applied to the cylindrical wall of the granite**, so the cylinder is free to expand in the radial direction, but the ends of the cylinder are prevented from being displaced in the axial direction. The normal strains are expressed by

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right] + \alpha(\Delta T)$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{zz} + \sigma_{xx}\right)\right] + \alpha(\Delta T)$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right] + \alpha(\Delta T)$$

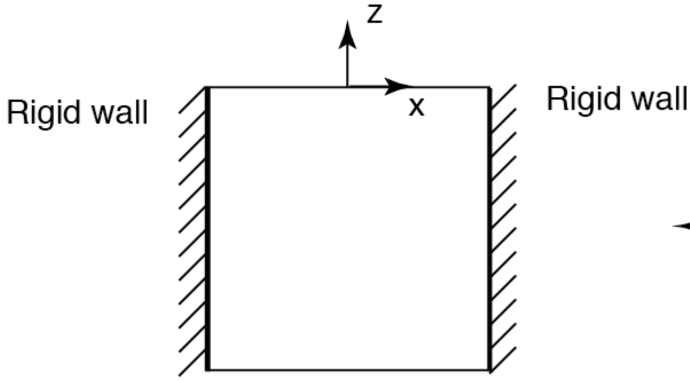
where α is the thermal coefficient of expansion (with dimensions of 1/°) and ΔT is the temperature change in degrees. The temperature change does not give rise to any shear strains. Show that the only non-vanishing normal stress and normal strain components in the granite are

$$\sigma_{xx} = -E\alpha(\Delta T)$$

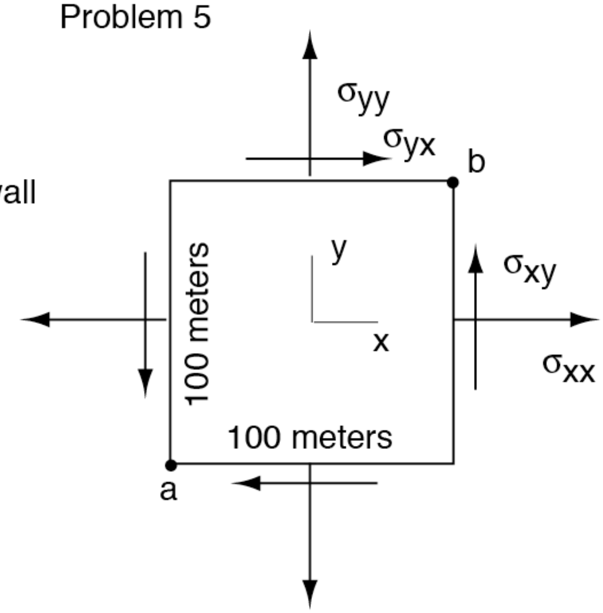
$$\varepsilon_{yy} = \varepsilon_{zz} = \alpha(\Delta T)(1+\nu)$$

Symmetry considerations can be invoked here to understand how to solve the problem. For example, if *r* is the radial distance from the axis of the cylinder and *a* is the radius of the cylinder, then $\varepsilon_{rr}(r=a) = \varepsilon_{zz}(z=a, y=0) = \varepsilon_{yy}(y=a, z=0)$, and $\sigma_{rr}(r=a) = \sigma_{zz}(z=a, y=0) = \sigma_{yy}(y=a, z=0)$. **(4 pts total)**

Problem 4



Problem 5



Problem 6