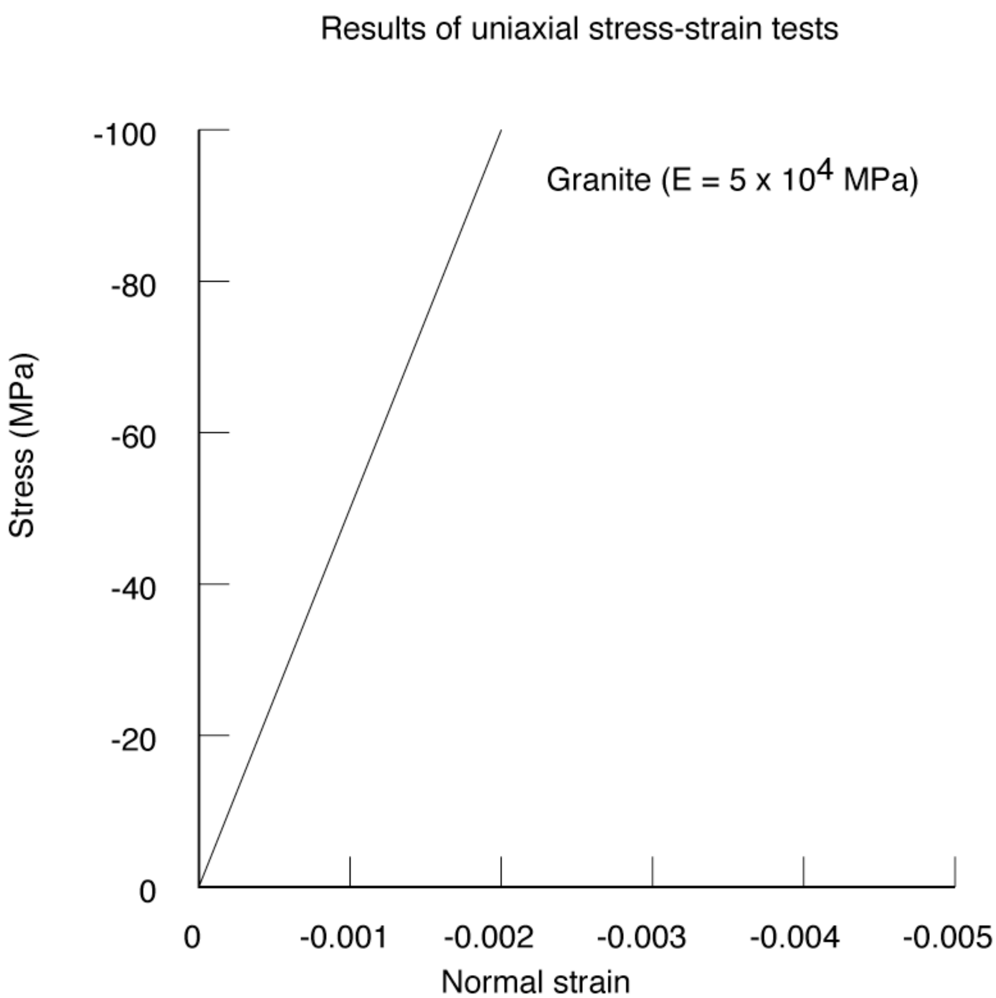
Introduction to Elasticity Theory (Stress-Strain relationships)

In all the questions here tension is considered positive and compression is considered negative.

1 How deep must a column of water be to exert a pressure of 1MPa at the base of the column? Assume gravitational acceleration = 10 m/sec^2 here. **(2 pts)**

2 The figure below is a stress-strain curve from a uniaxial compression test on a cylinder of Barre granite with a Young's modulus of 5 x 10^4 MPa. Draw the stress-strain curves for a sample of cemented Navajo sandstone that has a Young's modulus of 10^4 MPa and a weakly cemented Navajo sandstone that has a Young's modulus of 5 x 10^3 MPa. **(2 pts for each curve; 4 pts total)**



3 An incompressible material is one whose volume does not change during deformation (i.e., Δ = $\Delta V/V_0$ = 0). Rubber is nearly incompressible. Using the stress-strain relationships, show that Poisson's ratio (for infinitesimal strain) is 0.5 for incompressible materials. (**3 pts**)
Describe in words why this answer makes sense in the context of a uniaxial compression test. (**1 pt**). Hints: See the Poisson ratio definition from lecture 20, and recall that the dilation ($\Delta V/V$) for infinitesimal strain is $\varepsilon_1 + \varepsilon_2 + \varepsilon_3$ (see page 20-6). (**4 pts total**)

4 To a first approximation, the non-tectonic normal stress acting on a horizontal plane at shallow depths in the Earth's crust can be taken as

$$\sigma_{zz} = \rho g z$$

where ρ is the density of the rock, g is gravitational acceleration (9.8 m/sec^2), and z is vertical position (z= 0 corresponds to the ground surface and the positive z axis points up). To a first approximation, the non-tectonic normal stress acting on a vertical plane at shallow depths in the Earth's crust can be taken as

$$\sigma_{xx} = \frac{\nu}{1-\nu}\rho g z$$

where the x-axis is horizontal. Derive this expression from the equations for Hooke's law (page 20-5) if one assumes that the extension of all horizontal lines is zero at all depths (i.e., $\varepsilon_{xx} = \varepsilon_{yy} = 0$), that is, if the material is laterally confined (i.e., it acts as though it is in a container with rigid vertical walls). For a Poisson's ratio of 0.25, the typical value for many materials) this relationship is quite simple. A labeled picture can help here. (**4 pts total**)

5 A square plate of granite is loaded as shown on the last page, where $\sigma_{xx} = \sigma_{yy} = \sigma_{xy} = 200$ MPa. Using the Young's modulus for granite in problem 2 and a Poisson's ratio of 0.25, determine the change in length of the diagonal ab. Assume plane stress conditions (i.e., tractions are applied at the edges of the plate but not on its face)

Solve first by using the Mohr circle for stress to find the orientation (**2 pts**) and magnitude (**2 pts**) of the principal stresses, then use Hooke's law to find the principal strains (**2 pts**), and then use the principal strains to find the length change of diagonal ab (**2 pts**).

Check the answers for the principal stresses by using the eig function in Matlab (e.g., [V,D] = eig(sigmaxy) as in the last lab). State in words what the meaning of the terms in V and D are, and support your description with any drawings you find helpful. (**4 pts**)

6 Suppose a horizontal cylinder of granite (see the last page) is initially unstrained and is then subjected to a constant uniform positive temperature change (i.e, it is heated). **No tractions are applied to the cylindrical wall of the granite**, so the cylinder is free to expand radially), but the ends of the cylinder are prevented from being displaced in the axial direction. The normal strains are expressed by

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\left(\Delta T\right)$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] + \alpha\left(\Delta T\right)$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\left(\Delta T\right)$$

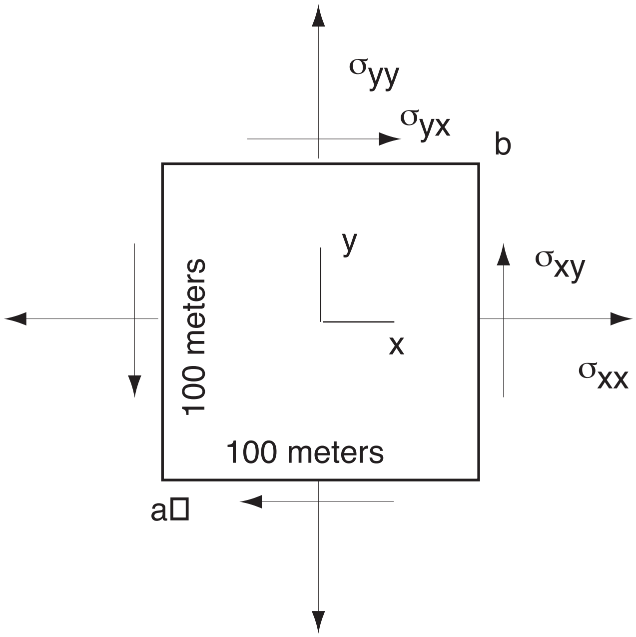
where α is the thermal coefficient of expansion (with dimensions of 1/°) and ΔT is the temperature change in degrees. The temperature change does not give rise to any shear strains. Show that the only nonvanishing normal stress and normal strain components in the granite are

$$\sigma_{xx} = -E\alpha\left(\Delta T\right)$$

$$\varepsilon_{yy} = \varepsilon_{zz} = \alpha\left(\Delta T\right)\left(1+\nu\right)$$

Symmetry considerations can be invoked here to simplify the problem. (**4 pts total**)

Problem 10-4



Problem 10-5