Stresses

Exercise 1 **(38 pts total this page)**

1a Write down the expressions for all the stress components in the x'y' coordinate system in terms of the stress components in the xy reference frame. The expression for $\sigma_{x'x'}$ is given as a guide.

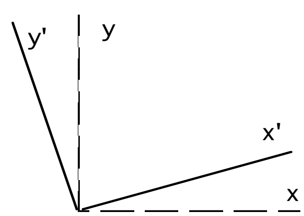
$$\sigma_{x'x'} = a_{x'x}a_{x'x}\sigma_{xx} + a_{x'x}a_{x'y}\sigma_{xy} + a_{x'y}a_{x'x}\sigma_{yx} + a_{x'y}a_{x'y}\sigma_{yy}$$

$\sigma_{x'y'} =$ **6pts**

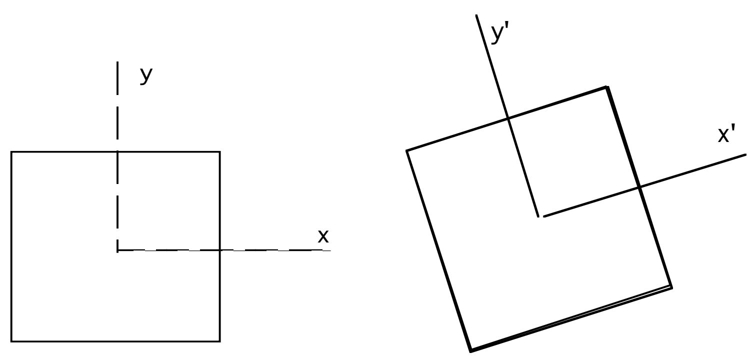
$\sigma_{y'x'} =$ **6pts**

$\sigma_{y'y'} =$ **6pts**

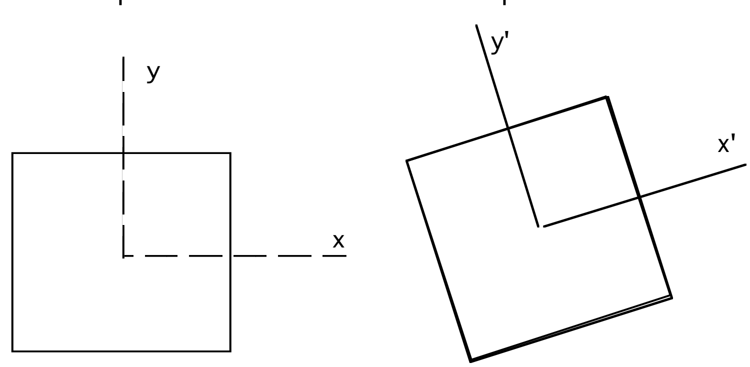
1b Using the picture below, describe the meaning of terms $a_{x'x}$, $a_{x'y}$, $a_{y'x}$, $a_{y'y}$. **4pts**



1c On each box below draw and label positive normal stresses and positive shear stresses using the tensor ("on-in") convention. **8pts**



1d On each box below draw positive normal tractions and positive shear tractions. **8pts**



Exercise 2: Suppose σ_{xx} = -3MPa, σ_{xy} = 8MPa, σ_{yx} = 8MPa, σ_{yy} = -15MPa.

2a On a separate piece of paper, draw a square box about 2" on a side showing the stresses with arrows acting on the sides of the square. Show arrows in the positive direction, and label them with the correct magnitudes. Put a north arrow on your box parallel to the y-axis. Make sure the box will be in equilibrium. **2pts**

2b On the same piece of paper, draw a new square box about 2" on a side showing the tractions (τ_{xn}, τ_{xs}, τ_{yn}, and τ_{ys}) with arrows acting on the sides of the square. Remember that the n- and s-directions for a traction are normal and parallel, respectively, to the side of the box the traction acts on. For each side of the box, show arrows in the positive n- and s-directions, and label them with the correct magnitudes. Put a north arrow on your box parallel to the y-axis. **2pts**

2c Inside the box draw a line representing a vertical fault that strikes 120°. **2pts**

2d Guess whether this fault will tend to slip left-laterally or right-laterally. **2pts**

2e Plot a Mohr circle describing this state of stress, labeling the point on the Mohr circle corresponding to the TRACTION COMPONENTS (see 2b) that act on the face that has a normal along the x-axis (i.e., τ_{xn}, τ_{xs}) and the point on the Mohr circle corresponding to the TRACTION COMPONENTS that act on the face that has a normal along the y-axis (i.e., τ_{yn}, τ_{ys}). **4pts**

2f Using the Mohr circle, find the magnitude of τ_1, the most tensile (i.e., most positive) traction, and τ_2, the least tensile (i.e., most negative) traction. **4pts**

2g Mark on the Mohr circle the points corresponding to τ_1 and τ_2. **2pts**

2h Augment the point marked "τ_1" with a second label "$\tau_{x'n'}$, $\tau_{x's'}$" **while retaining the original "τ_1" label.** Augment the point marked "τ_2" with a second label "$\tau_{y'n'}$, $t_{y's'}$" **while retaining the original "τ_2" label.** Use the Mohr circle to find the negative double angles between the x-axis and the x'-axis, and between the x-axis and the y'-axis, and label the angles $-2\theta_{xx'}$ and $-2\theta_{xy'}$, respectively. **4pts**

2i On another sheet of paper draw new x, x', y and y' axes in their correct orientation. Then draw a 2" square with sides normal to the x' and y' axes. Then show the principal stresses $\sigma_{x'x'}$ and $\sigma_{y'y'}$ acting on the sides of the square. **4pts**

2j Using the angle of (2h) and the formulas on the first page, calculate $\sigma_{x'x'}$, $\sigma_{x'y'}$, $\sigma_{y'x'}$, and $\sigma_{y'y'}$ to check your answer of (2f) – are they consistent? **16pts**

2k Inside this square draw a line representing a vertical fault that strikes 120°. **2pts**

2l Make a guess as to whether this fault will tend to slip left-laterally or right laterally. Is this guess the same as your first guess? **2pts**

2m On one more piece of paper, draw the fault and surround it with a box whose sides are parallel and perpendicular to the fault. Add an x" axis that is parallel to the fault and a y" axis perpendicular to the fault; make **sure** they are right-handed. **2pts**

2n Measure the angle from the x-axis to the y" axis and label it $+\theta_{xy''}$. **2pts**

2o Using the corresponding negative double angle, plot on the Mohr circle the point representing ($\tau_{y''n''}$, $\tau_{y''s''}$). Label the negative double angle $-2\theta_{xy''}$. **4pts**

2p Will the fault slip left-laterally or right laterally? How does this compare with (2d) and (2l)? **2pts**

2q Use the procedure of (2j) to calculate $\sigma_{y''y''}$ and $\sigma_{y''x''}$ to check your results. **8pts**

2r Use the Matlab command "[V,D] = eig" and the stress components at the top of the page to find the principal stresses to check your results **(2)**. Draw a picture showing the principal stresses acting on the sides of a square **(2)** and the **two** sets of direction cosines defining the eigenvectors that give the orientation of the two principal stresses **(2+2)**. Include a printout of your Matlab results. **8pts**

(72 pts total this page)

The following page might prove helpful in organizing your work

Stress Components (MPa)	Traction Components (MPa)
σ_{xx} =	τ_{xn} =
σ_{xy} =	τ_{xs} =
σ_{yx} =	τ_{yn} =
σ_{yy} =	τ_{ys} =

Stress Components (MPa)	Traction Components (MPa)
$\sigma_{x'x'}$ =	$\tau_{x'n'}$ =
$\sigma_{x'y'}$ =	$\tau_{x's'}$ =
$\sigma_{y'x'}$ =	$\tau_{y'n'}$ =
$\sigma_{y'y'}$ =	$\tau_{y's'}$ =

Stress Components (MPa)	Traction Components (MPa)
$\sigma_{x''x''}$ =	$\tau_{x''n''}$ =
$\sigma_{x''y''}$ =	$\tau_{x''s''}$ =
$\sigma_{y''x''}$ =	$\tau_{y''n''}$ =
$\sigma_{y''y''}$ =	$\tau_{y''s''}$ =