Homogenous Strain

This lab explores homogeneous deformation, principal directions, symmetry, and rotations for three (3) two-dimensional problems. During homogeneous deformation, vectors [X] that describe the positions of points on a black unit circle are transformed to vectors [X'] that describe the positions of points on a red ellipse using the F-matrix: [X'] = [F][X].

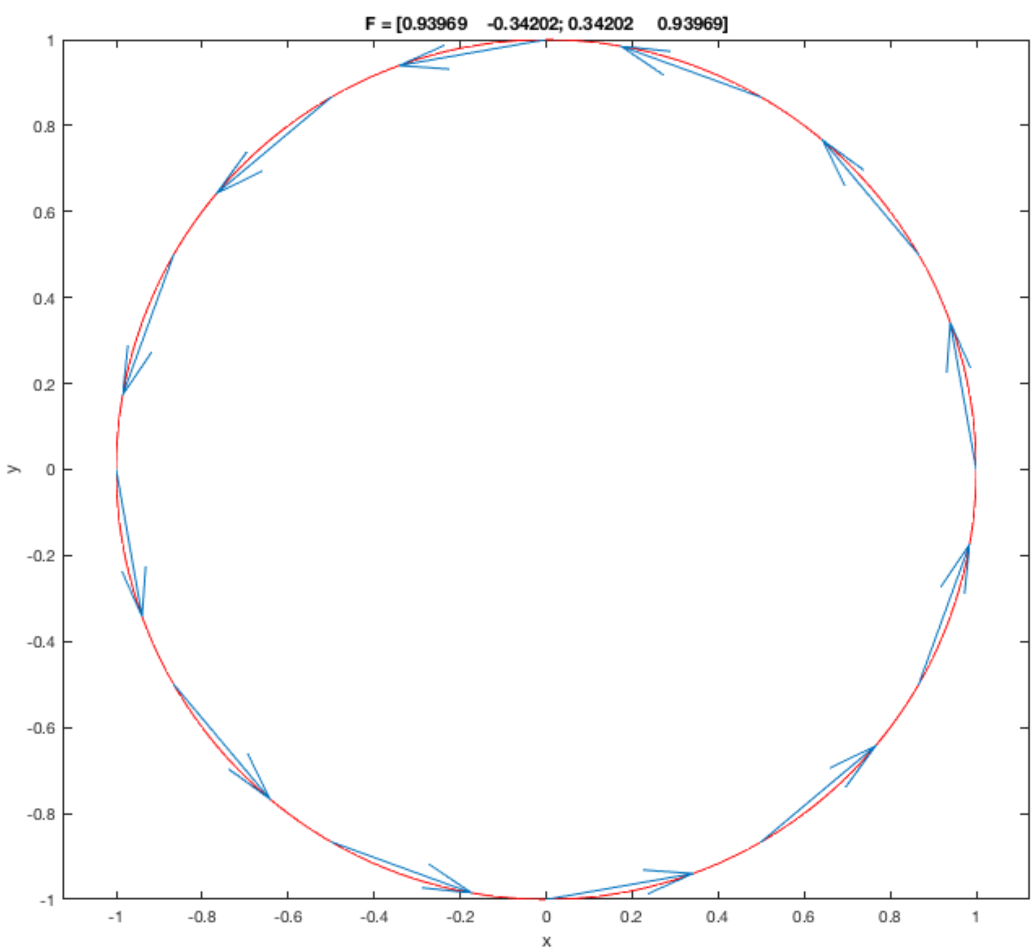
In this assignment, you will measure the principal elongations and rotations associated with three different F-matrices. You will also answer a set of questions that guide you through an exploration of the deformations. You will need to answer the set of questions for each of the three deformations. Show all your calculations, and include copies of your Matlab printouts.

Exercise 1: Pure rotation

The figure below shows a clockwise (positive) rotation of a unit circle about the z-axis, which points out of the page. The figure was prepared with the following two Matlab commands:

```
>> F = [cos(20*pi/180), -sin(20*pi/180); sin(20*pi/180), cos(20*pi/180)]
>> circle_to_ellipse(F)
```

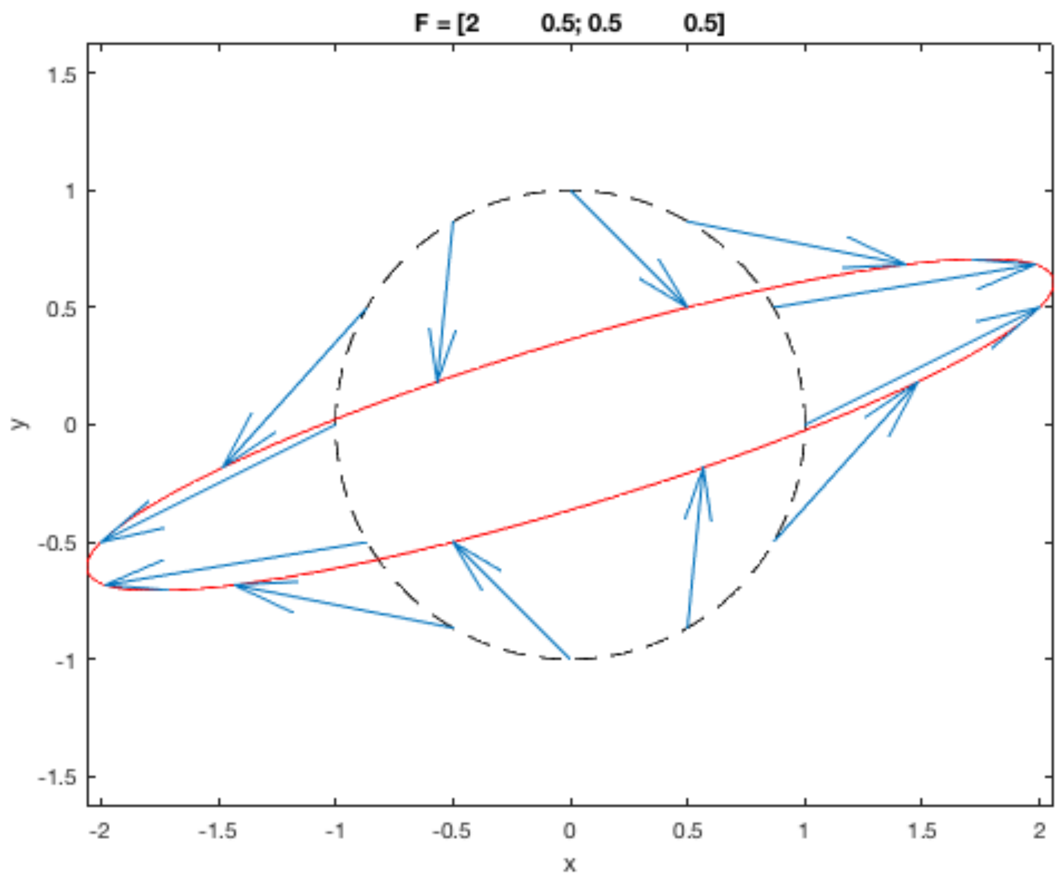
The black unit circle is completely hidden behind the red ellipse (the red ellipse is also a circle).



Exercise 2: Strain for a symmetric F-matrix

The figure below shows a black unit circle and its homogeneously deformed counterpart: a red ellipse. The figure was prepared with the following two Matlab commands:

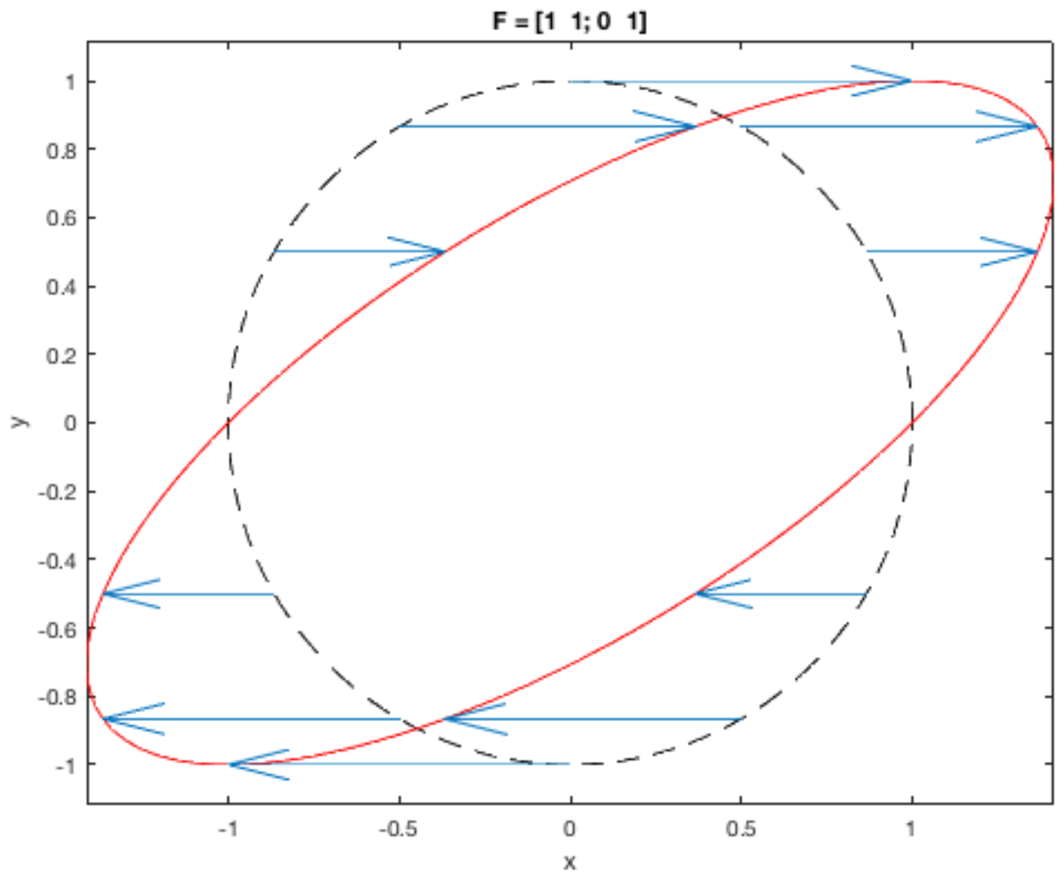
```
>> F = [2,0.5;0.5,0.5]
>> circle_to_ellipse(F)
```



Exercise 3: Simple Shear Strain

The figure below shows a black unit circle and its homogeneously deformed counterpart: a red ellipse. The figure was prepared with the following two Matlab commands:

```
>> F = [1,1;0,1]
>> circle_to_ellipse(F)
```



<u>Short answer questions and calculations (record answers in table on the next page)</u>

A) Record the F-matrix in the table and enter it in the Matalb command window.
B) Is the F-matrix symmetric?
C) Is the displacement field described by the arrows in the figure symmetric about any lines in the plane of the figure? If so, identify the axes of symmetry.
D) Carefully and accurately mark the center of the circle
E) In a solid line, draw the major axis of the ellipse. For exercise 1, draw the major axis through the arrowheads at positions ~5:30 and ~11:30.
F) In a solid line, draw the minor axis of the ellipse. For exercise 1, draw the minor axis through the arrowheads at positions ~2:30 and ~8:30.
G) Do these solid axes intersect at right angles?
H) Label the solid major semi-axis length as a'.
I) Label the solid minor semi-axis length as b'.
J) Measure and record the length *a'* to the nearest 0.1 mm.
K) Measure and record the length *b'* to the nearest 0.1 mm.
L) In a dashed line, draw the retro-deformed major axis (diameter). For exercise 1, draw the this through the arrow tails at positions ~6:00 and ~12:00.
M) In a dashed line, draw the retro-deformed minor axis (diameter). For exercise 1, draw the this through the arrow tails at positions ~3:00 and ~9:00.
N) Do these dashed retro-deformed axes intersect at right angles?
O) Label the dashed major semi-axis length as *a*.
P) Label the dashed minor semi-minor axis length as *b*.
Q) Measure and record the length *a* to the nearest 0.1 mm.
R) Measure and record the length *b* to the nearest 0.1 mm.
S) Measure, label, and record the angle ω from dashed major axis to the solid major axis. This angle ω is the rotation. Consider a counterclockwise rotation as positive.
T) Using your measurements, calculate the maximum elongation (ε_{max}), the maximum stretch (S_{max}), and the maximum quadratic elongation (λ_{max}); see Lec. 13 notes.
U) Using your measurements above, calculate the minimum elongation (ε_{min}), the minimum stretch (S_{min}), and the minimum quadratic elongation (λ_{min}).
V) Using your measurements, calculate the area of the circle; this equals πab.
W) Using your measurements, calculate the area of the ellipse; this equals $\pi a'b'$.
X) Using your measurements, calculate the dilation Δ;
Δ = (area of the ellipse – the area of the circle)/ area of the circle.
Y) Find the principal stretch matrix U in Matlab with the command U = (F'*F)^(1/2)
Z) Find the rotation matrix R with the command R = F*inv(U)
AA) Find the eigenvectors and eigenvalues of U with the command
[vec,val] = (F'*F)^(1/2)
BB) Plot and label the eigenvectors of U on the figure.
CC) Calculate and record the sine and cosine of the rotation angle ω.

Question	Exercise 1	Exercise 2	Exercise 3
A: F-matrix			
B: Is F symmetric?)			
C: Axis of symmetry			
G: Are axes orthogonal?			
J: *a'*			
K: *b'*			
N: Are axes orthogonal?			
Q: *a*			
R: *b*			
S: ω			
T	ε_{max} = S_{max} = λ_{max} =	ε_{max} = S_{max} = λ_{max} =	ε_{max} = S_{max} = λ_{max} =
U	ε_{min} = S_{min} = λ_{min} =	ε_{min} = S_{min} = λ_{min} =	ε_{min} = S_{min} = λ_{min} =
V: area of circle			
W: area of ellipse			
X: dilation Δ			
Y: stretch matrix [U]			
Z: rotation matrix [R]			
AA Eigenvectors of [U] Eigenvalues of [U]			
EE: sin(ω) cos(ω)			

DD) Explain in words what the eigen<u>vectors</u> of U mean.

EE) Explain in words what the eigen<u>values</u> of U mean.

FF) Explain whether or how sin(ω) and cos(ω) relate to each of the four entries in matrix R.