Lab 9 Finite strain **170pts total**

Here you measure the lengths and orientations of the axes of two similar strain ellipses (formed from deformed unit circles), calculate various finite strain quantities based on your measurements, and calculate various measures of the strain using Matlab.

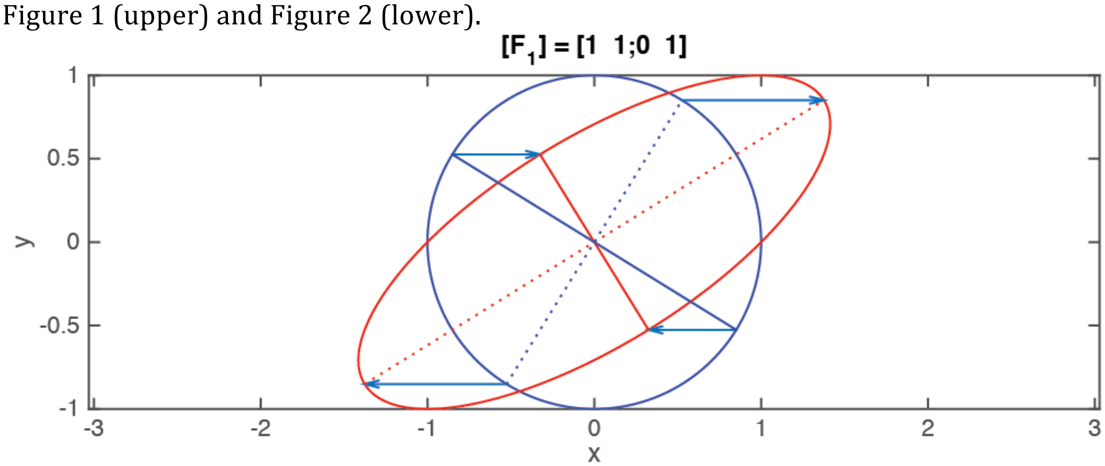
The plots below show unit circles, the associated strain ellipses, and displacement vectors for two deformations. The unit circles could represent idealized undeformed oolites (nearly spherical carbonate grains) as seen in cross section. The ellipses could represent deformed oolites. The two strain ellipses are identical, but the displacement fields that produce them are quite different.

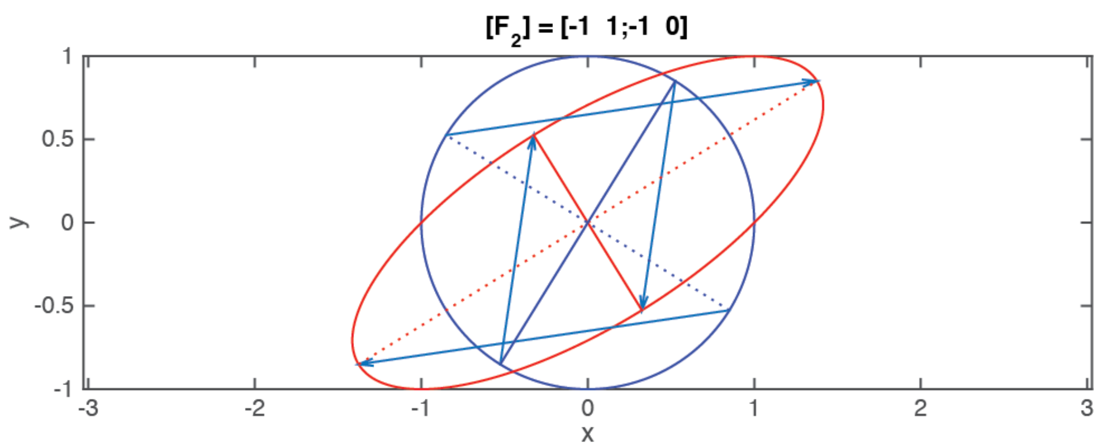
Fill out the two tables on page 2 using two different approaches:

1) A graphically based approach (Table 1).
2) An approach based on Matlab where you are given an F-matrix (Table 2).

Label the figures on p. 2 as requested. Include your long-hand calculations, your Matlab coding, and your Matlab results on separate attached sheets.

Figure 1 (upper) and Figure 2 (lower).



Fill in Table 1, and neatly **show your hand calculations on the bottom of this page**. Full credit requires showing your calculations and including the algebraic equations you are solving for. Give your answers to three significant figures. **Positive angles are counterclockwise**. (**30 pts total**)

| **Table 1 (2 pts/box)** | Figure 1 | Figure 2 |
|---|---|---|
| Label the major (M) and minor (m) semi-axes of the ellipse | | |
| Label the major ($M^{-1}$) and minor ($m^{-1}$) semi-axes of the reciprocal axes | | |
| Measure and label the rotation angle ω from $M^{-1}$ to M (show ω with an arc) | | |
| Major semi-axis length: circle radius | | Same as answer for Fig. 1 |
| Minor semi-axis length: circle radius | | Same as answer for Fig. 1 |
| $S_1$ (greatest stretch) | | Same as answer for Fig. 1 |
| $S_2$ (least stretch) | | Same as answer for Fig. 1 |
| $\varepsilon_1$ (greatest elongation) | | Same as answer for Fig. 1 |
| $\varepsilon_2$ (least elongation) | | Same as answer for Fig. 1 |
| $Q_1$ (greatest quadratic elongation) | | Same as answer for Fig. 1 |
| $Q_2$ (least quadratic elongation) | | Same as answer for Fig. 1 |
| Δ (dilation) | | Same as answer for Fig. 1 |

/30 pts

Now we investigate the displacements for the plots of Figure 3 (for [F1]) to understand better how different deformation paths can yield the same result. **56 pts total**

1 On Figure 3b, carefully draw the major and minor axes for the X' ellipse in the second row. Extend the lines of the axes all the way to the edge of the bounding rectangle. If you do this correctly, the axes with be perpendicular to the ellipse where they intersect the ellipse **(2 pts)**

2 On Figure 3b, neatly label the four points on the unit circle where it is intersected by the lines of symmetry as a, b, c, and d, starting with the upper left point and proceeding clockwise **(4pts)**. Then neatly label the four points on ellipse X' where it is intersected by the lines of symmetry as a', b', c', and d', starting with the upper left point and proceeding clockwise **(4pts)**.

3 On Figure 3b, draw the four displacement vectors from a to a', from b to b', from c to c', and from d to d'. Label these displacement vectors $u_a$, $u_b$, $u_c$, and $u_d$. The displacement vectors will be along a symmetry line. **(4pts)**

4 On Figure 3b, carefully draw the major and minor axes for the X'' ellipse in the second row. Extend the lines of the axes all the way to the edge of the bounding rectangle. If you do this correctly, the axes with be perpendicular to ellipse X'' where they intersect the ellipse **(2 pts)**

5 On Figure 3b, draw an arc (**1 pt**) from the major axis of ellipse X' to the major axis of ellipse X''. Label (**1 pt**) the rotation angle $\omega_1$ **(1 pt)** and measure the angle to the nearest degree and write it here: $\omega_1$ = ____________.

6 Fill in the following table **(1 pt/box).**

| | | | |
|---|---|---|---|
| $\cos(\omega_1)$= | $\sin(\omega_1)$= | $\cos(\omega_2)$= | $\sin(\omega_2)$= |

7 Describe how the answers for question 6 compare to the values in matrix [R] in Table 2 (**2 pts**).

8 On Figure 3c, neatly label the four points on ellipse X'' where it is intersected by the lines of symmetry drawn in step 7 as a'', b'', c'', and d'', starting with the upper left point and proceeding clockwise **(4pts)**.

9 On Figure 3c, carefully draw the major and minor axes for the X'' ellipse in the second row. Extend the lines of the axes all the way to the edge of the bounding rectangle. If you do this correctly, the axes with be perpendicular to the ellipse where they intersect the ellipse **(2 pts)**

10 On Figure 3c, neatly label the four points on the unit circle (i.e., ellipse X') where it is intersected by the lines of symmetry as a', b', c', and d', starting with the upper left point and proceeding clockwise **(4pts)**.

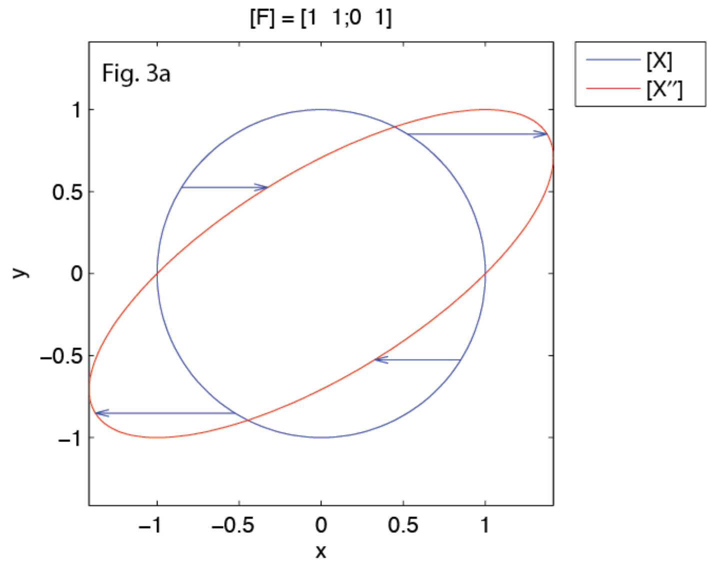
11 On Figure 3c, neatly plot **(4pts)** and label **(4pts)** the four points a, b, c, and d on the unit circle by transferring the plotted locations of points a, b, c, and d from Figure 3b.

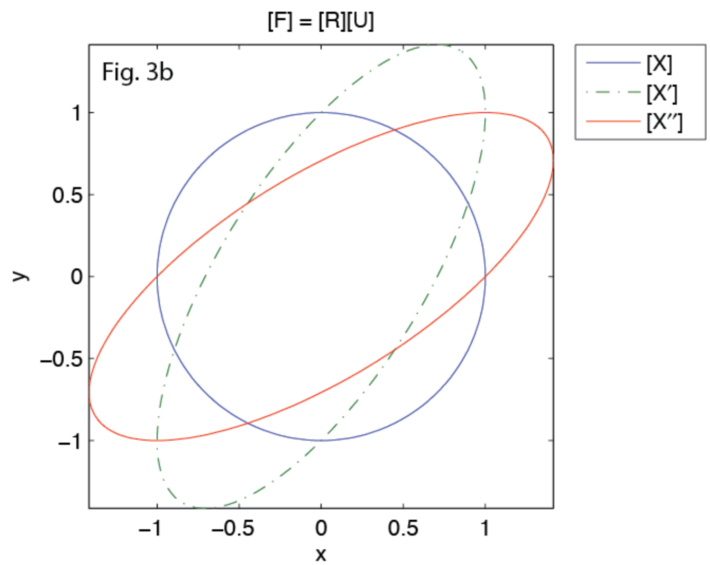
12 On Figure 3c, Connect points a and c with a straight line, and connect points b and d with a straight line **(2pts)**.

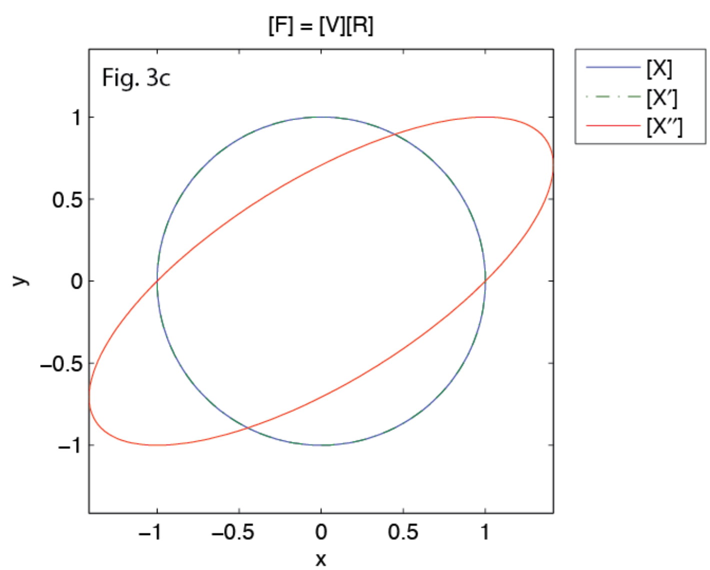
13 On Figure 3c, draw an arc (**1 pt**) from point a to point a'. Label (**1 pt**) the rotation angle $\omega_2$ and measure the angle to the nearest degree and write it here: $\omega_2$ = ____________**(1 pt)**.

One possible deformation path is shown in Fig. 3a. A second possible deformation path (shown in Fig. 3b) involves the symmetric displacement of points a, b, c, and d to a', b', c', and d', and then the rotation of points a', b', c', and d' to a'', b'', c'', and d'', respectively. A third possible deformation path (shown in Fig. 3c) involves the rotation of points a, b, c, and d to a', b', c', and d', and then the symmetric displacement of points a', b', c', and d' to a'', b'', c'', and d'', respectively.

/52 pts


[F] = [1 1;0 1]
Fig. 3a
[X]
[X″]
1
0.5
0
−0.5
−1
y
−1
−0.5
0
0.5
1
x


[F] = [R][U]
Fig. 3b
[X]
[X′]
[X″]
1
0.5
0
−0.5
−1
y
−1
−0.5
0
0.5
1
x


[F] = [V][R]
Fig. 3c
[X]
[X′]
[X″]
1
0.5
0
−0.5
−1
y
−1
−0.5
0
0.5
1
x

Now return to Figure 1. You will need to do Matlab calculations to fill out Table 2.

Neatly fill in the table below. Write all the matrices as 2x2 matrices inside brackets.
**Show your Matlab coding and results on pages you attach (no work, no credit)**. (**56 pts total**)

| **Table 2 (2 pts/box)** | Figure 1 | Figure 2 |
|---|---|---|
| [F] | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ |
| Is [F] symmetric? How do you know? | | |
| Does [F] involve a rotation? How do you know? | | |
| $[F^T][F]=[U]^2$ | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ |
| Symmetric stretch matrix [U] | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ |
| Greatest and least eigenvalues of [U] | $\lambda_1$ = <br> $\lambda_2$ = | $\lambda_1$ = <br> $\lambda_2$ = |
| x-, y- components of eigenvectors of [U] | $X_x(\lambda_1)$ = $X_x(\lambda_2)$ = <br> $X_y(\lambda_1)$ = $X_y(\lambda_2)$ = | $X_x(\lambda_1)$ = $X_x(\lambda_2)$ = <br> $X_y(\lambda_1)$ = $X_y(\lambda_2)$ = |
| Rotation matrix $[R]=[F][U]^{-1}$ | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ |
| $[F][F^T]=[V]^2$ | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ |
| Symmetric stretch matrix [V] | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ |
| Greatest and least eigenvalues of [V] | $\lambda_1$ = <br> $\lambda_2$ = | $\lambda_1$ = <br> $\lambda_2$ = |
| x-, y- components of eigenvectors of [V] | $X_x(\lambda_1)$ = $X_x(\lambda_2)$ = <br> $X_y(\lambda_1)$ = $X_y(\lambda_2)$ = | $X_x(\lambda_1)$ = $X_x(\lambda_2)$ = <br> $X_y(\lambda_1)$ = $X_y(\lambda_2)$ = |
| Rotation matrix $[R] = [V]^{-1}[F]$ | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ | $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ |
| Δ (dilation) | | |

/56 pts

Using [F] for Fig. 1, run the Matlab commands below to plot the eigenvectors of [U] and [V], title the plots, and attach the plots as pages 7 and 8. Then answer questions 14-16 (**4 pts each**)

```
figure(1);U=(F'*F)^(1/2);[vecU,valU] = eig(U);quiver([0,0],[0,0],vecU(1,:),vecU(2,:)); axis equal
figure(2);V=(F*F')^(1/2);[vecV,valV] = eig(V);quiver([0,0],[0,0],vecV(1,:),vecV(2,:)); axis equal
```

14 What directions do the eigenvectors of [U] point in relative to the axes of an ellipse in Fig. 3b?

15 What directions do the eigenvectors of [V] point in relative to the axes of the ellipse in Fig. 3c?

16 Compare the eigenvalues of [U] and [V] to the principal stretches $S_1$ and $S_2$ of Table 1.

/12 pts

Question (**4 points each**)

17 To calculate the elongation or stretch of a line segment (or vector), you need to know the initial length and final length of the line segment. What allows you to calculate the principal elongations for Figure 1 and Figure 2 by graphical means in Table 1? Explain.

18 If you knew the final size and shape of an ellipse, and you knew the ellipse was homogeneously deformed from a circle, but you did not know the size of the initial circle, could you calculate the **magnitudes** of the principal elongations? Explain.

19 If you knew the final size and shape of an ellipse, and you knew the ellipse was homogeneously deformed from a circle, but you did not know the size of the initial circle, could you calculate the **orientations** of the principal elongations? Explain.

20 If you knew the final size and shape of an ellipse, but you neither knew nor assumed the size and shape of the original object before it became transformed into the ellipse, could you calculate either the magnitudes or orientations of the principal elongations? Explain.

21 If you knew the final size and shape of an ellipse, and you assumed you knew the size and shape of the circle that became transformed into the ellipse, could you calculate either the magnitudes or orientations of the principal elongations? Explain.

/20 pts