

Exercise 1: Apparent dip problem (24 points total)

- 1a) An apparent dip of 62° to the southwest is measured for a bedding plane in a vertical cross section that strikes 230° (call this apparent dip vector 1). An apparent dip of 34° to the southeast is measured for a bedding plane in a vertical cross section that strikes 110° (call this apparent dip vector 2). What is the strike of the bedding plane and the true dip of the bedding plane? To solve this problem you find the common plane that contains two intersecting lines (which in this case are the two vectors showing apparent dips). Solve the problem using an equal-angle spherical projection (**7 points total; 2 points for plotting each line, 1 pt for graphically identifying the common plane, and 2 pts for getting the strike and dip of the common plane**)

Strike	Dip

- 1b) Solve the problem with cross products using Matlab. Include a copy of your Matlab printout. (**17 points total, 1 point per box**)

	Trend	Plunge	α	β	γ
Vector v1					
Vector v2					

Components of cross product **v2 x v1**

	N_x	N_y	N_z
N=v2xv1			

Pole N trend ($^\circ$)	Pole N plunge ($^\circ$)	Plane strike ($^\circ$)	Plane dip ($^\circ$)

Exercise 2: Rotation problem 2 (15 points total)

- 2) An outcrop displays a regular set of current ripple marks. The axes of the ripple marks pitch 32° northwest in the bedding plane, and the bedding plane strikes 325° and dips 20° NE. Determine the direction of the original paleocurrents responsible for the ripple marks by restoring the beds back to horizontal (assume the current flowed perpendicular to the axes of the ripples). Before you answer that question, first determine the orientation of the rotation axis **N** and the angle of rotation θ . Neatly label your stereonet to show how the relevant features rotate (i.e., the ripple axis and the pole to bedding). **(1 pt/box here; 5 subjective points for clarity of stereonet work)**

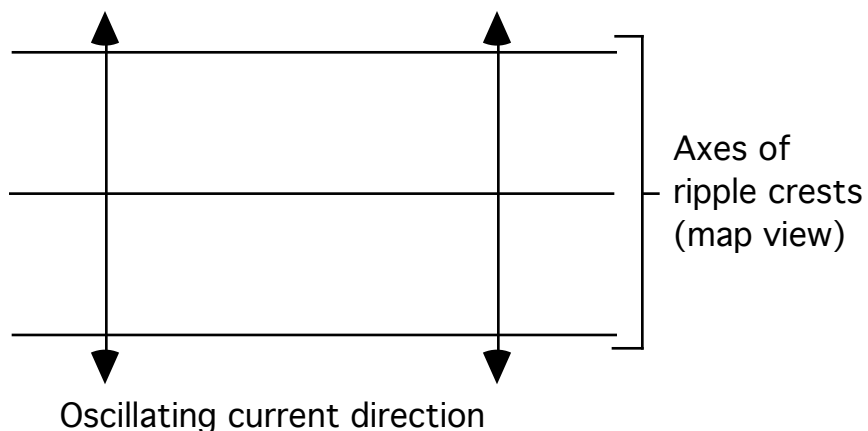
Trend of rotation axis θ ($^\circ$)	Plunge of rotation axis ϕ ($^\circ$)	Rotation angle ω ($^\circ$)

Existing ripple axis trend ($^\circ$)	Existing ripple axis plunge ($^\circ$)

Restored ripple axis trend ($^\circ$)	Restored ripple axis plunge ($^\circ$)

Original trend of current ($^\circ$)	Original plunge of current ($^\circ$)

Rake of restored ripple axis ($^\circ$)



Exercise 3: Rotations using linear algebra

A common problem in geology involves transforming data from one reference frame to another. A simple but informative example involves changing coordinates from an $x =$ north, $y =$ east, $z =$ down reference frame to an $x''' =$ east, $y''' =$ north, $z''' =$ up reference frame. This rotation problem usually is highly inconvenient to do using spherical projections but can be done rapidly using three matrix methods discussed in class.

- 1 Draw an oblique perspective 3D diagram showing each of the reference frame, labeling the axes of the two reference frames and the directions north, east, up, and down. **10 pts total; 5 pts for each reference frame, with 3 pts for axes and 2 pts for labels)**

- 2 Rotation using direction cosine matrices. Complete the following tables, entering the angles in degrees, between the axes in the unprimed and triple-primed reference frames. The angle $\theta_{xx''}$ is the angle from the x -axis to the x'' -axis, and $a_{xx''}$ is the cosine of the angle from the x -axis to the x'' -axis. **(36 pts total; 1 pt/box)**

$\theta_{xx''} =$	$\theta_{xy''} =$	$\theta_{xz''} =$
$\theta_{yx''} =$	$\theta_{yy''} =$	$\theta_{yz''} =$
$\theta_{zx''} =$	$\theta_{zy''} =$	$\theta_{zz''} =$

$\theta_{x'''x} =$	$\theta_{x'''y} =$	$\theta_{x'''z} =$
$\theta_{y'''x} =$	$\theta_{y'''y} =$	$\theta_{y'''z} =$
$\theta_{z'''x} =$	$\theta_{z'''y} =$	$\theta_{z'''z} =$

Rotation matrix for converting triple-primed coordinates to unprimed coordinates

$a_{xx''} =$	$a_{xy''} =$	$a_{xz''} =$
$a_{yx''} =$	$a_{yy''} =$	$a_{yz''} =$
$a_{zx''} =$	$a_{zy''} =$	$a_{zz''} =$

Rotation matrix for converting unprimed coordinates to triple-primed coordinates

$a_{x'''x} =$	$a_{x'''y} =$	$a_{x'''z} =$
$a_{y'''x} =$	$a_{y'''y} =$	$a_{y'''z} =$
$a_{z'''x} =$	$a_{z'''y} =$	$a_{z'''z} =$

3 Construction of rotation matrix using consecutive rotations. (15+54+9 = 78 pts)

- A On a separate page, draw a sequence of three drawings showing the three sequential rotations about the z -, x' -, and y'' -axes needed to rotate the x,y,z axes to the x''',y''',z''' axes. The sequence should illustrate the rotations described below, and it should state the angle of rotation. The drawings should be neat and clearly labeled (15 points total, 5 points/drawing).

First rotation: about **z -axis** to bring trend of y' -axis into alignment with trend of y''' -axis.

$\theta_{x'x} =$	$\theta_{x'y} =$	$\theta_{x'z} =$
$\theta_{y'x} =$	$\theta_{y'y} =$	$\theta_{y'z} =$
$\theta_{z'x} =$	$\theta_{z'y} =$	$\theta_{z'z} =$

First rotation matrix R1; converts unprimed () coordinates to single-primed (') coordinates

$a_{x'x} =$	$a_{x'y} =$	$a_{x'z} =$
$a_{y'x} =$	$a_{y'y} =$	$a_{y'z} =$
$a_{z'x} =$	$a_{z'y} =$	$a_{z'z} =$

Second rotation: about **x' -axis** to bring y' -axis into alignment with y''' -axis.

$\theta_{x''x'} =$	$\theta_{x''y'} =$	$\theta_{x''z'} =$
$\theta_{y''x'} =$	$\theta_{y''y'} =$	$\theta_{y''z'} =$
$\theta_{z''x'} =$	$\theta_{z''y'} =$	$\theta_{z''z'} =$

Second rotation matrix R2; converts (') coordinates to (') coordinates

$a_{x''x'} =$	$a_{x''y'} =$	$a_{x''z'} =$
$a_{y''x'} =$	$a_{y''y'} =$	$a_{y''z'} =$
$a_{z''x'} =$	$a_{z''y'} =$	$a_{z''z'} =$

Third rotation: about **y'' -axis** to bring all (') axes into alignment with all (') axes.

$\theta_{x'''x''} =$	$\theta_{x'''y''} =$	$\theta_{x'''z''} =$
$\theta_{y'''x''} =$	$\theta_{y'''y''} =$	$\theta_{y'''z''} =$
$\theta_{z'''x''} =$	$\theta_{z'''y''} =$	$\theta_{z'''z''} =$

Third rotation matrix R3; converts (') coordinates to (') coordinates

$a_{x'''x''} =$	$a_{x'''y''} =$	$a_{x'''z''} =$
$a_{y'''x''} =$	$a_{y'''y''} =$	$a_{y'''z''} =$
$a_{z'''x''} =$	$a_{z'''y''} =$	$a_{z'''z''} =$

Single matrix for doing rotation using one rotation: $R = R3 * R2 * R1$

Include a copy of your Matlab work to do this next step.

R	=	$R3$	*	$R2$	*	$R1$
	=		*		*	
	=		*		*	
	=		*		*	

4 Construction of rotation matrix using consecutive rotations. (30 pts)

In order to bring the unprimed reference frame into coincidence with the triple-primed reference frame in one step, what are the trend and plunge of the rotation axis, and what is the angle of rotation? You should be able to visualize this. Make sure you get the sign of the rotation angle correct using a right-hand rule for a positive angle.

Trend (°)	Plunge (°)	Rotation angle (°)

To four significant figures, what are the direction cosines (η_x, η_y, η_z) of the rotation axis?

η_x	η_y	η_z

Using the formulas on page 11 of lecture 10,

$a_{x''x} = \eta_x \eta_x (1 - \cos\theta) + \cos\theta$	$a_{x''y} = \eta_x \eta_y (1 - \cos\theta) + \eta_z \sin\theta$	$a_{x''z} = \eta_x \eta_z (1 - \cos\theta) - \eta_y \sin\theta$
$a_{y''x} = \eta_y \eta_x (1 - \cos\theta) - \eta_z \sin\theta$	$a_{y''y} = \eta_y \eta_y (1 - \cos\theta) + \cos\theta$	$a_{y''z} = \eta_y \eta_z (1 - \cos\theta) + \eta_x \sin\theta$
$a_{z''x} = \eta_z \eta_x (1 - \cos\theta) + \eta_y \sin\theta$	$a_{z''y} = \eta_z \eta_y (1 - \cos\theta) - \eta_x \sin\theta$	$a_{z''z} = \eta_z \eta_z (1 - \cos\theta) + \cos\theta$

Calculate the single-step rotation matrix R for converting unprimed coordinates to triple-primed coordinates. Show your work on a separate page; a Matlab printout is preferred.

$a_{x'''x} =$	$a_{x'''y} =$	$a_{x'''z} =$
$a_{y'''x} =$	$a_{y'''y} =$	$a_{y'''z} =$
$a_{z'''x} =$	$a_{z'''y} =$	$a_{z'''z} =$

4 Conversion of coordinates. Consider the x,y,z coordinates listed column-by-column below in matrix X (each of the 5 points has x,y,z coordinates)

X

	Point 1	Point 2	Point 3	Point 4	Point 5
x	1	2	-1	1	0
y	1	2	-1	2	0
z	1	2	-1	3	0

Use the one-step rotation matrix to convert the x,y,z coordinates to x''' , y''' , z''' coordinates by pre-multiplying them en masse using the rotation matrix and Matlab (i.e., $X''' = RX$).

Show your Matlab work

X'''

	Point 1	Point 2	Point 3	Point 4	Point 5
x'''					
y'''					
z'''					