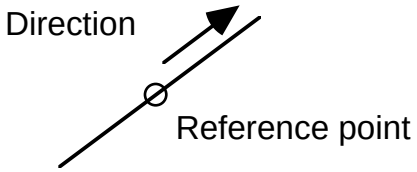
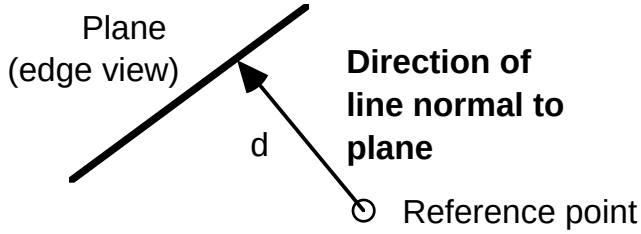
ORIENTATIONS OF LINES AND PLANES IN SPACE

I Main Topics
- A Definitions of points, lines, and planes
- B Geologic methods for describing lines and planes
- C Attitude symbols for geologic maps
- D Reference Frames

II Definitions of points, lines, and planes
- A Point
  - 1 Defined by one set of coordinates (an ordered triple in 3-D)
  - 2 Defined by distance and direction from a reference point
  - 3 Intersection of two lines
  - 4 Intersection of three planes
- B Line
  - 1 Defined by two sets of coordinates
  - 2 Defined by two points
  - 3 Defined by distance from a reference point and the direction of the line



  - 4 Intersection of two planes
- C Plane
  - 1 Defined by three sets of coordinates
  - 2 Defined by three points
  - 3 Defined by distance and direction from a reference point



  - 4 Defined by two intersecting or two parallel lines

III Geologic methods for describing lines and planes

A Orientations of lines

1 Trend & plunge

a **Trend**: Direction (azimuth) of a vertical plane containing the line of interest.

i Azimuth (compass bearing): direction of a horizontal line contained in a vertical plane. Measured by quadrant or (°). Examples: N90°E, N90°W, S90°W, 270°.

i i The trend "points" in the direction a line plunges

b **Plunge**: The inclination of a line below the horizontal

2 Pitch (or rake): the angle, measured in a plane of specified orientation, between one line and a horizontal line (see handout)

B **Orientations of planes**

1 Orientation of two intersecting lines in the plane

Strike & dip

a **Strike**: direction of the line of intersection between an inclined plane and a horizontal plane (e.g., a lake);

b **Dip**: inclination of a plane below the horizontal; $0° \leq dip \leq 90°$

c The azimuth directions of strike and dip are perpendicular

d Good idea to specify the direction of dip to eliminate ambiguity, but right hand rule (see handout) can also be used.

e Examples: Strike N90°W Dip 45°N right-handed
Strike N90°W Dip 45°S left-handed: don't use

f NOTE: **Trend and plunge refer to lines; strike and dip refer to planes**

2 Orientation of one special line in the plane

Dip & dip direction (azimuth of dip)

a Used mostly in Europe

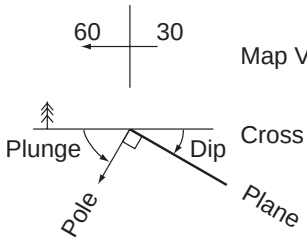
b Water runs down the dip direction

3 Trend & plunge of pole (unit normal) to plane

1 Pole is a line traditionally taken to point down

2 Pole trend = strike - 90°; pole plunge = 90° - dip

Attitude Symbol for a Plane and its Pole



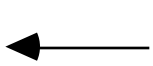
The pole trends 180^o from the direction the plane dips

The plane dip and pole plunge sum to 90^o

The plane appears as a line if viewed in the direction of strike

IV Attitude symbols (strike and dip of a plane; trend and plunge of a line)

Plane strikes SW
dips 45° to NW



Line trends west;
plunges 20°

V Reference frames

A Cartesian coordinates

1 Points are described by their x, y, z coordinates

2 The x,y, and z axes are right-handed and mutually perpendicular

3 Direction of a line

a Given by the coordinates of pairs of points

b Given by the difference in coordinates of pairs of points

i $\Delta x = x_2 - x_1$

ii $\Delta y = y_2 - y_1$

iii $\Delta z = z_2 - z_1$

b Given by the angles ω_x, ω_y, and ω_z. These are the angles between a line of unit length and the x, y, and z axes, respectively. **The respective cosines of the these angles (α, β, γ) are called the direction cosines**.

Angle	ω_x	ω_y	ω_z
Direction cosine	α	β	γ

4 **Length of a line**

$$L = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2} = \sqrt{\left(\Delta x\right)^2 + \left(\Delta y\right)^2 + \left(\Delta z\right)^2}$$

A line of unit length has a length of one.

B Cylindrical coordinates (3-D version of polar coordinates)

1 Point P is described by its ρ, θ, z coordinates

a ρ = distance from the origin to P', where P' is the projection of point P onto x,y plane

b θ = angle between x-axis and the OP'

c z = distance from origin to projection of point P onto z-axis

C Spherical coordinates

1 Points are described by their r, θ, ϕ coordinates

a r = distance from the origin to point P

Note: r here is different than ρ for cylindrical coordinates

b θ = angle between x-axis and the OP', where P' is the projection of point P onto x,y plane

c ϕ = angle between OP' and OP

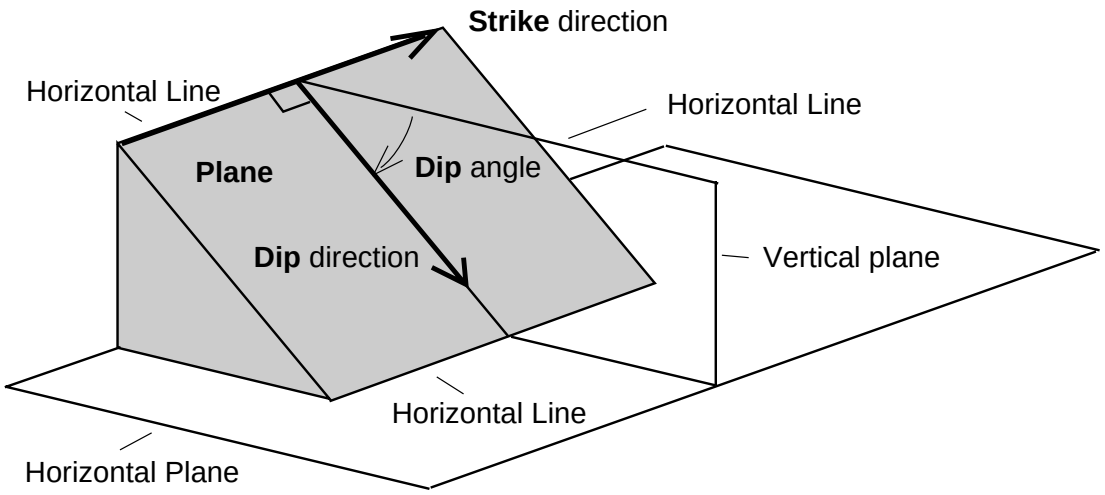
Note: In some spherical schemes, the angle between OP and the z-axis is used as the second angle.

D Conversions between coordinate systems

Cartesian ← Spherical	**Spherical ← Cartesian**
$x = r \cos\phi \cos\theta$ $\quad \alpha = \cos\phi \cos\theta$	$r = \sqrt{x^2 + y^2 + z^2}$ $\quad 1 = \sqrt{\alpha^2 + \beta^2 + \gamma^2}$
$y = r \cos\phi \sin\theta$ $\quad \beta = \cos\phi \sin\theta$	$\theta = \tan^{-1}(y/x)$ $\quad \theta = \tan^{-1}(\beta/\alpha)$
$z = r \sin\phi$ $\quad \gamma = \sin\phi$	$\phi = \sin^{-1}(z/r)$ $\quad \phi = \sin^{-1}(\gamma)$
Cartesian ← Cylindrical	**Cylindrical ← Cartesian**
$x = r \cos\theta$	$r = \sqrt{x^2 + y^2}$
$y = r \sin\theta$	$\theta = \tan^{-1}(y/x)$
$z = z$	$z = z$
Cylindrical ← Spherical	**Spherical ← Cylindrical**

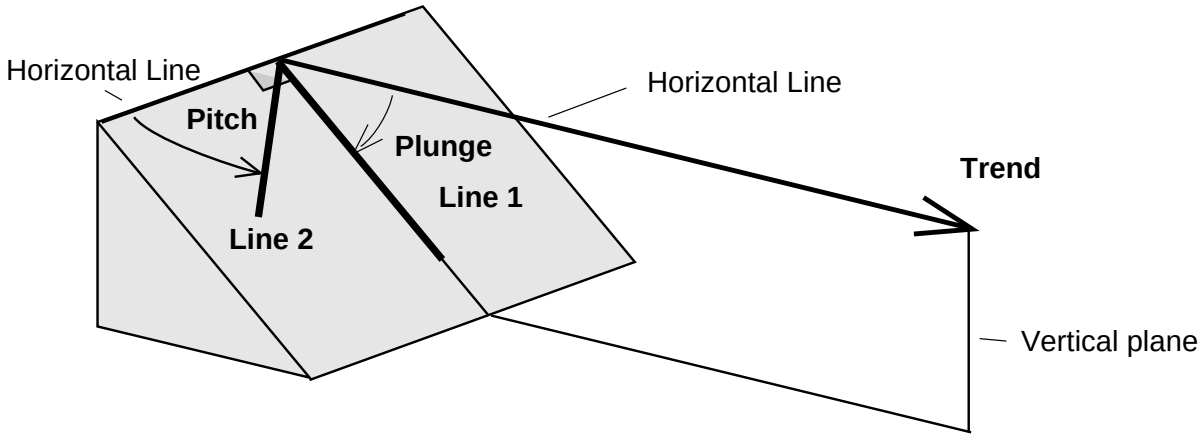
ORIENTATION OF LINES AND PLANES

PLANES



Right hand rule for strike and dip directions: If thumb on right hand points in the direction of strike the fingers on the right hand should point in the direction of dip

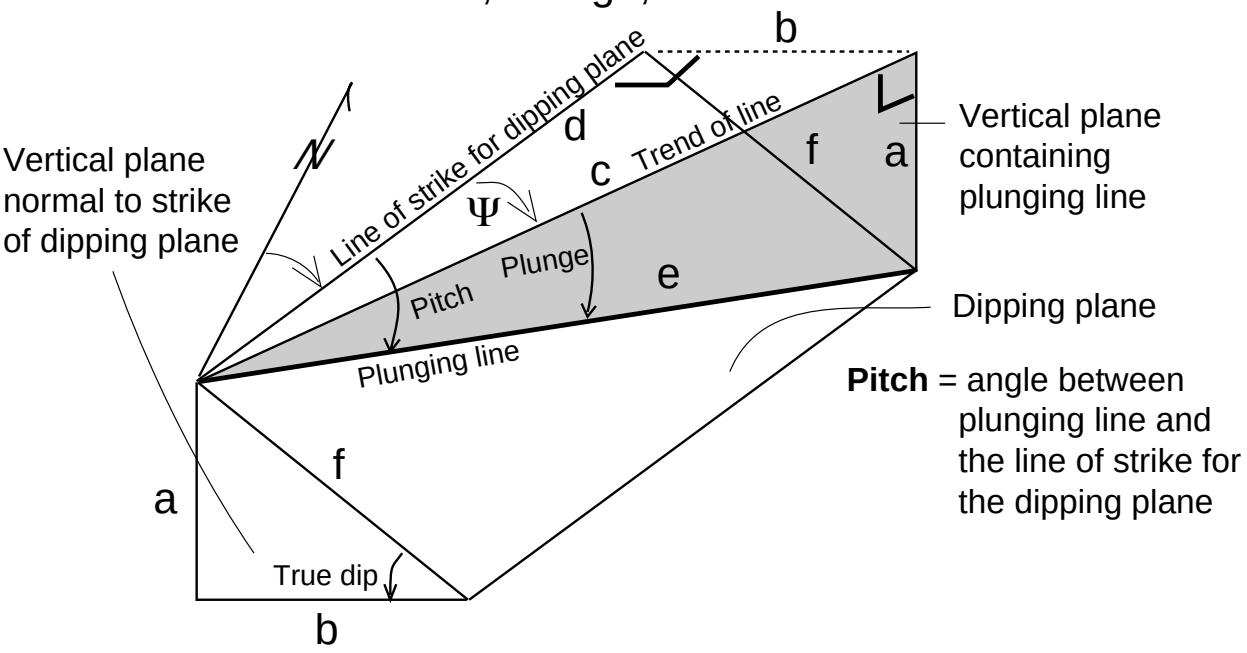
LINES



Need to define orientation of plane for the pitch (rake) to have meaning

The POLE to a plane is a line that is perpendicular to the plane.
The trend of the pole is opposite the direction a plane dips.
The plunge of a pole and the dip of a plane sum to 90¡.

Trend, Plunge, & Pitch



	plunge (ϕ)	pitch (Ω)	dip (δ)	Ψ
sin	$\frac{a}{e}$	$\frac{f}{e}$	$\frac{a}{f}$	$\frac{b}{c}$
cos	$\frac{c}{e}$	$\frac{d}{e}$	$\frac{b}{f}$	$\frac{d}{c}$
tan	$\frac{a}{c}$	$\frac{f}{d}$	$\frac{a}{b}$	$\frac{b}{d}$

(1) Trend = θ = strike + Ψ = strike + $\cos^{-1}$ (d/c) = strike + $\cos^{-1}\{(\cos \Omega)/(\cos \phi)\}$

(2) Trend = θ = strike + Ψ = strike + $\tan^{-1}$ (b/d) = strike + $\tan^{-1}\{(\cos \delta)(\tan\Omega)\}$

(3) Plunge = ϕ = $\sin^{-1}$ (a/e) = $\sin^{-1}$ $\{(\sin \delta)(\sin\Omega)\}$

(4) Pitch = Ω = $\sin^{-1}$ (f/e) = $\sin^{-1}$ $\{(\sin\phi) / (\sin \delta)\}$

Geologic Conventions for Measuring Orientations

Compass Bearings

By quadrant (relative to north or south). The angle does not exceed 90°

By 360° azimuth (0° - 360°)

Examples

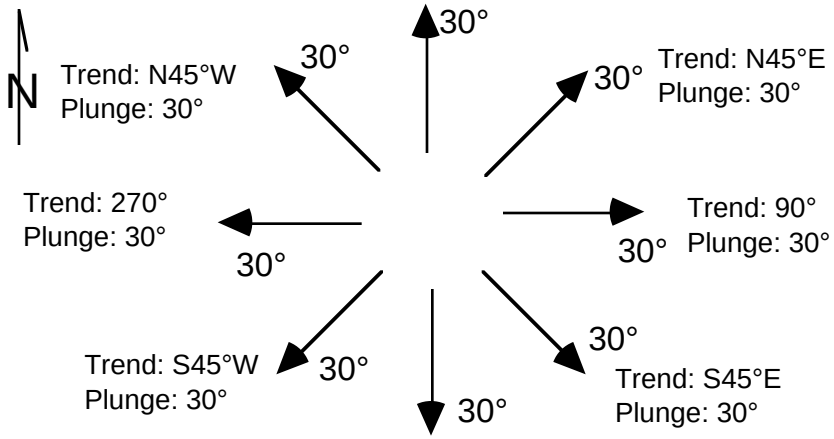
N0°E	N45°E	N90°E	S45°E	S0°E	S45°W	S90°W	N45°W
0°	45°	90°	135°	180°	225°	270°	315°

Lines

Trend: A compass bearing

Plunge: An inclination below horizontal

Examples: The lines below all plunge at 30°. Their trends vary according to the table above

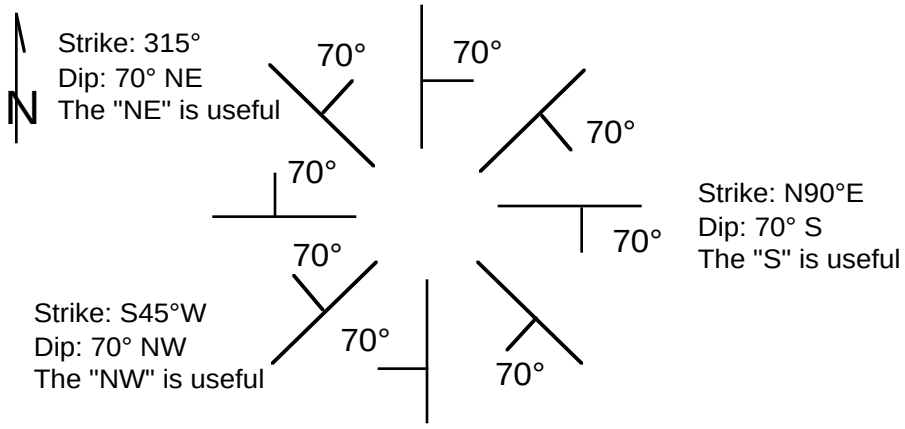


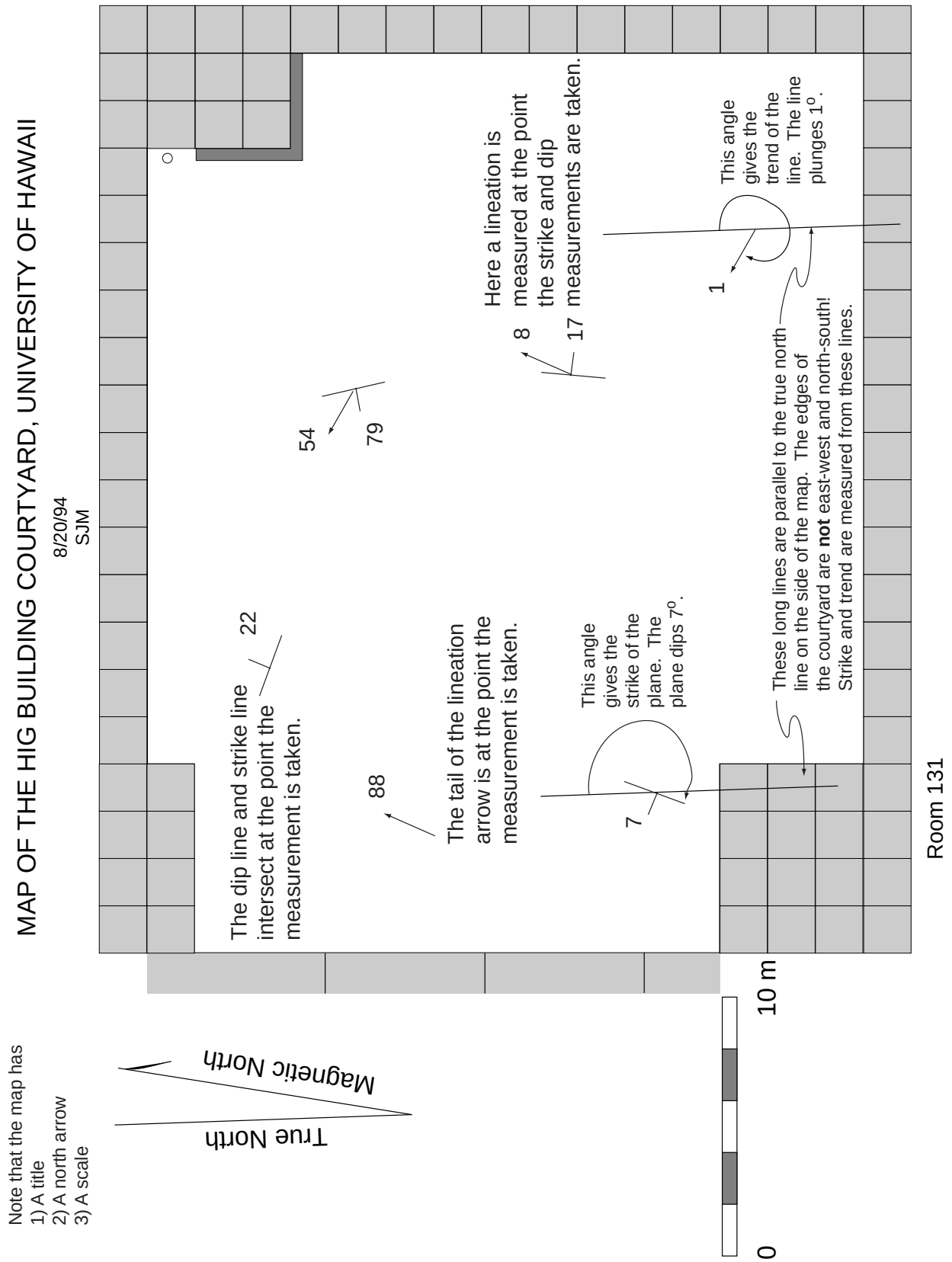
Planes

Strike: A compass bearing

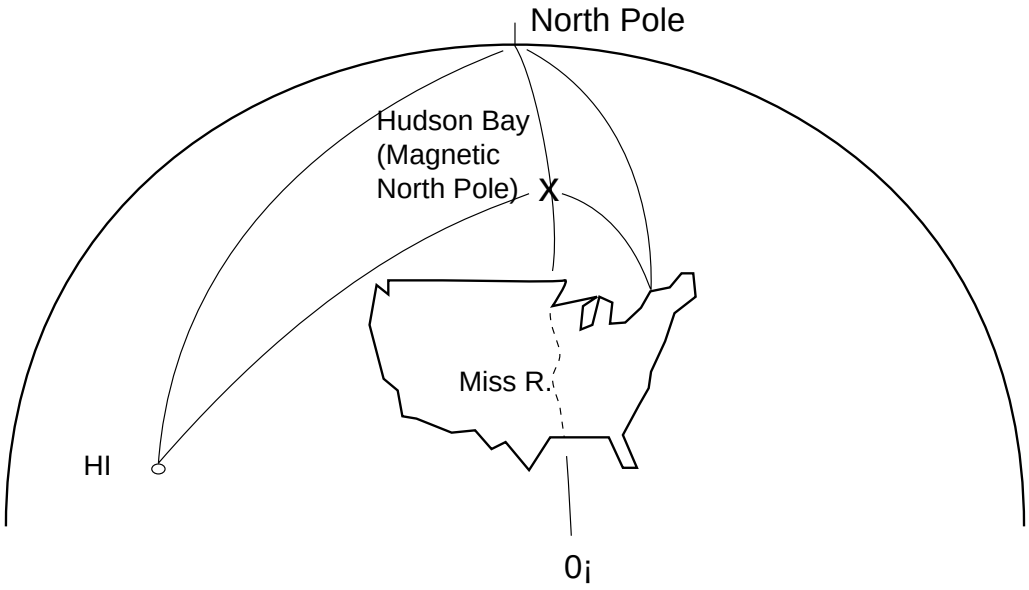
Dip: An inclination below horizontal

Examples: The planes below all dip at 70°. Their strikes vary according to the table above

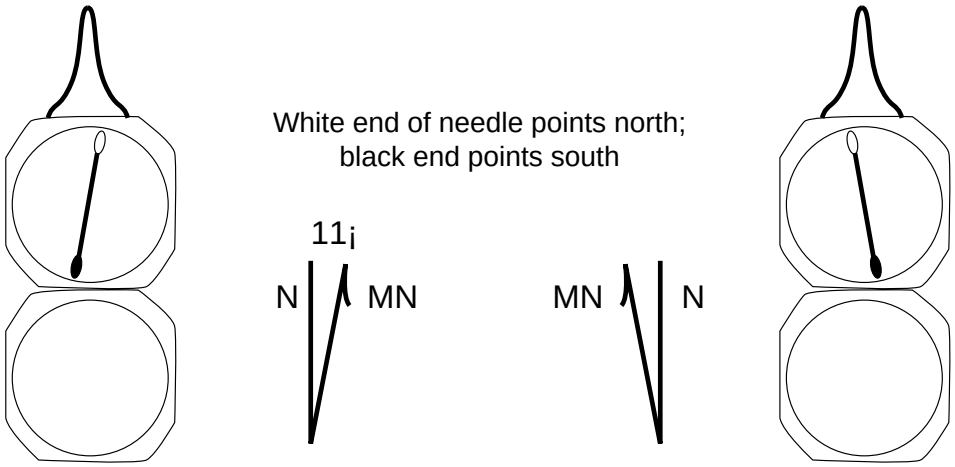



MAP OF THE HIG BUILDING COURTYARD, UNIVERSITY OF HAWAII
8/20/94
SJM
Note that the map has
1) A title
2) A north arrow
3) A scale
True North
Magnetic North
0
10 m
Room 131
The dip line and strike line intersect at the point the measurement is taken.
22
88
The tail of the lineation arrow is at the point the measurement is taken.
This angle gives the strike of the plane. The plane dips 7°.
7
These long lines are parallel to the true north line on the side of the map. The edges of the courtyard are **not** east-west and north-south! Strike and trend are measured from these lines.
54
79
Here a lineation is measured at the point the strike and dip 17 measurements are taken.
8
This angle gives the trend of the line. The line plunges 1°.
1

Magnetic Declination and a Geologic Compass



When the declination is set correctly,
if the compass body points to true north,
then the magnetic needle of the compass points to magnetic north
and the compass reads "0"



Direction Cosines from Geologic Angle Measurements (Spherical coordinates)

Positive z-axis up
y = north; x = east

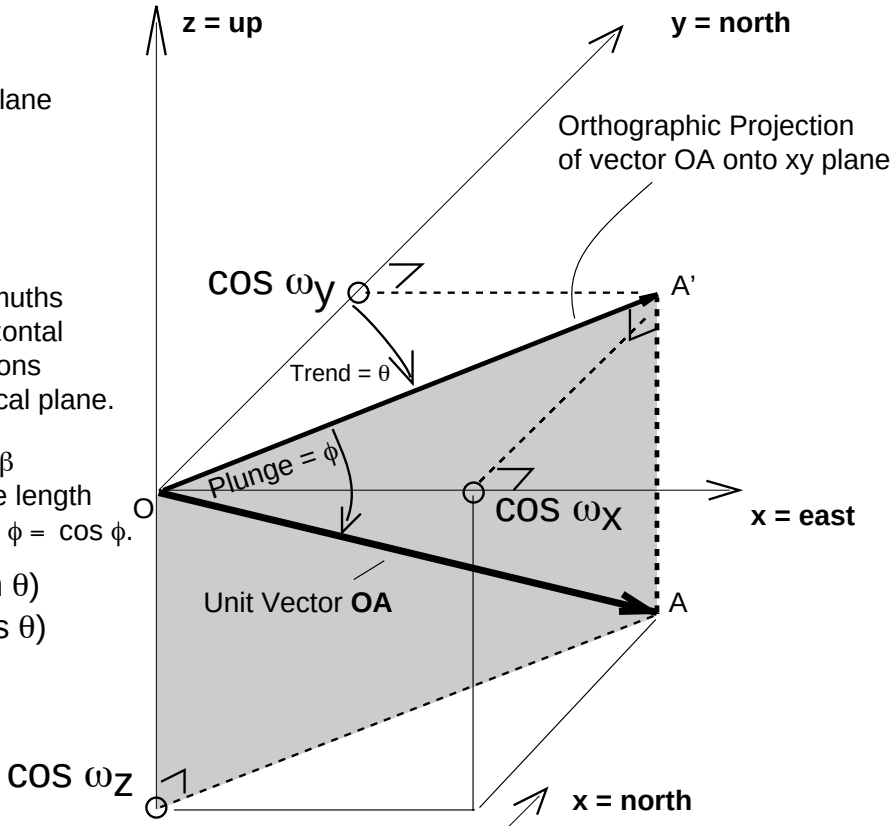
xy plane is horizontal plane

RIGHT-HANDED COORDINATES

Remember: Trends are azimuths and are measured in a horizontal plane. Plunges are inclinations and are measured in a vertical plane.

The direction cosines α and β are determined from OA', the length of OA' being $|OA'| = |OA| \cos\phi = \cos\phi$.

$\alpha = \cos\omega_x = (\cos\phi)(\sin\theta)$
$\beta = \cos\omega_y = (\cos\phi)(\cos\theta)$
$\gamma = \cos\omega_z = -(\sin\phi)$
$\alpha^2 + \beta^2 + \gamma^2 = 1$



Positive z-axis down
x = north; y = east

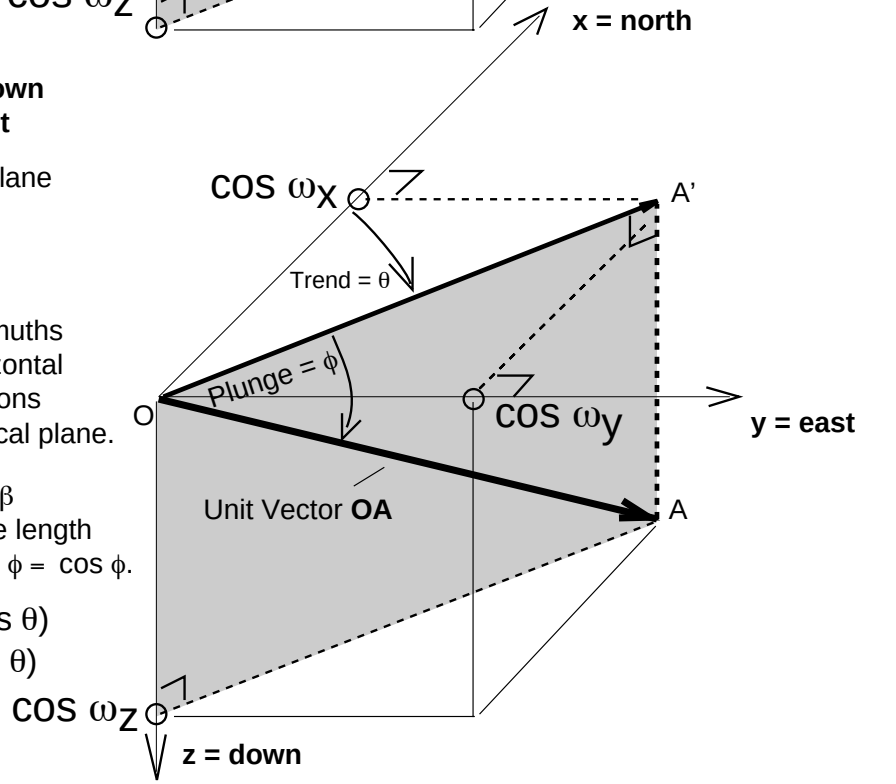
xy plane is horizontal plane

RIGHT-HANDED COORDINATES

Remember: Trends are azimuths and are measured in a horizontal plane. Plunges are inclinations and are measured in a vertical plane.

The direction cosines α and β are determined from OA', the length of OA' being $|OA'| = |OA| \cos\phi = \cos\phi$.

$\alpha = \cos\omega_x = (\cos\phi)(\cos\theta)$
$\beta = \cos\omega_y = (\cos\phi)(\sin\theta)$
$\gamma = \cos\omega_z = +(\sin\phi)$
$\alpha^2 + \beta^2 + \gamma^2 = 1$



Lab 1

1 Measure the strikes and dips of **4** of the planes in the HIG courtyard, and write them in the table below. (**16 points total**)

Scoring: Strikes:1 pt if within 3°, 1/2 pt if within 8°; 1 pt for syntax
Dips: 1 pt if within 3°, 1/2 pt if within 8°

2 Determine the trend and plunge of the pole to each of the four planes in step 1, and write them in the table below. (**16 points total**)

Scoring: 1 pt for correct numerical calculation; 1 pt for correct syntax

Plane ID#	Strike **(2 pts/box)**	Dip **(2 pts/box)**	Pole trend **(2 pts/box)**	Pole plunge **(2 pts/box)**
A				
B				
C				
D				
E				
F				

3 Plot the attitudes of the planes on the courtyard map. (**8 pts total**)

Scoring: 1 pt for correct symbol and orientation
1 pt for correct location (± 1/2 box)

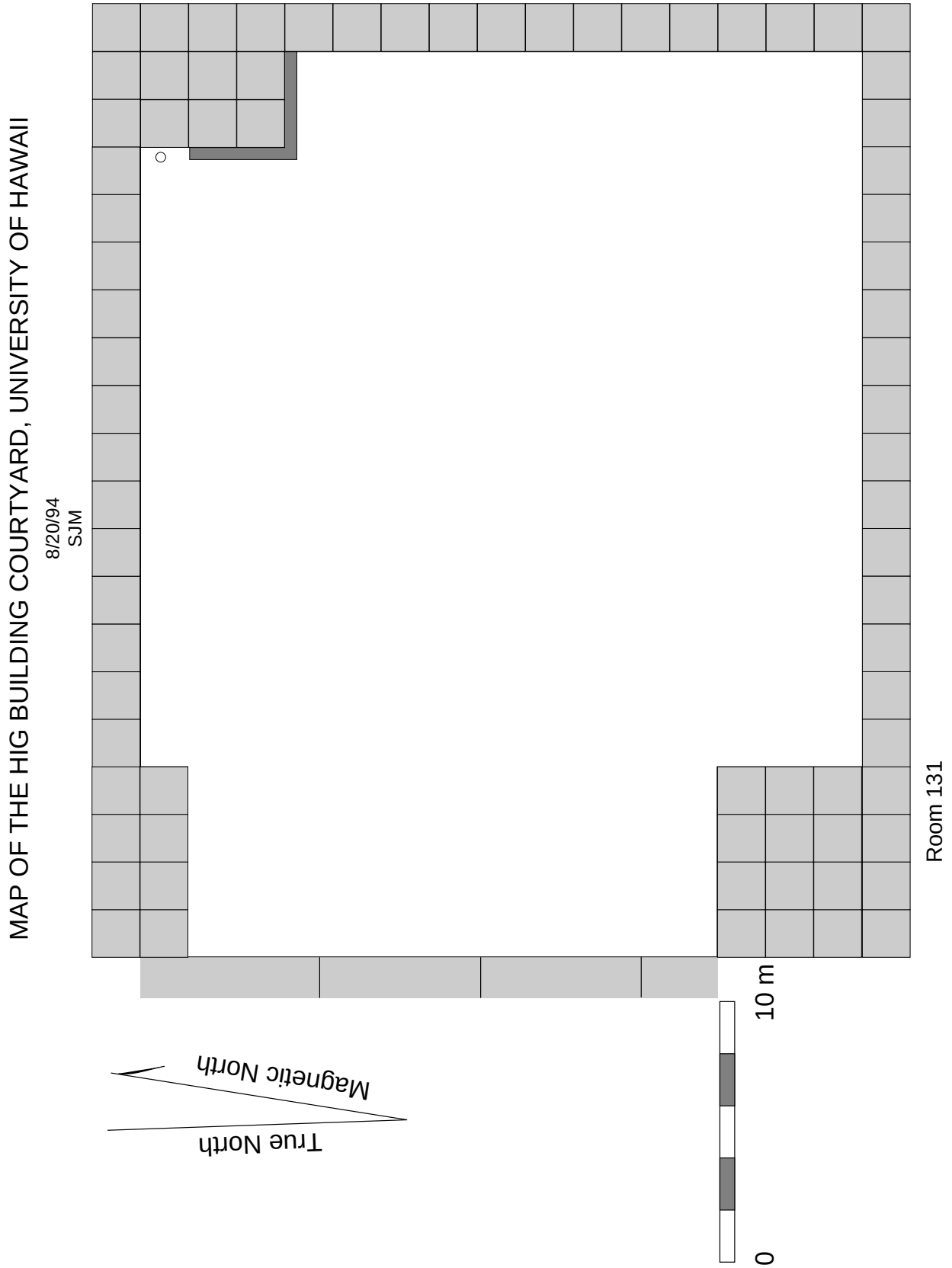
4 Measure the trends and plunges of **4** of the lines in the HIG courtyard, and write them in the table below. (**16 points total**)

Scoring: Trends: 1 pt if within 3°, 1/2 pt if within 8°; 1 pt for syntax
Plunges: 2 pts if within 3°, 1 pt if within 8°

Line ID #	Trend	Plunge	Line ID #	Trend	Plunge
A1			A2		
B1			B2		
C1			C2		
D1			D2		
E1			E2		
F1			F2		

5 Plot the attitudes of the lines on the courtyard map. (**8 pts total**)

Scoring: 1 pt for correct symbol and orientation
1 pt for correct location (± 1/2 box)


MAP OF THE HIG BUILDING COURTYARD, UNIVERSITY OF HAWAII
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10 m
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