14. Folds

I Main Topics
- A Local geometry of a plane curve (cylindrical fold)
- B Local geometry of a curved surface (3D fold)
- C Numerical evaluation of curvature (geometry)
- D Kinematics of folding
- E Fold terminology and classification (geometry)

14. Folds

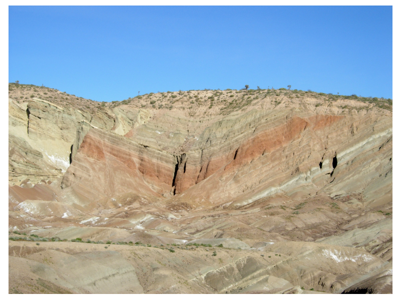
http://upload.wikimedia.org/wikipedia/commons/a/ae/Caledonian_orogeny_fold_in_King_Oscar_Fjord.jpg

14. Folds

Anticline, New Jersey

http://en.wikipedia.org/wiki/File:NJ_Route_23_anticline.jpg

Syncline, Rainbow Basin, California



http://en.wikipedia.org/wiki/File:Rainbow_Basin.JPG

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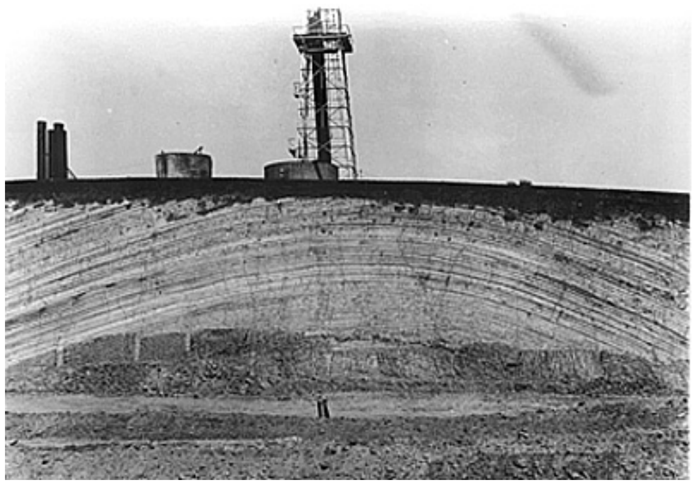
Folds, New South Wales, Australia

http://en.wikipedia.org/wiki/File:Folded_Rock.jpg

Folds in granite, Sierra Nevada, California

14. Folds

Energy Resources and an Anticline

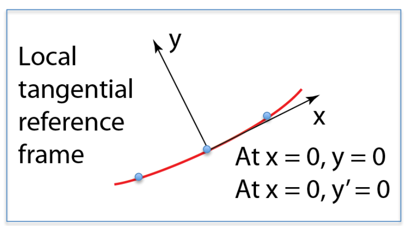


http://www.wou.edu/las/physci/Energy/graphics/OilAnticline.jpg

14. Folds

II Local geometry of a plane curve (cylindrical fold) in a tangential reference frame

A Express the plane curve as a power series:



1 $y = \left[\ldots + C_{-2}x^{-2} + C_{-1}x^{-1}\right] + \left[C_0 x^0\right] + \left[C_1 x^1 + C_2 x^2 + C_3 x^3 + \ldots\right]$

At x= 0, y = 0, so all the coefficients for terms with non-positive exponents must be zero

2 $y = C_1 x^1 + C_2 x^2 + C_3 x^3 + \ldots$

14. Folds

II Local geometry of a plane curve (cylindrical fold) in a tangential reference frame

2 $y = C_1 x^1 + C_2 x^2 + C_3 x^3 + \ldots$

Now examine y'

3 $y' = C_1 x^0 + 2C_2 x^1 + 3C_3 x^2 + \ldots = 0$

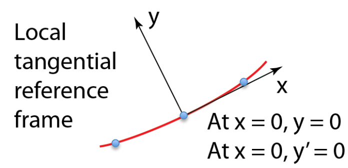
At x = 0, y' = 0, so C_1 = 0, so

4 $y = C_2 x^2 + C_3 x^3 + \ldots = 0$

As x ➔ 0, higher-order terms vanish

5 $\lim_{x \to 0} y = C_2 x^2$

6 $\lim_{x \to 0} k = |y(s)''| = |y(x)''| = 2C_2$



So all plane curves are locally second-order (parabolic).

14. Folds

III Local geometry of a curved surface in a tangential reference frame

A Plane curves are formed by intersecting a curved surface with a plane containing the surface normal

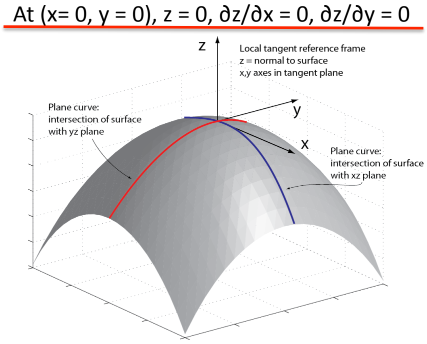
B These plane curves z = z(x,y) are locally all of second-order, so any continuous surface is locally 2nd order. The general form of such a surface in a tangential frame is

$z = Ax^2 + Bxy + Cy^2$

This is the equation of a paraboloid: all surfaces are locally either ellliptic or hyperbolic paraboloids

C Example: curve (*normal section*) in the arbitrary plane y = mx

At (x= 0, y = 0), z = 0, ∂z/∂x = 0, ∂z/∂y = 0



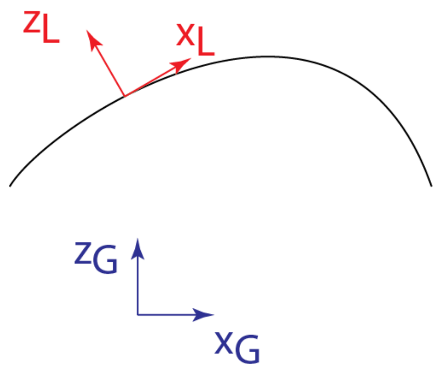
Parabolic plane curves

$y = \lim_{x \to 0, y \to 0} z = Ax^2 + Bx(mx) + C(mx)^2 = (A + Bm + Cm^2)x^2$

Sum of constants

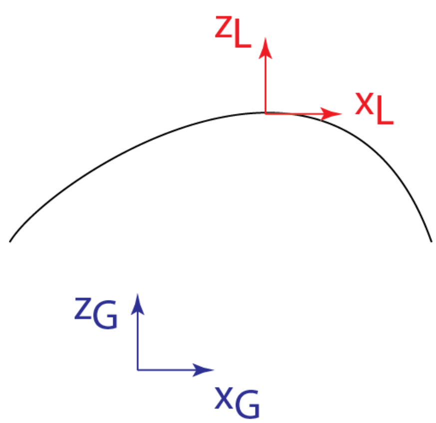
14. Folds

III Local geometry of a curved surface … (cont.)
- D Dilemma
  1. Evaluating curvatures of a surface $z_L=z_L(x_L,y_L)$, where "z_L" is normal to the surface, is easy
  2. The "global" reference frame, $z_G= z_G(x_G,y_G)$, in which data are collected are usually misaligned with the tangential local reference frame
  3. Alignment is generally difficult



14. Folds

III Local geometry of a curved surface … (cont.)
- E "resolution"
  1. At certain places the local and global reference frames are easily aligned though: at the summits or bottoms of folds
  2. We will evaluate the curvatures there, leaving the more general problem to "later"



14. Folds

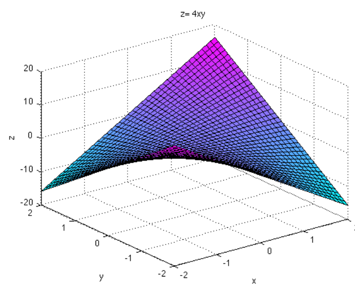
III Local geometry of a curved surface … (cont.)

F Example (analytical)

$z_G = 4x_G y_G$

1 First plot and evaluate z_G near (0,0)

```
>> [X,Y] = meshgrid([-2:0.1:2]);
>> Z=4*X.*Y;
>> surf(X,Y,Z);
>> xlabel('x'); ylabel('y');
>> zlabel('z'); title('z= 4xy')
```



This is a saddle

14. Folds

F Example (analytical) (cont.)

$z_G = 4x_G y_G$

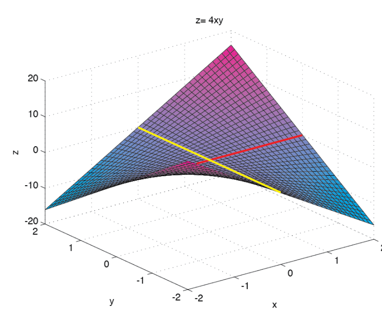
2 Now evaluate the first derivatives

a $\partial z_G/\partial x_G = 4y_G$

b $\partial z_G/\partial y_G = 4x_G$

c Both derivatives equal zero at (0,0)

3 The local tangential and global reference frames are aligned at (0,0)




```
>> hold on
>> plot3(X(21,:),Y(21,:),Z(21,:),'r')
>> plot3(X(:,21),Y(:,21),Z(:,21),'y')
```

14. Folds

F Example (analytical) (cont.)

$z_G = 4x_Gy_G$

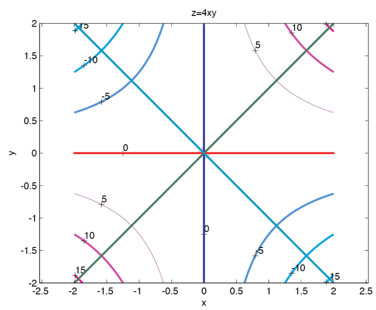
4 Now evaluate the second derivatives

a $\partial^2 z_G/\partial x_G^2 = 0$

b $\partial^2 z_G/\partial x_G \partial y_G = 4$

c $\partial^2 z_G/\partial y_G \partial x_G = 4$

d $\partial^2 z_G/\partial y_G^2 = 0$




```
>> c=contour(X,Y,Z); clabel(c);
>> xlabel('x'); ylabel('y');
>> title('z=4xy')
>> hold on
>> plot([0 0],[-2 2],[-2 2],[-2,2],[-2 2],[0 0],[-2 2],[2 -2])
>> axis equal
```


14. Folds

F Example (analytical) (cont.)

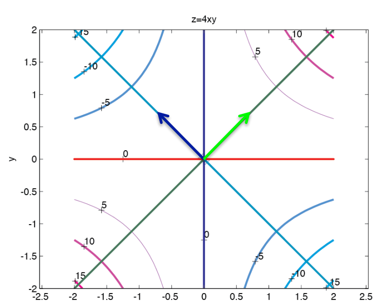
$z_G = 4x_Gy_G$

5 Now form the Hessian matrix

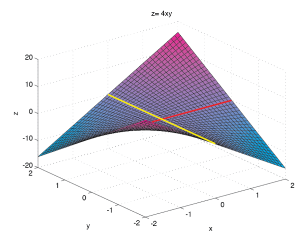
$$H = \begin{bmatrix} \partial^2 z/\partial x^2 & \partial^2 z/\partial x \partial y \\ \partial^2 z/\partial y \partial x & \partial^2 z/\partial y^2 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$$

6 Find its eigenvectors and eigenvalues

```
>> H=[0 4;4 0]
H =
0   -4
-4    0
>> [v,k]=eig(H)
v =
-0.7071   0.7071
0.7071    0.7071
k =
-4    0
 0    4
```



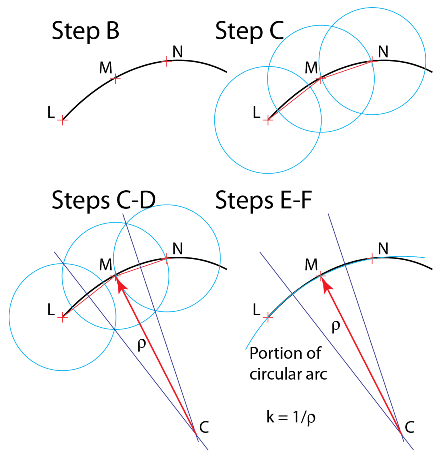
Saddle geometry much more clear in principal reference frame



14. Folds

IV Evaluation of curvature from discrete data (geometry)

- A Three (non-colinear) points define a plane – and a circle.
- B Locate three discrete non-colinear points along a curve (e.g., L, M, N)
- C Draw the perpendicular bisectors to line segments LM and MN
- D Intersect perpendicular bisectors at the center of curvature C.
- E The radius of curvature (ρ) equals the distance from C to L, M, or N.
- F The curvature is reciprocal of the radius of curvature ($k = 1/\rho$)
- G Local geometry of a curve also is circular!



14. Folds

IV Kinematics of folding (strain)

- A Curvature of a plane curve
  - $k = d\phi/ds$, where
  - ϕ = orientation of tangent ***t*** to curve
  - *s* = distance along curve
- B Curvature of a circular arc

$$\phi = \theta + 90°, \text{ so } d\phi = d\theta$$

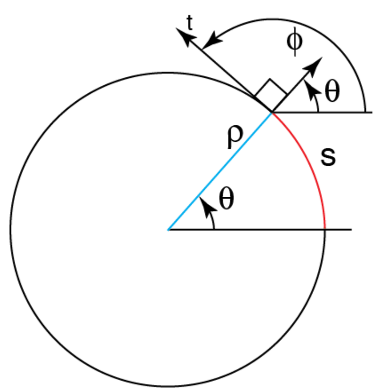
$$s = \rho\theta, \quad \text{so } ds = \rho d\theta$$

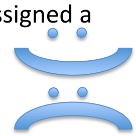
$$k = d\phi/ds = d\theta/ds = 1/\rho$$

Large curvature = small radius

Small curvature = large radius

- C Curvature can be assigned a sign
  - \+ = concave up
  - \- = concave down





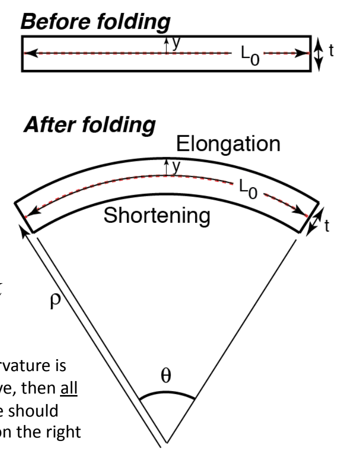
14. Folds

V Kinematics of folding (cont.)

D Layer-parallel normal strain ($\varepsilon_{\theta\theta}$) for cylindrical folds

1 Mid-plane of layer (y = 0) maintains length L_0

2 Layer maintains thickness *t* during folding



$$\varepsilon_{\theta\theta} = \frac{\Delta L}{L_0} = \frac{L_1 - L_0}{L_0} = \frac{(\rho + y)\theta - \rho\theta}{\rho\theta} = \frac{y}{\rho} = yk$$

$$\varepsilon_{\theta\theta}\left(y = \frac{t}{2}\right) = \frac{+tk}{2}\ (elongation)$$

$$\varepsilon_{\theta\theta}\left(y = \frac{-t}{2}\right) = \frac{-tk}{2}\ (contraction)$$

Note: If convex curvature is considered negative, then all the equations here should have minus signs on the right side

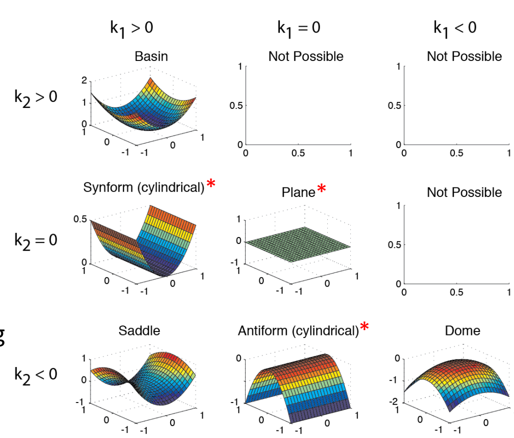
14. Folds

V Kinematics of folding (cont.)

E Layer-parallel normal strain for three-dimensional folds

1 Gauss' Theorem: If the *product* of the principal curvatures (i.e., the Gaussian curvature K = $k_1 k_2$) is constant, then a deformed *surface* remains unstrained*

2 For geologic folds, the Gaussian curvature invariably changes during folding, so layer-parallel strains will occur on the surfaces, as well as interiors, of folded layers



Curvature-based classification scheme for 3D folds

14. Folds

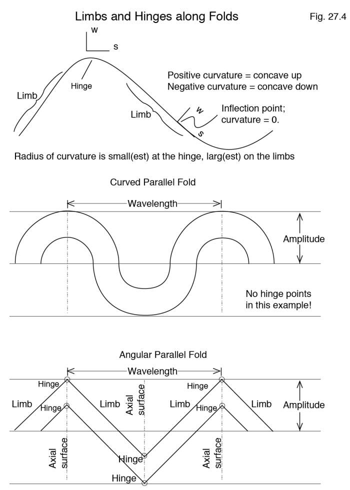
VI Fold terminology and classification

A ***Hinge point***: point of local maximum curvature.

B ***Hinge line***: connects hinge points along a given layer.

C ***Axial surface***: locus of hinge points in ***all*** the folded layers.

D ***Limb***: surface of low curvature.



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14. Folds

VI Fold terminology and classification (cont.)

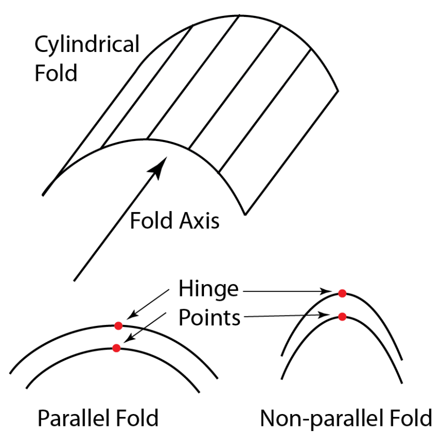
D ***Cylindrical fold***: a surface swept out by moving a straight line parallel to itself

1 **Fold axis**: line that can generate a cylindrical fold

2 Parallel fold: top and bottom of layers are parallel and layer thickness is preserved*

3 Non-parallel fold: top and bottom of layers are not parallel; layer thickness is not preserved*

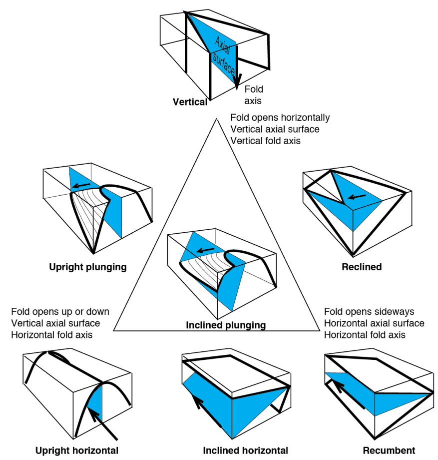
* Assumption: bottom and top of layer were originally parallel



11/27/13 GG303 20

14. Folds

VI Fold terminology and classification (cont.)
- E Fleuty's Classification
  - 1 Based on orientation of axial surface and fold axis
  - 2 First modifier (e.g., "upright") describes orientation of axial surface
  - 3 Second modifier (e.g., "horizontal") describes orientation of fold axis

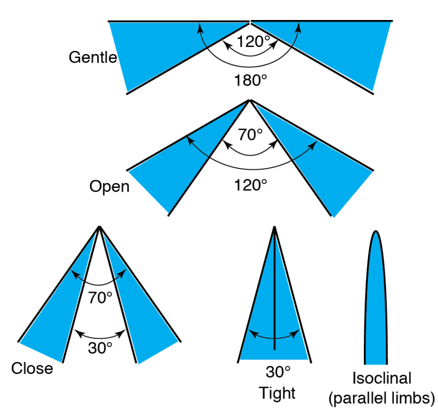


11/27/13 GG303 21

14. Folds

Fold terminology and classification (cont.)
- F Inter-limb angle

Interlimb angle	Classification
180° - 120°	Gentle
120° - 70°	Open
70° - 30°	Close
30° - 0°	Tight
"0°"	Isoclinal
Negative	Mushroom



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