1. ORIENTATIONS OF LINES AND PLANES IN SPACE

I Main Topics

A Definitions of points, lines, and planes

B Geologic methods for describing lines and planes

C Attitude symbols for geologic maps

D Reference frames

1. ORIENTATIONS OF LINES AND PLANES IN SPACE

II Definitions of points, lines, and planes

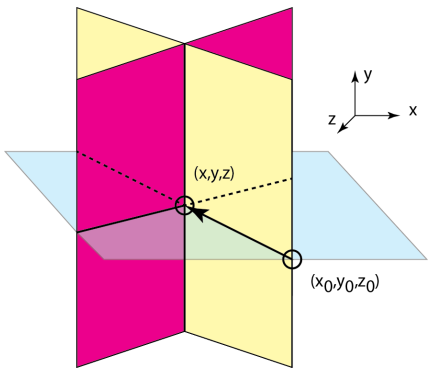
A Point

1 Defined by one set of coordinates (an ordered triple in 3-D)

2 Defined by distance and direction from a reference point

3 Intersection of two lines

4 Intersection of three planes



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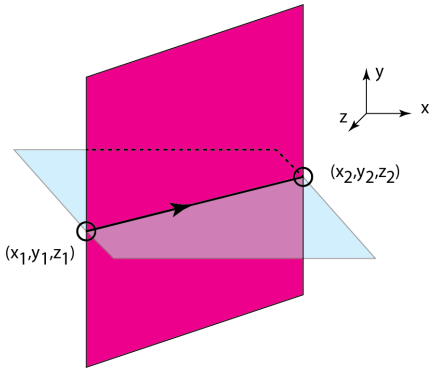
B Line

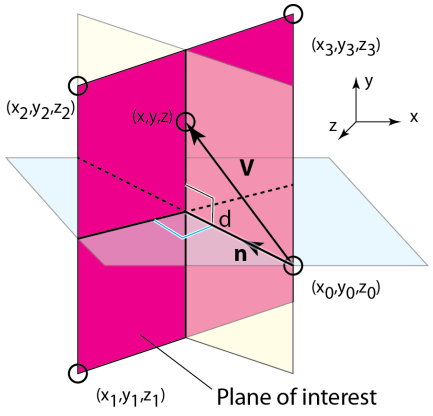
1 Defined by two sets of coordinates
2 Defined by two points
3 Defined by a direction from a reference point
4 Intersection of two planes



1. ORIENTATIONS OF LINES AND PLANES IN SPACE

C Plane

1 Defined by three sets of coordinates
2 Defined by three points
3 Defined by distance and direction from a reference point and the direction of the line
4 Defined by two intersecting or two parallel lines



1. ORIENTATIONS OF LINES AND PLANES IN SPACE

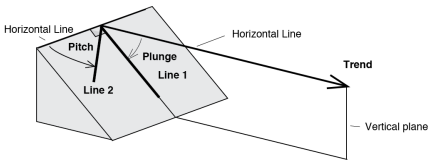
III Geologic methods for describing lines and planes
- A Orientation of a line
  - 1 Trend & plunge
    - a **Trend**: Direction (azimuth) of a vertical plane containing the line of interest.
      - i Azimuth (compass bearing): direction of a horizontal line contained in a vertical plane. Measured by quadrant or (°).

        Examples:

        90° or N90°E,

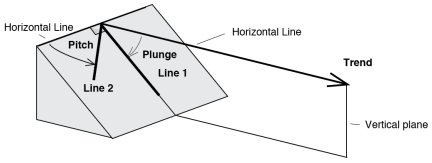
        270° or N90°W,

        270° or S90°W.
      - Ii The trend "points" in the direction a line plunges
    - b **Plunge**: The inclination of a line below the horizontal (e.g., 35°)



1. ORIENTATIONS OF LINES AND PLANES IN SPACE

III Geologic methods for describing lines and planes
- A Orientation of a line
  - 2 Pitch (or rake): the angle, measured in a plane of specified orientation, between one line and a horizontal line



1. ORIENTATIONS OF LINES AND PLANES IN SPACE

B **Orientation of a plane**

1 Orientation of two intersecting lines in the plane

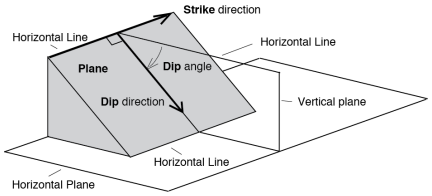
Strike & dip

a **Strike**: direction of the line of intersection between an inclined plane and a horizontal plane (e.g., a lake);

b **Dip**: inclination of a plane below the horizontal; 0°≤dip≤90°

c The azimuth directions of strike and dip are perpendicular

d Good idea to specify the direction of dip to eliminate ambiguity, but right hand rule can also be used.



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e Examples:

Strike 270°	Dip 45°N	right-handed
Strike N90°W	Dip 45°N	right-handed
Strike 270°W	Dip 45°S	left-handed: don't use!

f NOTE: **Trend and plunge refer to lines; strike and dip refer to planes**

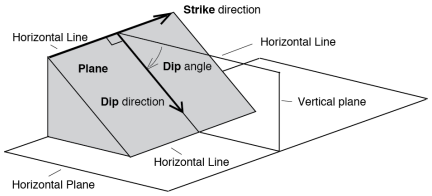
1. ORIENTATIONS OF LINES AND PLANES IN SPACE

2 Orientation of one special line in a plane

Dip & dip direction (azimuth of dip)

a Used mostly in Europe

b Water runs down the dip direction



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3 Trend & plunge of pole (unit normal) to plane

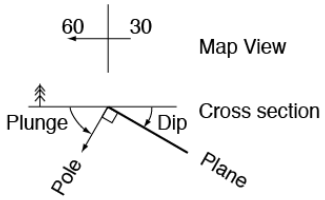
a Pole is a line traditionally taken to point down

b The pole trends 180° from the direction the plane dips;

c Pole trend = strike -90°

d Pole plunge + dip = 90°; Pole plunge = 90° - dip;

Attitude Symbol for a Plane and its Pole



A plane appears as a line if viewed in the direction of strike

Note: In the map-view diagram, the plane dips to the east, and its pole trends to the west

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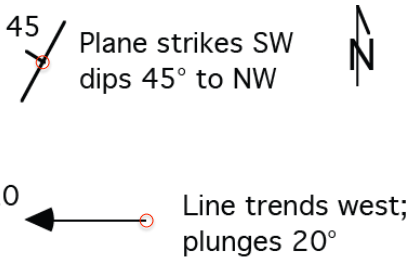
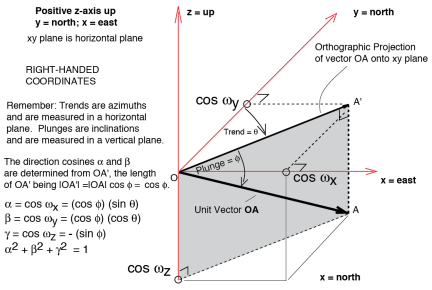
IV Attitude symbols (strike and dip of a plane; trend and plunge of a line)



The symbols should be plotted on a map with the circled part of the the symbols at the point where the measurements are made

1. ORIENTATIONS OF LINES AND PLANES IN SPACE

V Reference frames
- A <u>Cartesian coordinates</u>
  - 1 Points are described by their x, y, z coordinates
  - 2 The x,y, and z axes are right-handed and mutually perpendicular
  - 3 Direction of a line
    - a Given by the coordinates of pairs of points
    - b Given by the <u>difference</u> in coordinates of pairs of points
      - i $\Delta x = x_2 - x_1$
      - ii $\Delta y = y_2 - y_1$
      - iii $\Delta z = z_2 - z_1$
    - c Given by the angles ω_x, ω_y, and ω_z. These are the angles between a line of unit length and the x, y, and z axes, respectively.
    - d The respective cosines (α, β, γ) of these angles are called the direction cosines.



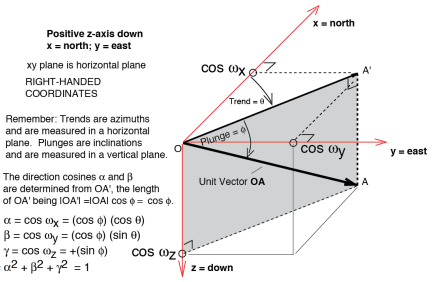
Angle	ω_x	ω_y	ω_z
Direction cosine	α	β	γ

1. ORIENTATIONS OF LINES AND PLANES IN SPACE

4 Length of a line segment (distance between two points)

$$L = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

A line of unit length has a length of one.



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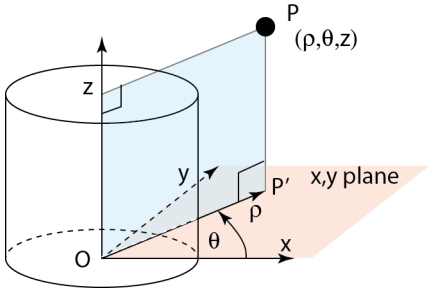
B Cylindrical coordinates (3-D version of polar coordinates)

1 Point P is described by its ρ, θ, z coordinates
   a ρ = distance from the origin to P', where P' is the projection of point P onto x,y plane
   b θ = angle between x-axis and OP'
   c z = distance from origin to projection of point P onto z-axis



1. ORIENTATIONS OF LINES AND PLANES IN SPACE

C Spherical coordinates

1 Points are described by their r, θ, ϕ coordinates

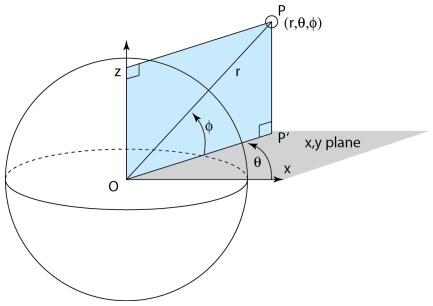
a r = distance from the origin <u>to point P</u>

<u>Note</u>: r here is different than ρ for cylindrical coordinates

b θ = angle between x-axis and the OP', where P' is the <u>projection</u> of point P onto x,y plane

c ϕ = angle between OP' and OP

Note: In some spherical schemes, the angle between OP and the z-axis is used as the second angle.

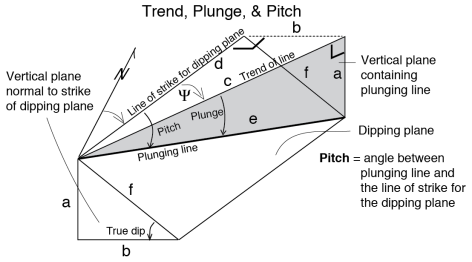


1. ORIENTATIONS OF LINES AND PLANES IN SPACE

Cartesian (x,y,z) ← Spherical (r,θ,ϕ) $x = r\cos\phi\cos\theta$ $\alpha = \cos\varphi\cos\theta$ $y = r\cos\phi\sin\theta$ $\beta = \cos\varphi\cos\theta$ $z = r\sin\theta$ $\gamma = \sin\theta$	Spherical (r,θ,ϕ) ← Cartesian (x,y,z) $r = (x^2 + y^2 + z^2)^{1/2}$ $1 = (\alpha^2 + \beta^2 + \gamma^2)^{1/2}$ $\theta = \tan^{-1}(y/x)$ $\theta = \tan^{-1}(\beta/\alpha)$ $\phi = \tan^{-1}[z/(y^2 + z^2)^{1/2}]$ $\phi = \sin^{-1}(\gamma)$
Cartesian (x,y,z) ← Cylindrical (ρ,θ,z) $x = \rho\cos\theta$ $y = \rho\sin\theta$ $z = z$	Cylindrical (ρ,θ,z) ← Cartesian (x,y,z) $\rho = (x^2 + y^2)^{1/2}$ $\theta = \tan^{-1}(y/x)$ $z = z$
Cylindrical (ρ,θ,z) ← Spherical (r, θ,ϕ) $\rho = r\cos\theta$ $\theta = \theta$ $z = r\sin\theta$	Spherical (r, θ,ϕ) ← Cylindrical (ρ,θ,z) $r = (\rho^2 + z^2)^{1/2}$ $\theta = \theta$ $\phi = \tan^{-1}(z/\rho)$

Note: some references define the angles in cylindrical and spherical coordinates differently. Exercise care, and check to ensure that the equations are consistent with the graphics. The definitions here are consistent with Matlab.

1. ORIENTATIONS OF LINES AND PLANES IN SPACE



	plunge (ϕ)	pitch (Ω)	dip (δ)	Ψ
sin	a/e	f/e	a/f	b/c
cos	c/e	d/e	b/f	d/c
tan	a/c	f/d	a/b	b/d

(1) Trend = θ = strike + Ψ = strike + $\cos^{-1}$ (d/c) = strike + $\cos^{-1}${(cos Ω)/(cos ϕ)}

(2) Trend = θ = strike + Ψ = strike + $\tan^{-1}$ (b/d) = strike + $\tan^{-1}${(cos δ)(tanΩ)}

(3) Plunge = ϕ = $\sin^{-1}$ (a/e) = $\sin^{-1}$ {(sin δ)(sinΩ)}

(4) Pitch = Ω = $\sin^{-1}$ (f/e) = $\sin^{-1}$ {(sinϕ) / (sin δ)}

1. ORIENTATIONS OF LINES AND PLANES IN SPACE

Geologic Conventions for Measuring Orientations

Compass Bearings

By 360° azimuth (0°- 360°)

By quadrant (relative to north or south). The angle does not exceed 90°

Examples

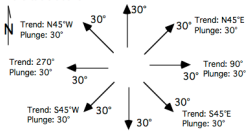
0°	45°	90°	135°	180°	225°	270°	315°
N0°E	N45°E	N90°E	S45°E	S0°E	S45°W	S90°W	N45°W

Lines

Trend: A compass bearing

Plunge: An inclination below horizontal

Examples: The lines below all plunge at 30°. Their trends vary according to the table above

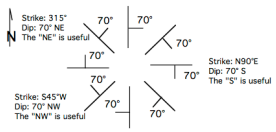


Planes

Strike: A compass bearing

Dip: An inclination below horizontal

Examples: The planes below all dip at 70°. Their strikes vary according to the table above



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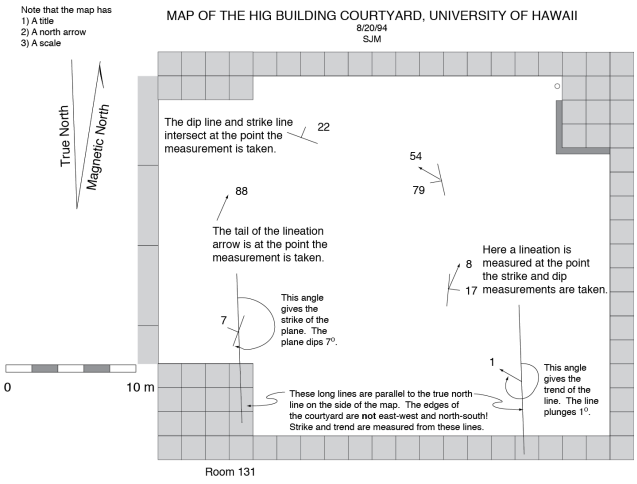

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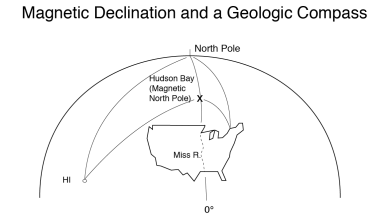

When the declination is set correctly,
if the compass body points to true north,
then the magnetic needle of the compass points to magnetic north
and the compass reads "0"