Auxiliary Material for paper 2010GL042252

Transient and persistent shoreline position change from a storm

Tiffany R. Anderson, L. Neil Frazer, and Charles H. Fletcher

Department of Geology and Geophysics, School of Ocean and Earth Science and Technology, University of Hawaii at Manoa, Honolulu, HI 96822.

A. Model Probabilities

The posterior probability of a model is proportional to the product of its likelihood and prior probability. As we give all models with non-zero prior the same prior probability, the model with the largest likelihood has the largest posterior probability; this is the model with the lowest IC score.

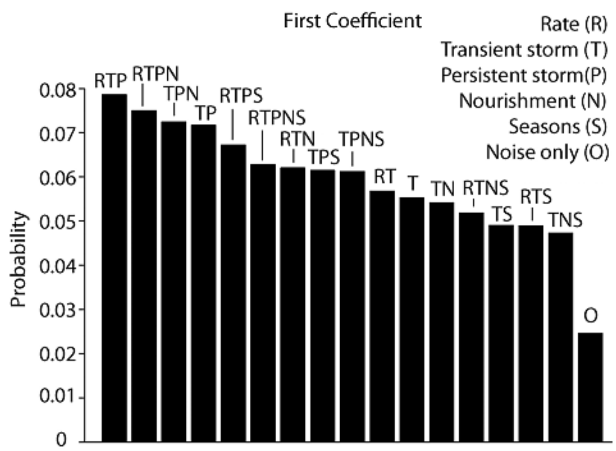


Figure A1. Posterior probabilities of models for the temporal coefficient of mode 1.

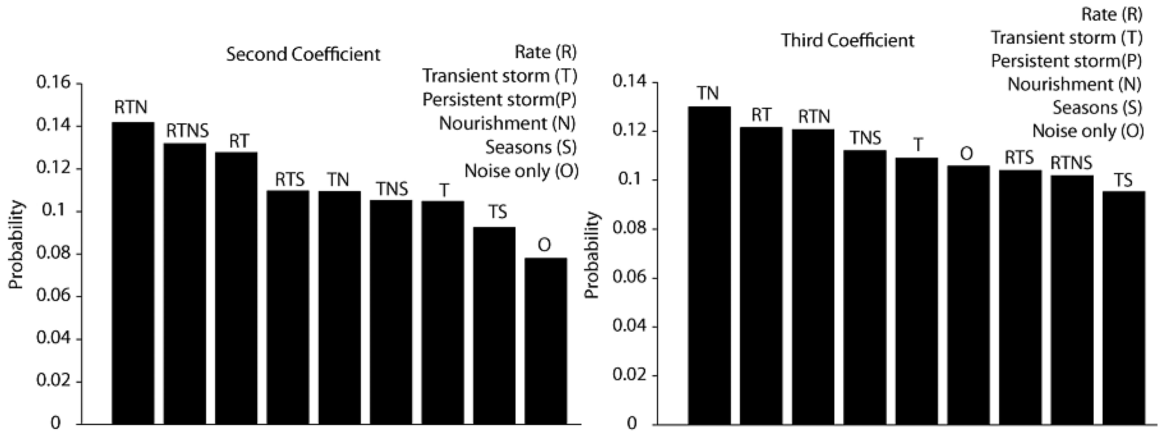


Figure A2. Posterior probabilities of models for the temporal coefficient of modes 2 and mode 3. Models assigned zero prior probability (see main text) are not displayed.

B. Temporal Coefficients of Modes 2 and 3.

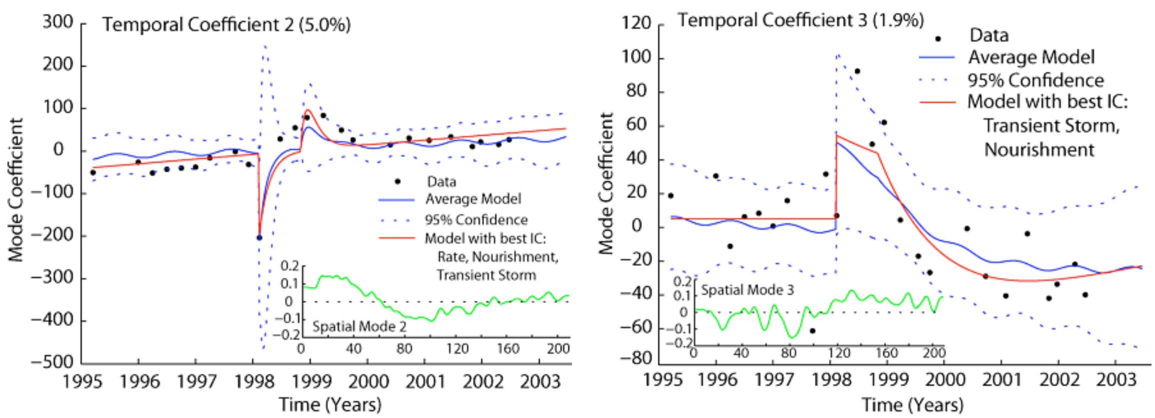


Figure B1. Left panel: mode 2. Right panel: mode 3. The red lines are the models with the lowest IC score (largest likelihood, largest posterior probability). The blue lines are the probability-weighted average models.

C. Modal Contributions

Table C1. Percent data variance contribution of each mode and cumulative percent.

Key: R-rate, T-transient storm, P-persistent storm, N-nourishment, S-seasons.

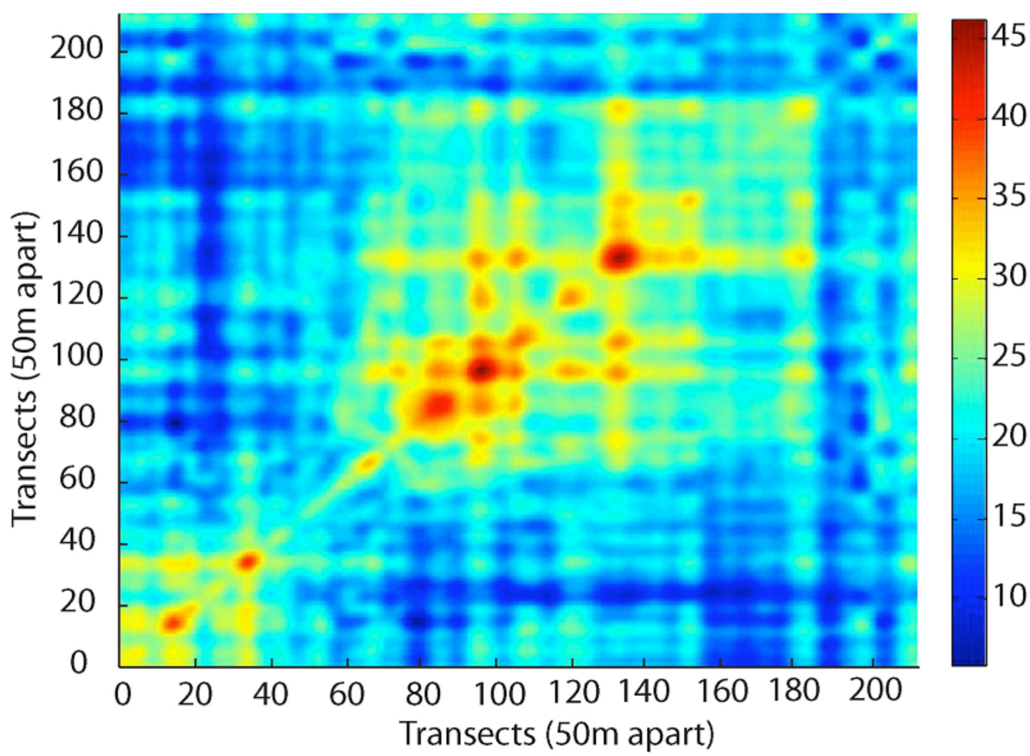
Mode	Data variance Contribution (%)	Cumulative Contribution (%)	Best-fit model
1	90.67	90.67	R,T,P
2	5.01	95.68	R,T,N
3	1.92	97.61	T,N
4	0.69	98.30	Noise
5	0.45	98.75	R,T,N
6	0.26	99.01	R,T,N
7	0.21	99.23	Noise
8	0.15	99.38	Noise
9	0.11	99.49	Noise
10	0.09	99.58	Noise
11	0.09	99.67	Noise
12	0.07	99.74	Noise
13	0.05	99.79	Noise
14	0.04	99.83	Noise
15	0.04	99.87	Noise
16	0.03	99.90	Noise
17	0.07	99.93	Noise
18	0.02	99.95	Noise
19	0.02	99.97	Noise
20	0.01	99.98	Noise
21	0.01	99.99	Noise
22	0.01	100	Noise
23	0	100	Noise
24	0	100	Noise

D. Prediction Error

Figure D1 shows the time averaged prediction error covariance matrix

$$\frac{1}{24}\sum_{k=1}^{24}\left[y_p(x_i,t_k)-y(x_i,t_k)\right]\left[y_p(x_j,t_k)-y(x_j,t_k)\right],$$

in which y is the data and y_p is the prediction from modeling. If our model for noise and signal corresponded exactly to reality, this matrix would be diagonal (a bright line along the diagonal of the figure with darkness off the diagonal), meaning that the prediction error at any alongshore location is uncorrelated with the error at any other alongshore location.



E. Error Estimation

In the Auxiliary Material for Frazer et al. [2009b] (paper 1) there is a Section D, entitled "Variance of a model-averaged estimator," in which we derived an expression (equation D9) for the variance of a model-averaged estimate. Briefly, for any quantity of interest ϕ, we calculated the variance, $\sigma^2_{\hat{\phi}}$, of the model-averaged estimate $\hat{\phi} = \sum_j p_j \phi_j$ in which ϕ_j is the estimate from the j^{th} model, and p_j is the posterior probability of the j^{th} model. (The posterior probability of a model is obtained by combining the prior probability of that model with the model likelihood computed from the data.). We calculated the variance $\sigma^2_{\hat{\phi}}$ by first calculating the variation $\delta\hat{\phi}$ with respect to a variation of the data δy, not neglecting the variation δp_j. The standard error indicated by the inner ticks on Figure 2(h) of this paper is the square root of that variance. That method of estimating errors was used in this paper and in paper 1 because it is analogous to the standard method of estimating error for any particular model and is thus comparable to most error estimates in the literature.

In this paper we also calculate model selection error [Buckland et al. 1997], which tends to be larger and is thus more conservative. We derive model selection error as follows by using probability density functions (pdf). Let $p(\phi \mid y)$ be the pdf of ϕ given data y. Then we may write

$$p(\phi \mid y) = \sum_i p(\phi \mid M_i, y) p_i \tag{E1}$$

in which $p(\phi \mid M_i, y)$ is the pdf of ϕ conditioned on M_i being the correct model. The expected value of ϕ for model M_i is thus

$$\phi_i = \int \phi \, p(\phi \mid M_i, y) d\phi \tag{E2}$$

and its variance, needed below, is

$$
\begin{aligned}
\sigma_{\phi_i}^2 &= \int (\phi - \phi_i)^2 p(\phi \mid M_i, y) d\phi \\
&= \int (\phi^2 - 2\phi\phi_i + \phi_i^2) p(\phi \mid M_i, y) d\phi \\
&= \left\langle \phi^2 \right\rangle_i - \phi_i^2.
\end{aligned} \tag{E3}
$$

The model-averaged estimate of ϕ is

$$
\begin{aligned}
\widehat{\phi} &= \int \phi p(\phi \mid y) d\phi \\
&= \int \phi \sum_i p(\phi \mid M_i, y) p_i d\phi \\
&= \sum_i p_i \int \phi p(\phi \mid M_i, y) d\phi \\
&= \sum_i p_i \, \phi_i
\end{aligned} \tag{E4}
$$

and the variance that includes model selection error is

$$
\begin{aligned}
\sigma_{\widehat{\phi}}^2 &= \int (\phi - \widehat{\phi})^2 p(\phi \mid y) d\phi \\
&= \int (\phi - \widehat{\phi})^2 \sum_i p(\phi \mid M_i, y) p_i d\phi \\
&= \sum_i p_i \int (\phi - \widehat{\phi})^2 p(\phi \mid M_i, y) d\phi \\
&= \sum_i p_i \int (\phi^2 - 2\phi\widehat{\phi} + \widehat{\phi}^2) p(\phi \mid M_i, y) d\phi \\
&= \sum_i p_i \left\{ \left\langle \phi^2 \right\rangle_i - 2\phi_i\widehat{\phi} + \widehat{\phi}^2 \right\} \\
&= \sum_i p_i \left\langle \phi^2 \right\rangle_i - 2\widehat{\phi} \sum_i p_i \phi_i + \widehat{\phi}^2 \sum_i p_i \\
&= \sum_i p_i \left\langle \phi^2 \right\rangle_i - 2\widehat{\phi}^2 + \widehat{\phi}^2 \\
&= \sum_i p_i \left\langle \phi^2 \right\rangle_i - \widehat{\phi}^2 \\
&= \sum_i p_i \sigma_{\phi_i}^2 + \sum_i p_i \phi_i^2 - \widehat{\phi}^2.
\end{aligned} \tag{E5}
$$

The square root of this variance is indicated by the outer ticks on the error bars of Figure 2(h) of the main text.

F. References for Auxiliary Material

Buckland, S.T., K.P. Burnham, and N.H. Augustin (1997), Model selection: an integral part of inference, *Biometrics, 53,* 603–618.

Frazer, L.N., T. R. Anderson, and C.H. Fletcher (2009b), Modeling storms improves estimates of shoreline change, *Geophys. Res. Lett., 36*, L20404, doi:10.1029/2009GL40061.